

Téléphone : +41 21 693 24 27
Fax : +41 21 693 28 68
E-mail : dimitrios.lignos@epfl.ch
Site web : <http://resslab.epfl.ch>
Address: EPFL ENAC IIC RESSLAB
GC B3 485, Station 18,
CH-1015, Lausanne

In-class Exercise – Week #9: Displacement-based elements

The rectangular cross section shown in Figure 1 is made of a material that is described with the constitutive law according to Equation 1. The cross section is subjected to the strain distribution shown in Figure 1.

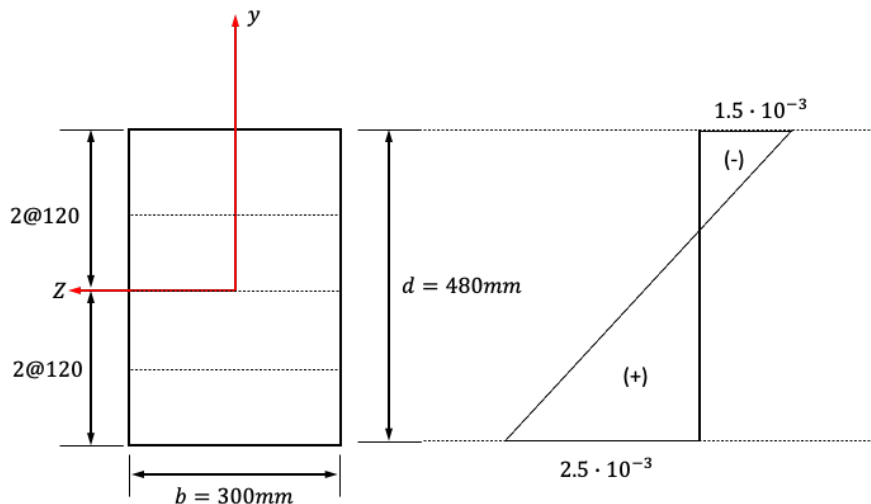


Figure 1. Cross section and strain distribution

Stress-strain constitutive law,

$$\sigma(\varepsilon) = \text{signum}(\varepsilon) \cdot f_y \cdot \left(1 - e^{-0.5 \frac{\text{signum}(\varepsilon) \cdot \varepsilon}{\varepsilon_y}} \right) \quad (1)$$

Where,

- $\text{signum}(\varepsilon)$ provides the sign depending on the sign of the longitudinal strain.
- Yield strength, $f_y = 275 \text{MPa}$
- Reference modulus, $E = 135 \text{GPa}$

Answer the following questions:

1. Compute the section forces
2. Compute the tangent stiffness matrix of the cross section

Solution:

First, the yield strain ε_y should be computed:

$$\varepsilon_y = \frac{f_y}{E} = 0.002 \quad (1.2)$$

The material tangent modulus corresponds to the derivative of stress with respect to strain is given by:

$$E_t = \frac{d\sigma}{d\varepsilon} = \frac{0.5f_y}{\varepsilon_y} \cdot e^{-0.5 \cdot \frac{\text{signum}(\varepsilon) \cdot \varepsilon}{\varepsilon_y}} \quad (1.3)$$

The given strain distribution consists of an axial strain ε_a at the neutral axis ($y=0$) and a curvature κ .

The total strain ε is obtained as follows:

$$\varepsilon(y) = \varepsilon_a + y \cdot \kappa \quad (1.4)$$

Using the strain values at the section extremities, the values $\varepsilon_a = 0.5 \cdot 10^{-3}$ and $\kappa = -\frac{1}{120} \cdot 10^{-3}$ are obtained.

Determine the material state (stress and tangent modulus) at each fiber

In order to determine the stress and tangent modulus at each fiber, the strain ε at the fiber centroid should be determined using Equation 1.4. The obtained fiber strain can then be inputted into the material stress-strain constitutive law in order to determine the stress (using Equation 1.1) and the tangent modulus (using Equation 1.3).

The following results are obtained:

Point	Location	Axial strain $\varepsilon_k = \varepsilon_a + y_k \cdot \kappa$	Axial stress $\sigma_k = \sigma(\varepsilon_k) [MPa]$	Tangent Modulus $E_k = E(\varepsilon_k) [GPa]$
$k = 1$	$y_1 = 180$	$\varepsilon_1 = -1.0 \cdot 10^{-3}$	$\sigma_1 = -59.9$	$E_1 = 52.8$
$k = 2$	$y_2 = 60$	$\varepsilon_2 = 0$	$\sigma_2 = 0$	$E_2 = 67.5$
$k = 3$	$y_3 = -60$	$\varepsilon_3 = 1.0 \cdot 10^{-3}$	$\sigma_3 = 59.9$	$E_3 = 52.8$
$k = 4$	$y_4 = -180$	$\varepsilon_4 = 2.0 \cdot 10^{-3}$	$\sigma_4 = 106.7$	$E_4 = 40.9$

Determination of section forces and tangent stiffness by integration

The fiber stresses and tangent modulus are integrated throughout the cross-section to determine the section forces \mathbf{s} and tangent stiffness \mathbf{k}_s :

$$\mathbf{s} = \int_A \begin{pmatrix} 1 \\ -y \end{pmatrix} \sigma(y) dA \quad (1.5)$$

$$\frac{\partial \mathbf{s}}{\partial \mathbf{e}} = \mathbf{k}_s(x) = \int_A \frac{d\sigma(y)}{d\varepsilon} \begin{bmatrix} 1 & -y \\ -y & y^2 \end{bmatrix} dA \quad (1.6)$$

The integrals can be evaluated for different fibers by using the midpoint integration rule:

$$\mathbf{s} = \sum_k \begin{pmatrix} 1 \\ -y_k \end{pmatrix} \sigma_k A_k \quad (1.7)$$

$$\mathbf{k}_s(x) = \sum_k E_k \begin{bmatrix} 1 & -y_k \\ -y_k & y_k^2 \end{bmatrix} A_k \quad (1.8)$$

Where E_k is the tangent material modulus and A_k is the fiber area.

Equations 1.7 and 1.8 are evaluated using the proposed fiber discretization as follows:

$$\mathbf{s} = \sum_{k=1}^4 \begin{pmatrix} 1 \\ -y_k \end{pmatrix} \sigma_k A_k = \begin{pmatrix} 3.84 \\ 1208 \end{pmatrix} 10^6 \quad (1.9)$$

$$\mathbf{k}_s(x) = \sum_k E_k \begin{bmatrix} 1 & -y_k \\ -y_k & y_k^2 \end{bmatrix} A_k = \begin{bmatrix} 7.70 & -109.2 \\ -109.2 & 124910 \end{bmatrix} 10^9 \quad (1.10)$$