



# CIVIL 449: Nonlinear Analysis of Structures

School of Architecture, Civil & Environmental Engineering  
Civil Engineering Institute

## Introduction

Prof. Dr. Dimitrios Lignos  
EPFL, ENAC, IIC, RESSLab

Dr. Savvas Saloustros  
EPFL, ENAC, IIC, EESD

# EPFL Objectives of today's lecture

To introduce:

- Course personnel
- Course logistics
- Learning objectives
- Nonlinear analysis of structures

## EPFL Course personnel

### Instructors:

- Prof. Dr. Dimitrios G. Lignos (GC B3 485)
- Dr. Savvas Saloustros (GC B2 485)

### Teaching Assistants:

- Mr. Lucas Kuhn (GC B3 475, [lucas.kuhn@epfl.ch](mailto:lucas.kuhn@epfl.ch) )
- Mr. Hnat Lesiv (GC B2 484, [hnat.lesiv@epfl.ch](mailto:hnat.lesiv@epfl.ch) )

## EPFL Assessment methods

### Graded Assignments (40% of the total grade)

- Four written assignments
- You are allowed to work in groups of 2

### Final Examination (60% of the total grade)

- Covers the entire course material
- Location & time to be announced later this year
- Closed notes:
  - standard calculator and 2 sheets, i.e., **4 pages of your own notes**

## EPFL Resources and other information

- Lecture notes will be distributed through *Moodle* the week before the lecture.
- Other reading material will be provided through *Moodle*.

## EPFL Lectures & announcements

- Lectures: ***Mondays: 14h15 to 16h00, Wednesdays 9h15 to 10h00***
- Exercise: ***Wednesdays: 10h15 to 12h00***
  
- All announcements will be made through *Moodle*

## EPFL In-Class exercise session

- Exercises will be posted ahead of time together with lecture notes
- An online forum has been posted in *Moodle* per exercise for Q & A
- Detailed solutions will be posted in *Moodle* within a week after the exercise session

## **EPFL** What is this course about?

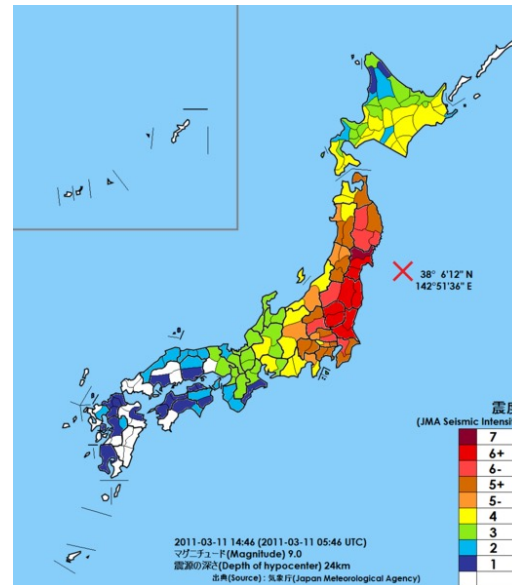
- Behavior of linear versus nonlinear structures
- Idealization of structures and their elements for nonlinear analysis
- Constitutive modeling of construction materials
  - Plasticity
  - Continuum damage mechanics
- Geometric transformations (geometric nonlinearities)
- Solution techniques for nonlinear problems
- Zero length and beam-column elements
- Integration techniques for nonlinear elements
- Mesh sensitivity
- Nonlinear static versus dynamic analysis
- Applications

# EPFL Motivation: Large earthquakes in recent years...

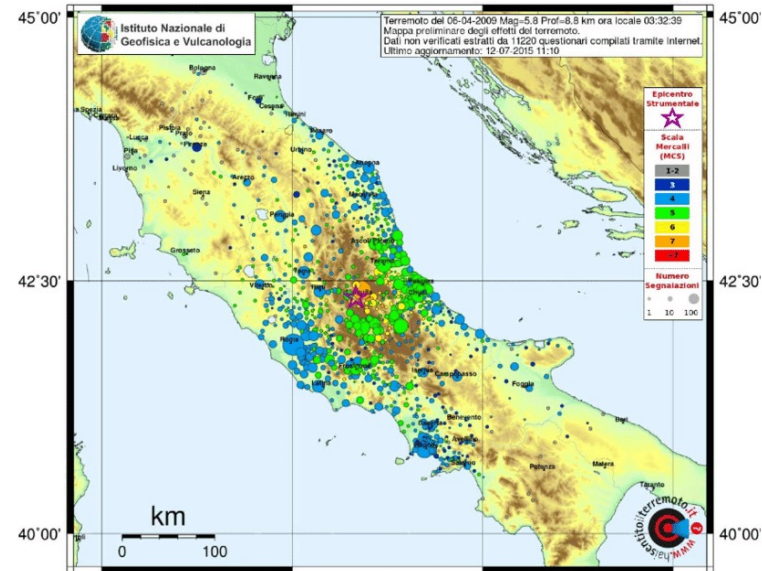
Demonstrated that the magnitude of earthquake forces applied to buildings were (or can be) significantly larger than the corresponding design earthquake loads based on regional seismic design.



Chile 2010



Japan 2011



Italy 2009

Images Source: EERI Reconnaissance Reports

# EPFL Collapse risk during large earthquakes



Hyogoken-Nanbu 1995



Taiwan 1999



Loma Prieta 1989



Northridge 1994

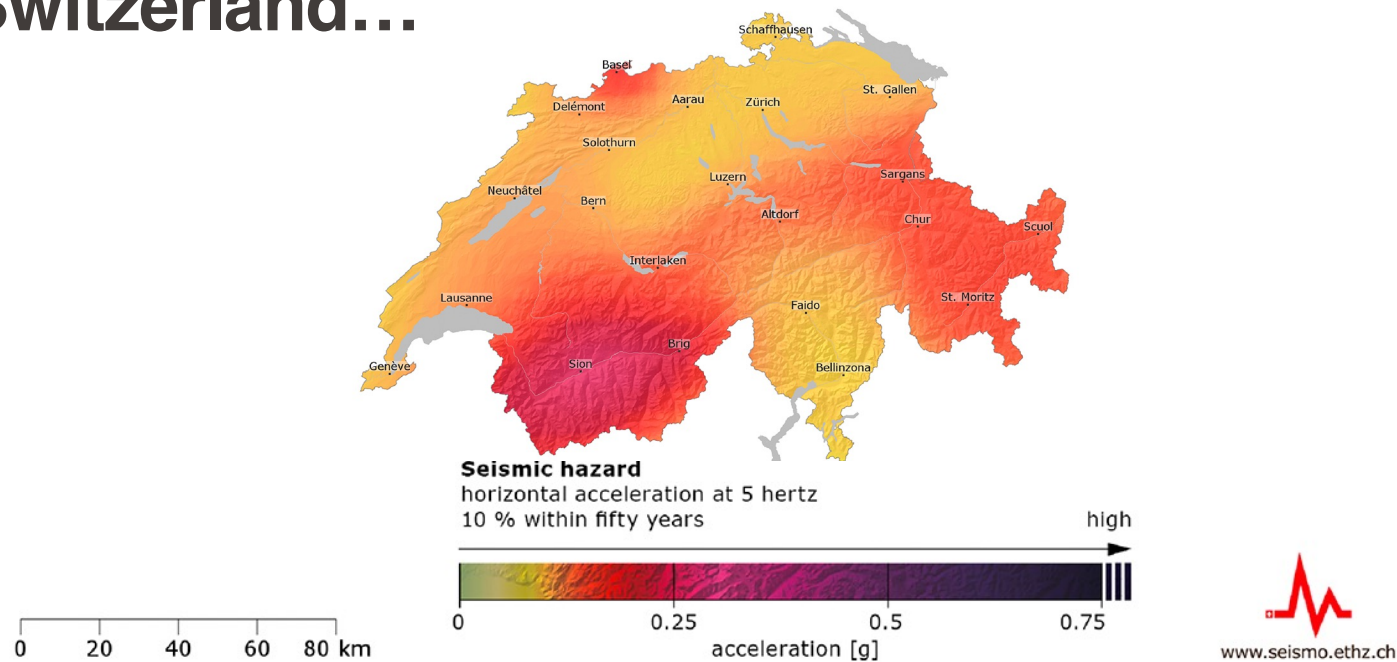


Hyogoken-Nanbu 1995



Tohoku 2011

*Images Source: NISEE, E-Library*



- Seismic risk: ranked 3<sup>rd</sup> after pandemics & electric shortages
- Scenario  $M_w = 6.5$  earthquake
  - collapse risk is high
  - Up to 6000 casualties, over 60000 injured
  - Expected financial losses: CHF50 to 100Billion

Sources: Fäh et al. (2000), Federal Office for Civil Protection (2020)

# EPFL Nonlinear behavior of structures

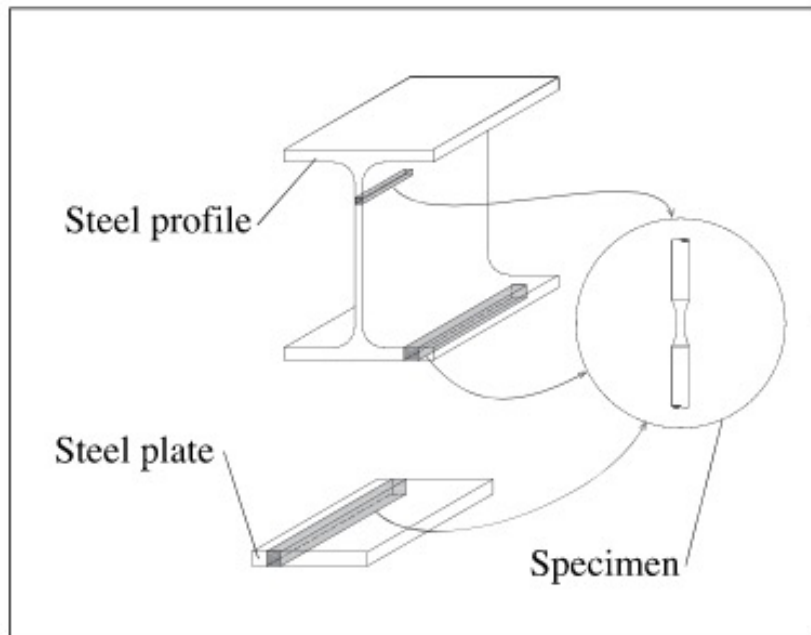


## Coupling of nonlinear phenomena

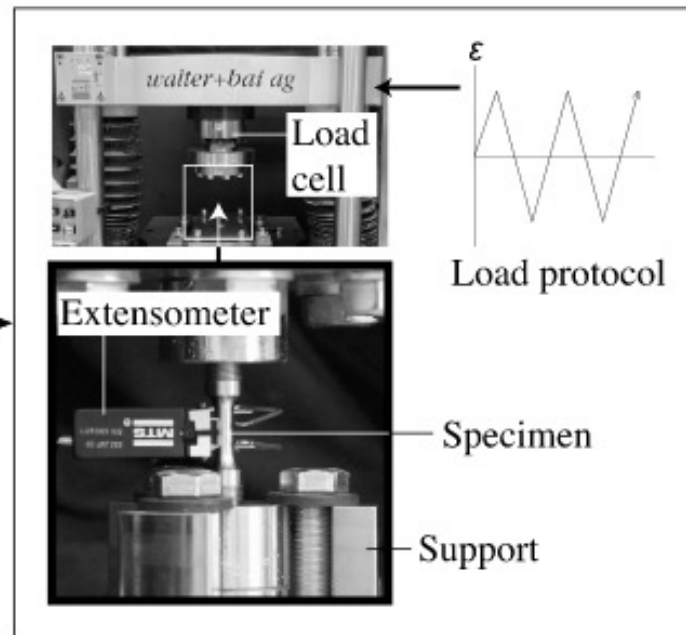
- Material nonlinearity (inelastic straining)
- Nonlinear geometric instabilities (large deformations)

# EPFL Material nonlinearity

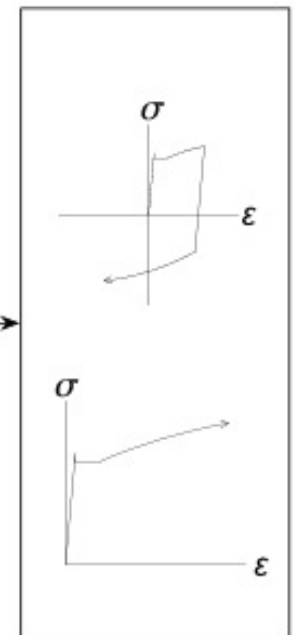
## Coupon Extraction and Manufacturing



## Testing



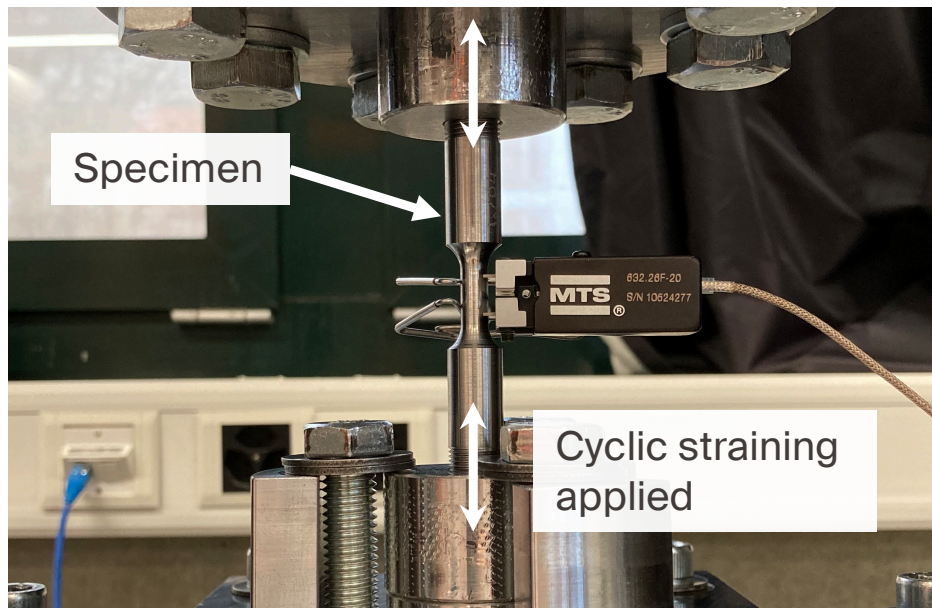
## Post-Processing



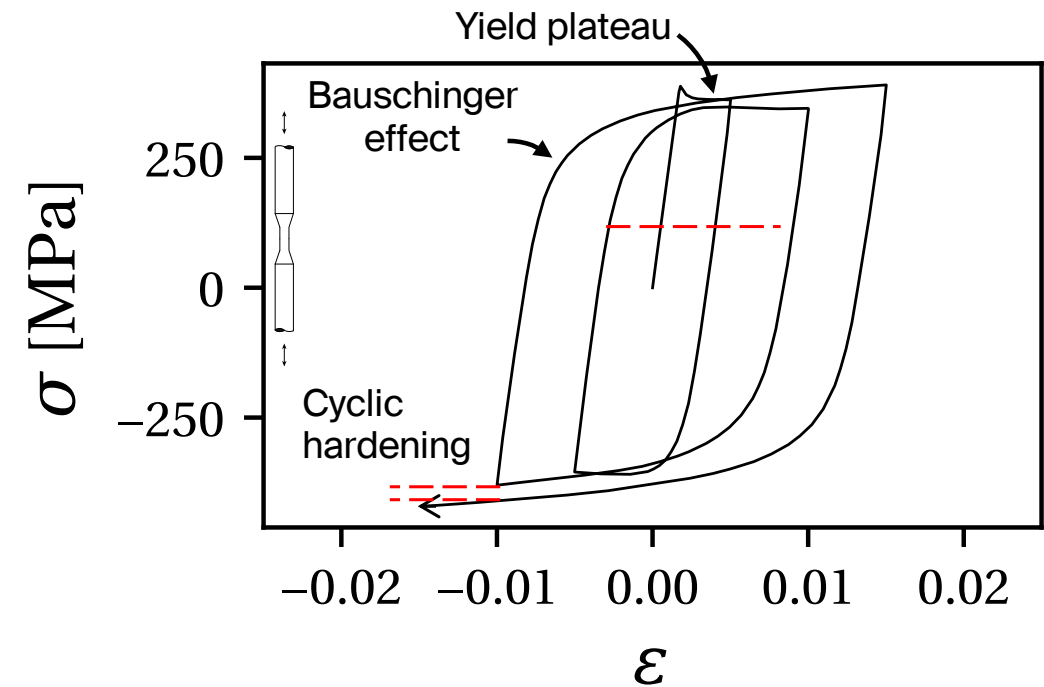
Source: Hartloper et al. (2023)

# EPFL Material nonlinearity

## Material testing



## Resulting stress-strain response



Source: Hartloper, de Castro e Sousa and Lignos (2020)

# Geometric nonlinearities



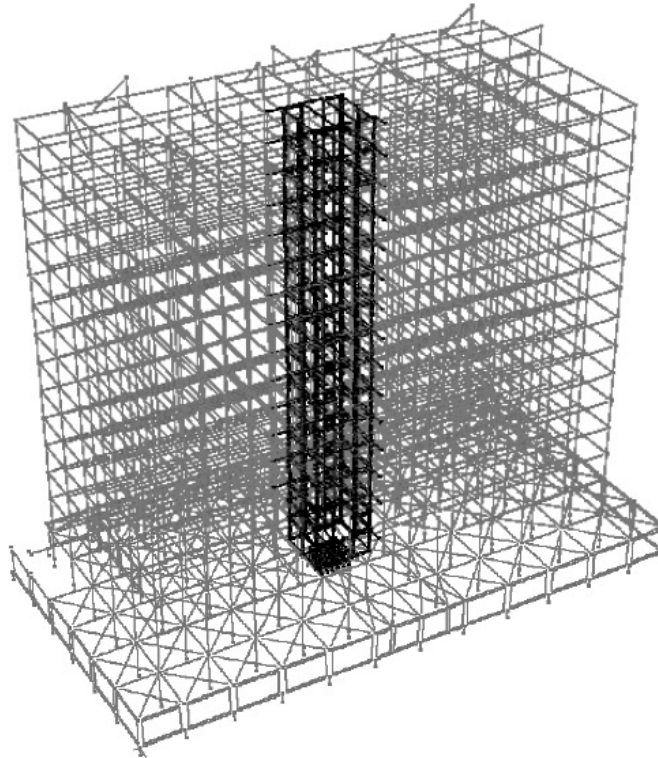
# Geometric nonlinearities



# EPFL Seismic interventions of existing buildings

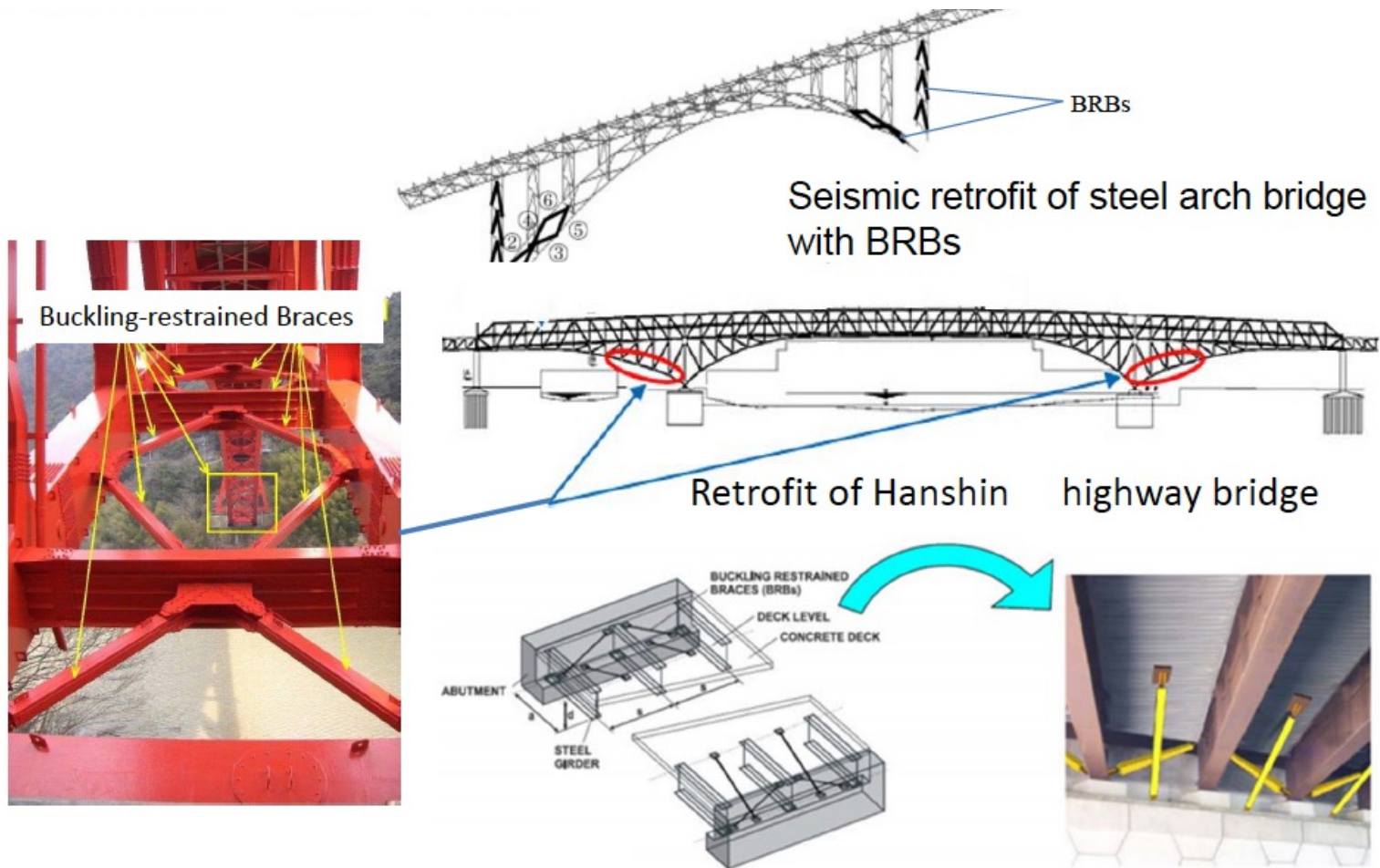


680 Folsom Street, San Francisco, USA



Source: Janhunen et al. (2013)

# Seismic interventions of existing bridges



Source: Celik and Bruneau (2011)

# EPFL Why nonlinear analysis of structures

## -Performance-based design



### Seismic Performance Assessment of Buildings

Volume 1 – Methodology

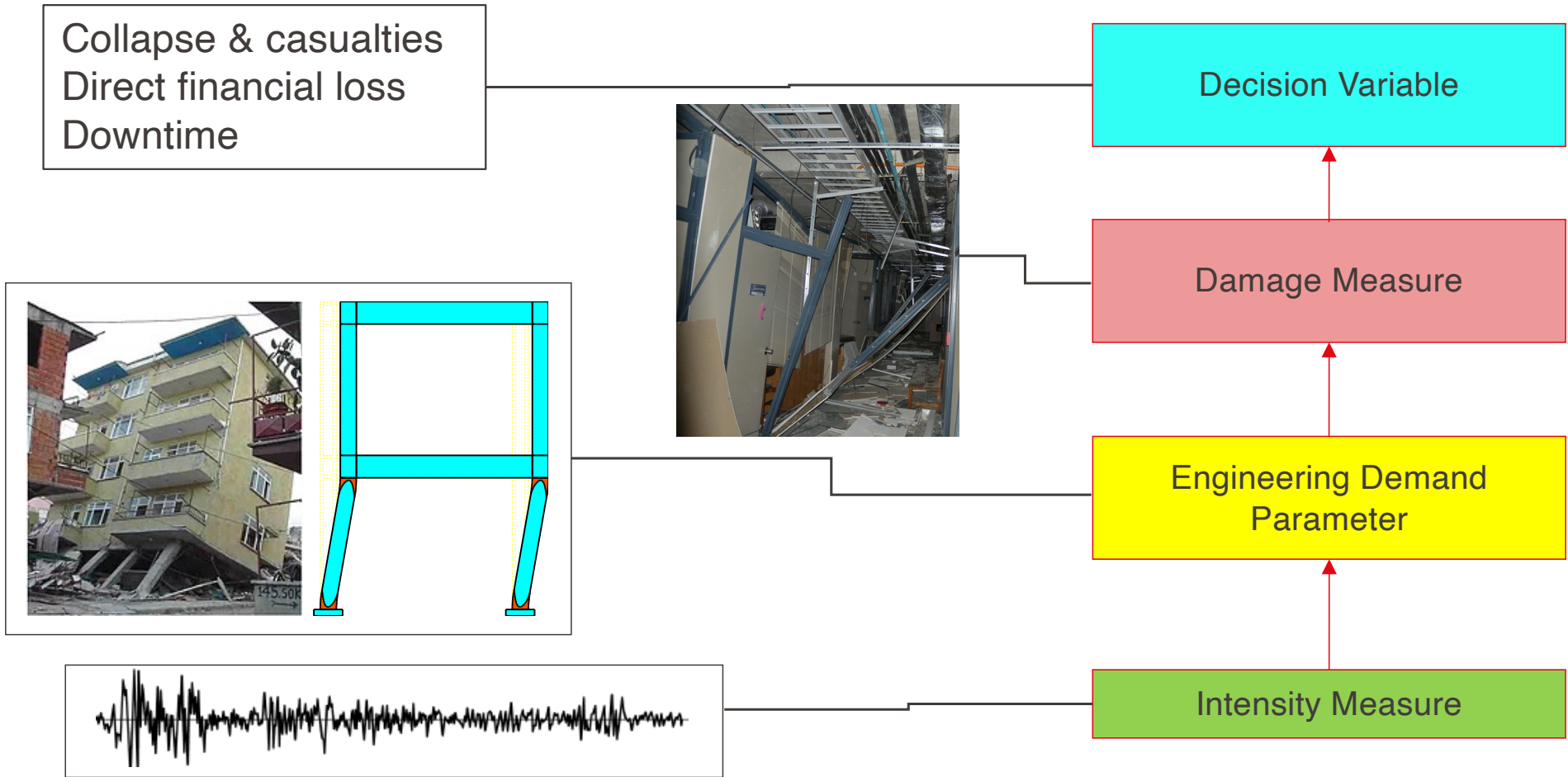
FEMA P-58-1 / September 2012



### Performance levels

- **Operational:** Building suitable for normal intended occupancy.
- **Immediate Occupancy:** Building may be used for intended purpose, albeit in an impaired mode.
- **Life Safety:** Occupancy may be prevented until repairs can be instituted.
- **Collapse Prevention:** Little margin against collapse.

# Performance-based earthquake engineering



Source: Prof. Dr. D.Lignos, CIVIL 714 (PBEE), EPFL

# Performance-based earthquake engineering

$$\lambda(DV) = \int_{\text{all IMs}} \int_{\text{all EDPs}} \int_{\text{all DMs}} G(DV|DM) dG(DM|EDP) dG(EDP|IM) d\lambda(IM)$$

**Impact**

**Performance (Loss) Models and Simulation**

**Hazard**

IM: Intensity Measure

EDP: Engineering Demand Parameter

DM: Damage Variable

DV: Decision Variable

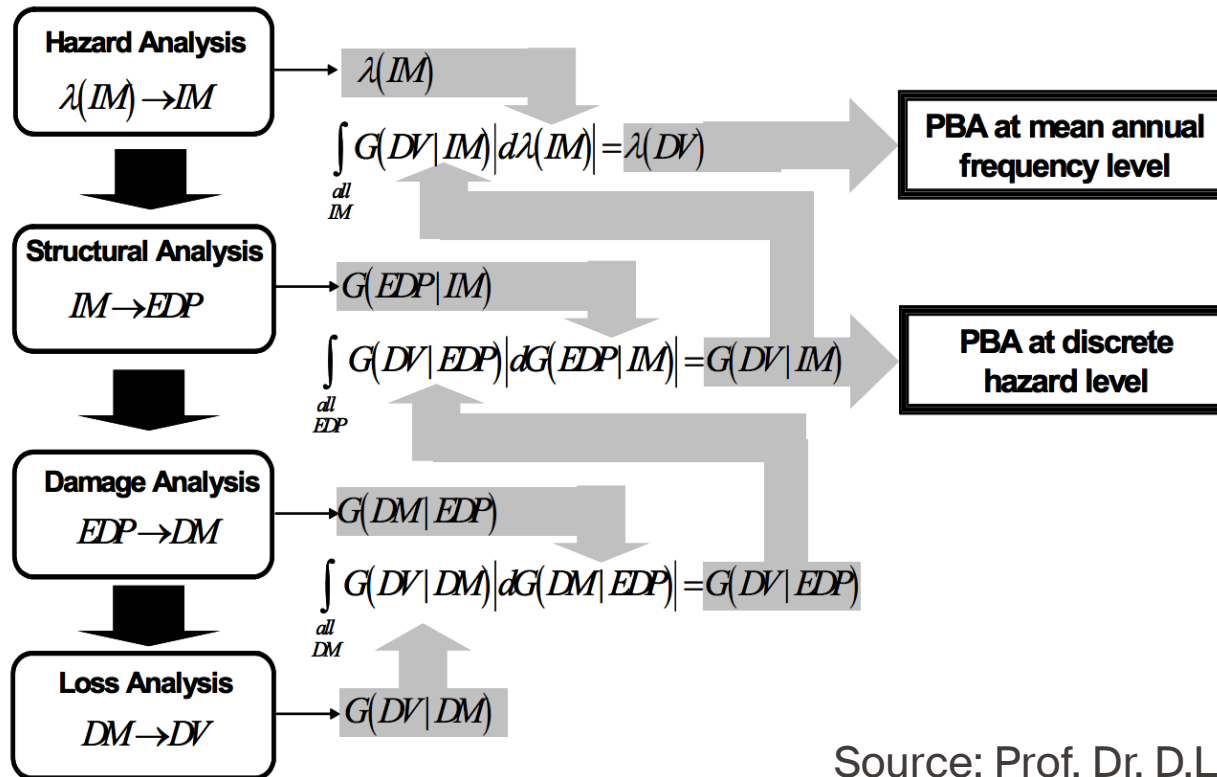
$\lambda(DV)$  : Probabilistic description of decision variable

(e.g., Mean Annual Frequency of Collapse,  $\lambda_c$  over a structure's life-expectancy)

Source: Prof. Dr. D.Lignos, CIVIL 714 (PBEE), EPFL

# Performance-based earthquake engineering

$$\lambda(DV) = \int_{\text{all } IMs} \int_{\text{all } EDPs} \int_{\text{all } DMs} G(DV|DM) dG(DM|EDP) dG(EDP|IM) d\lambda(IM)$$



Source: Prof. Dr. D.Lignos, CIVIL 714 (PBEE), EPFL

# EPFL New lateral load-resisting systems



- ✧ For implementation
  - ✧ q-factor (R-) quantification
  - ✧ System Overstrength ( $\Omega$ ) quantification

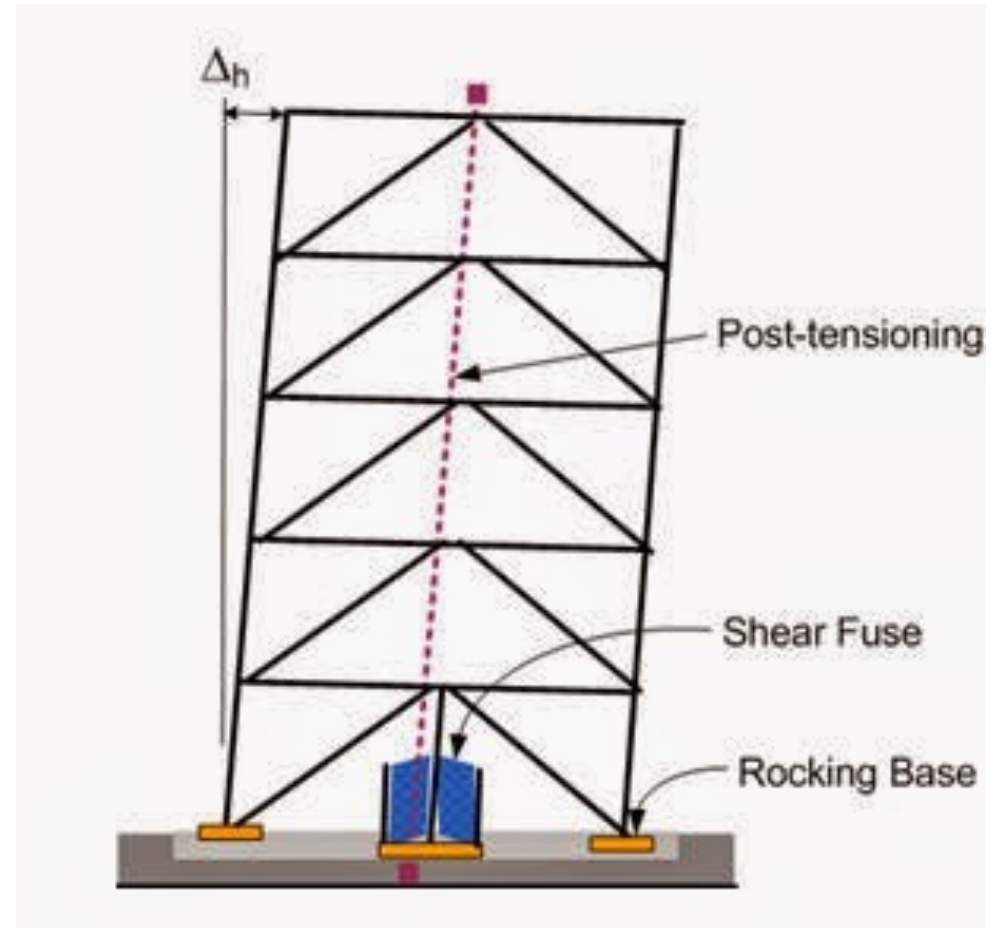


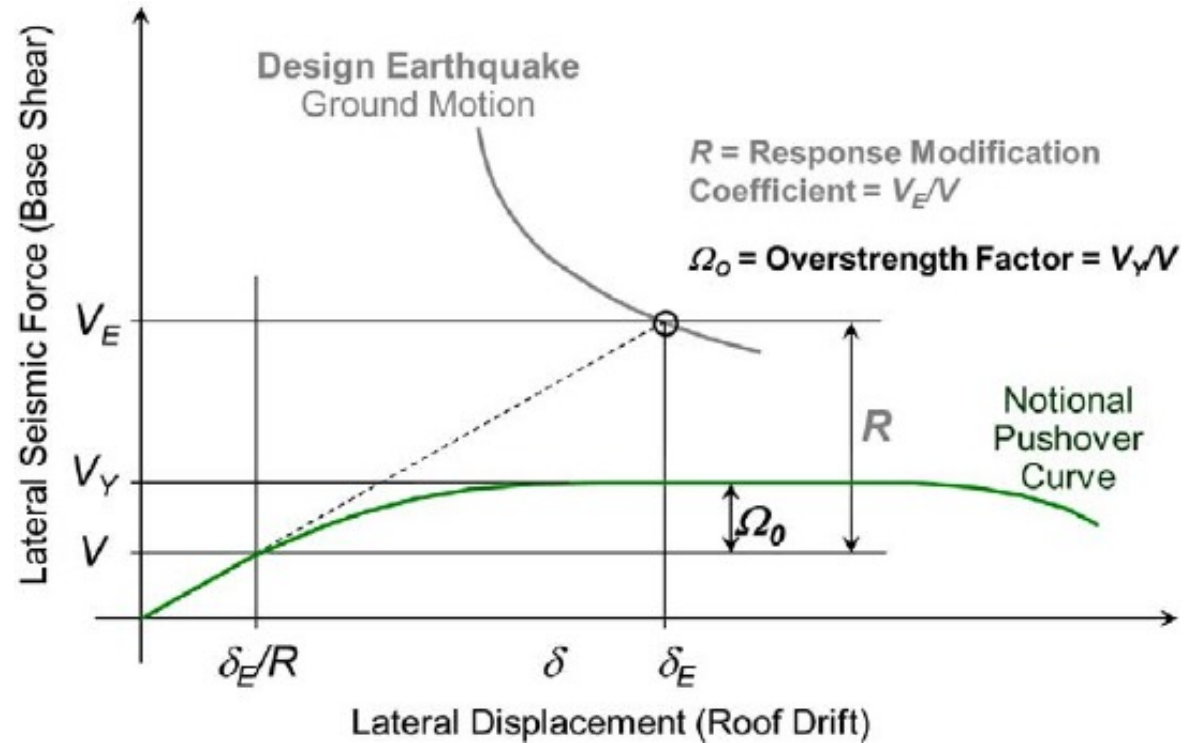
Image source: Prof. Ricles

# EPFL New lateral load-resisting systems

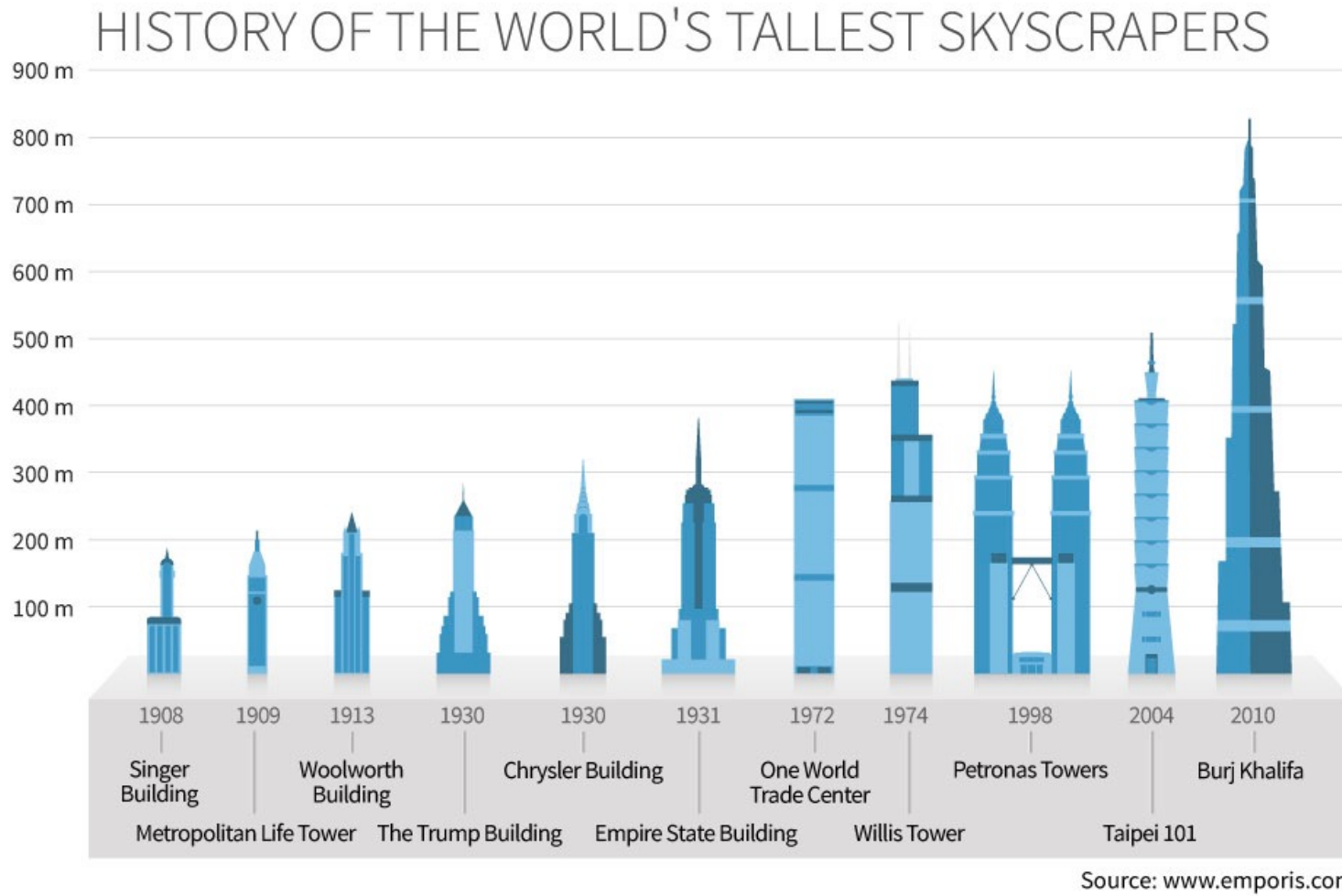


## Quantification of Building Seismic Performance Factors

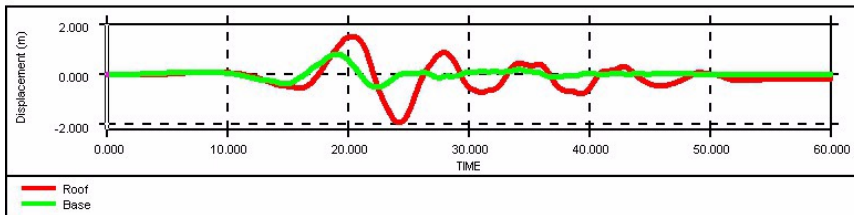
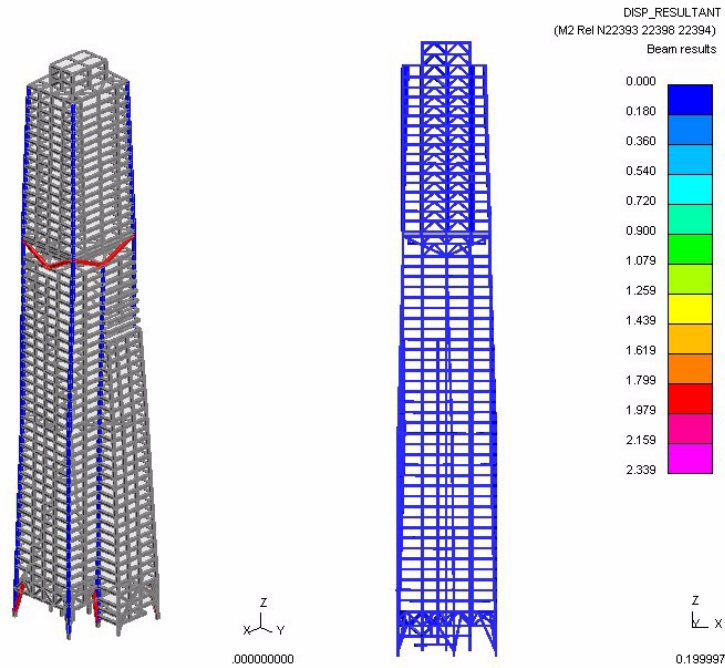
FEMA P695 / June 2009



# Use of nonlinear analysis in special projects

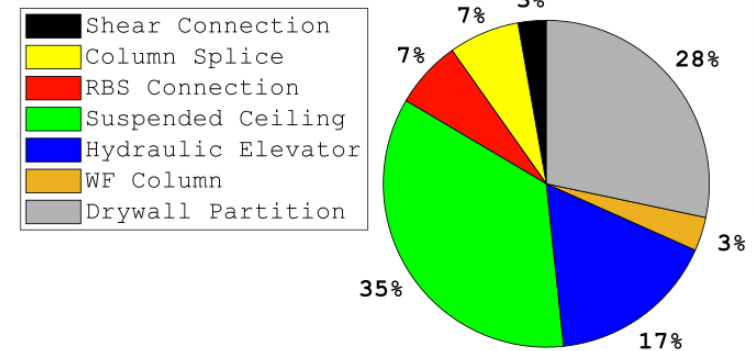


# Use of nonlinear analysis in special projects



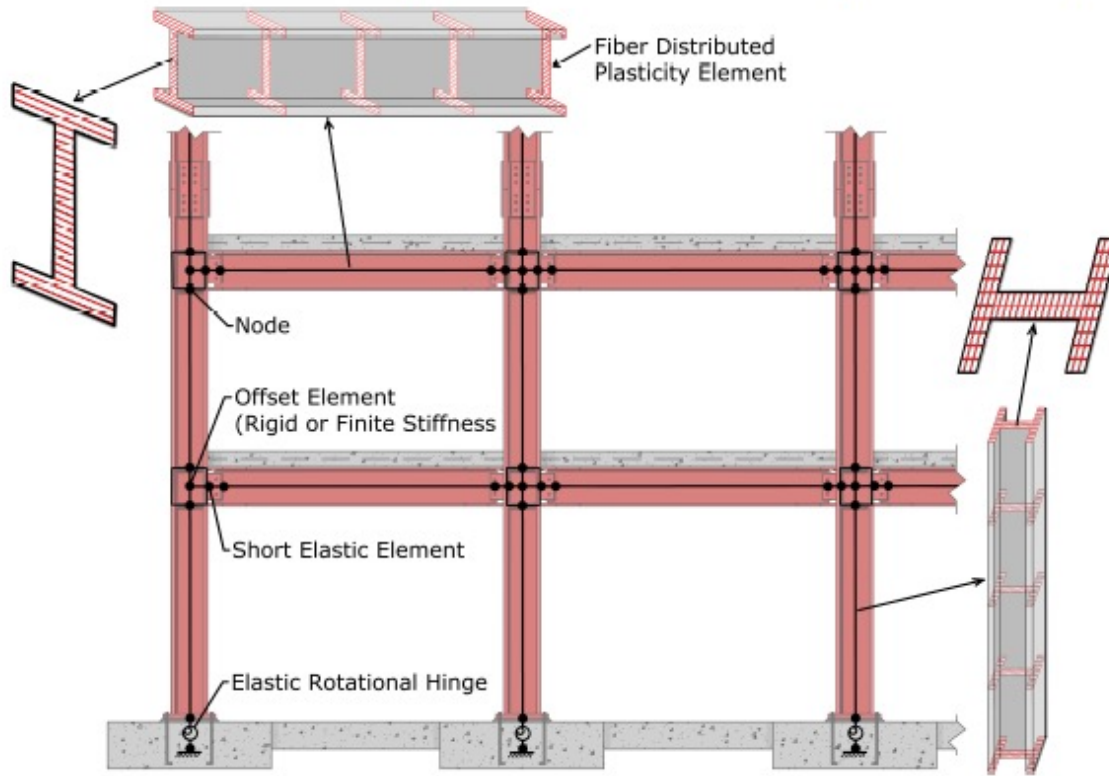
## Financial loss disaggregation For scenario-based events & planning

Asset components Total Losses: 2.1M\$

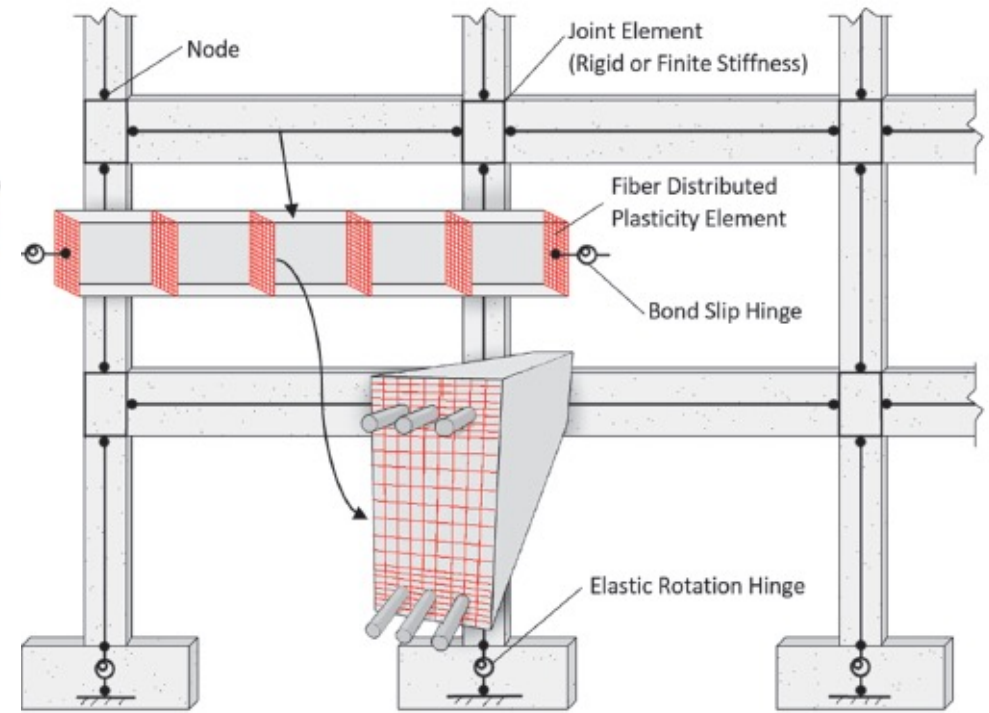


Source: Elkady and Lignos (2020)

**EPFL** Elements for nonlinear modeling of frame structures

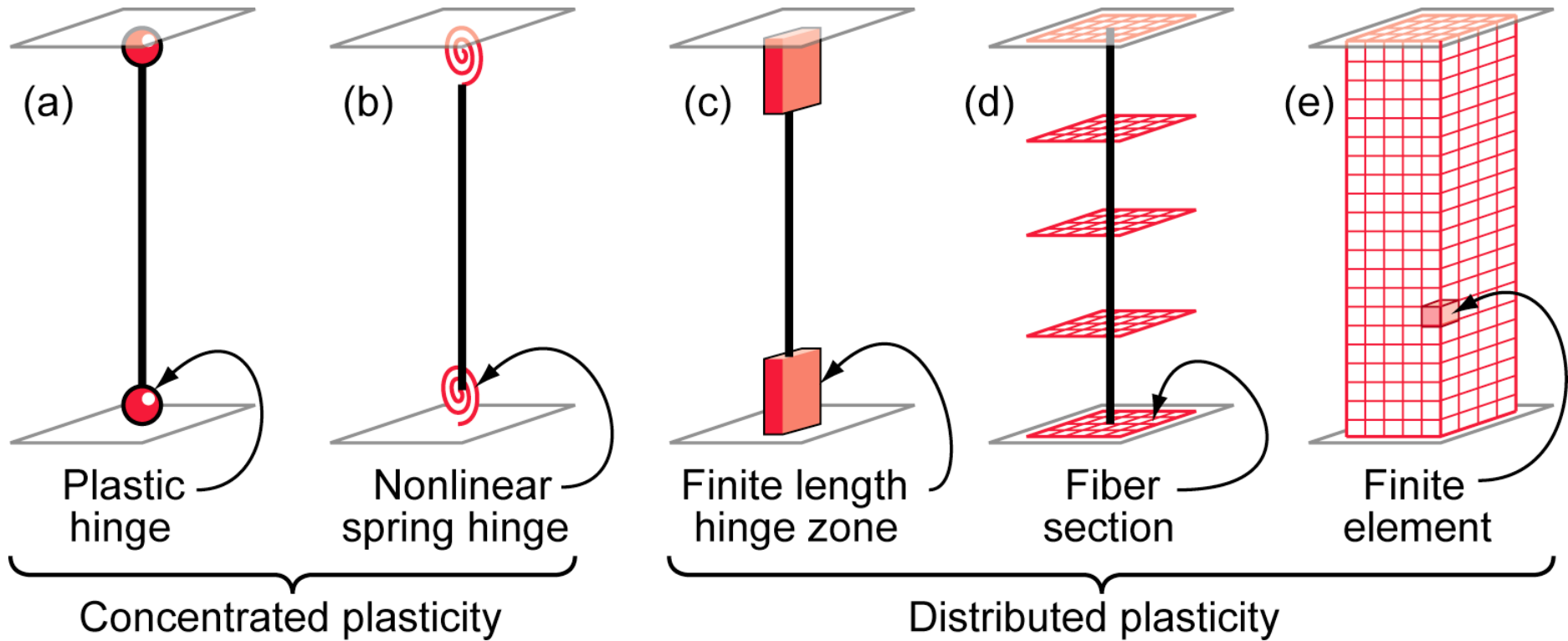


Source: GCR 17-917-46v2



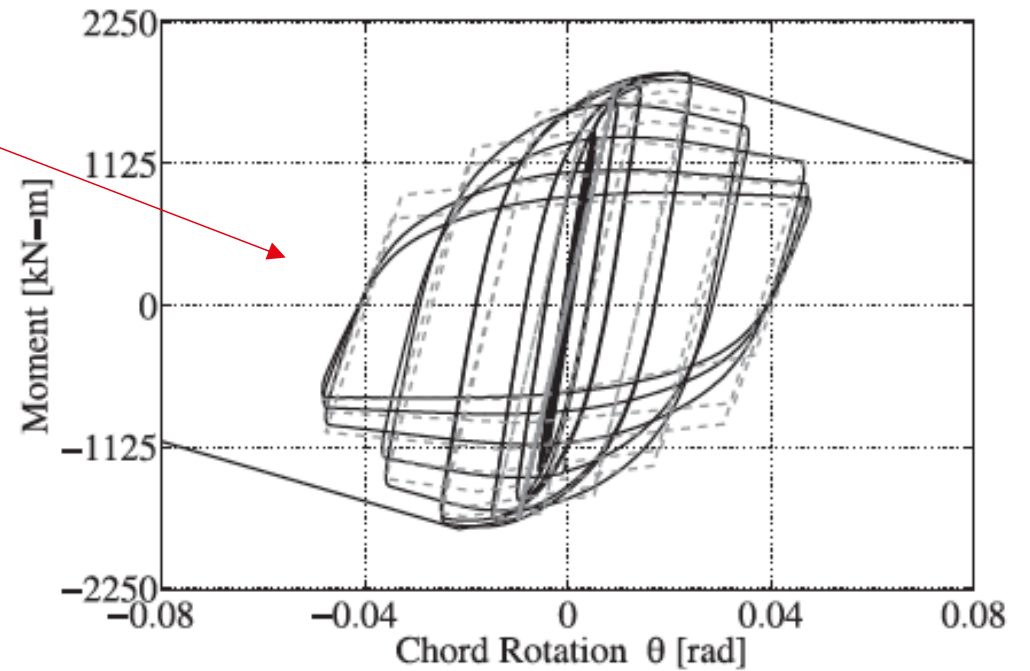
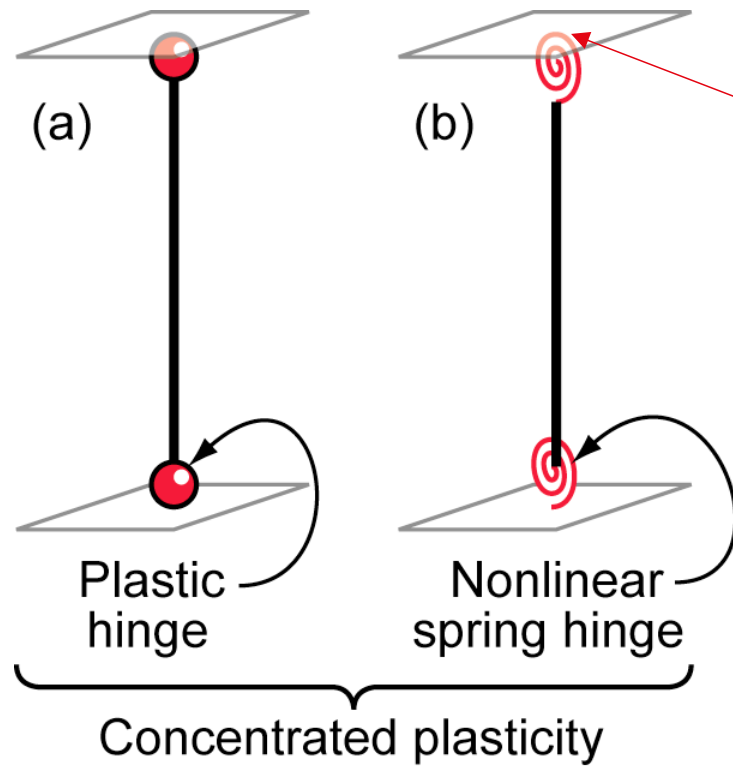
Source: GCR 17-917-46v3

# EPFL Model fidelity



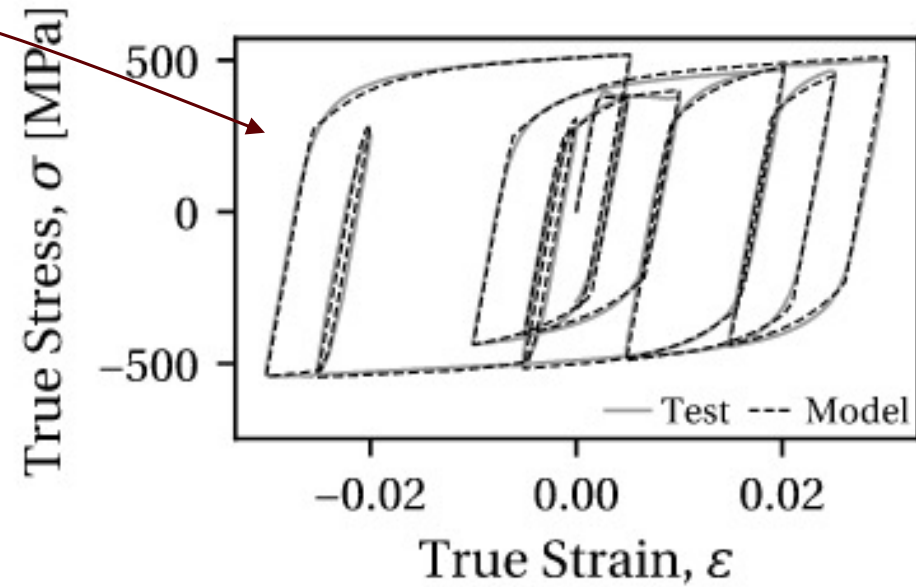
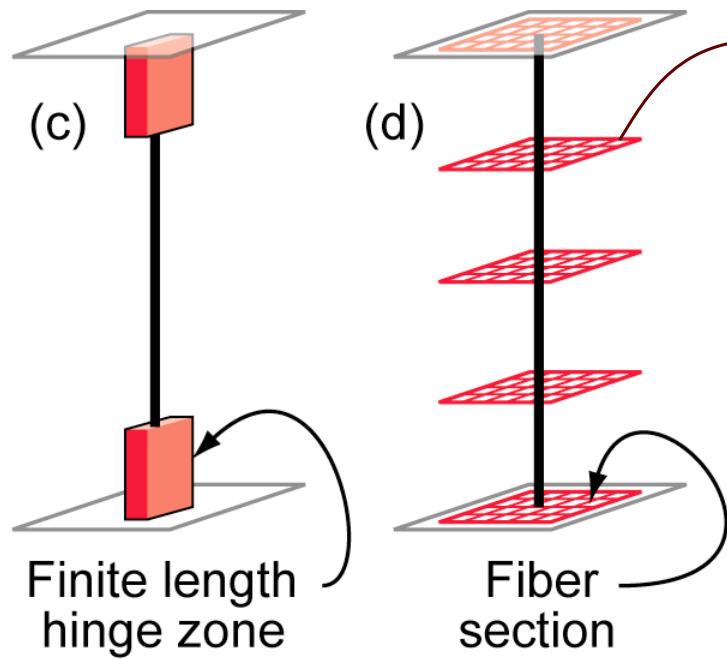
Source: NIST 2010

# EPFL Concentrated plasticity models



(Source: Lignos and Krawinkler 2011)

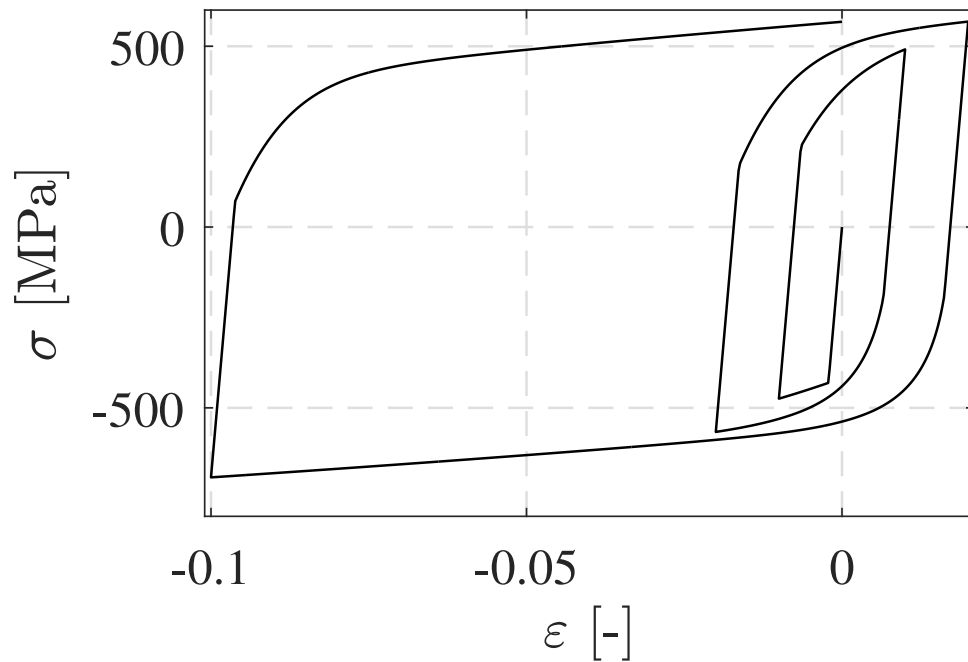
# EPFL Distributed plasticity models



(Source: Hartloper et al. 2023)

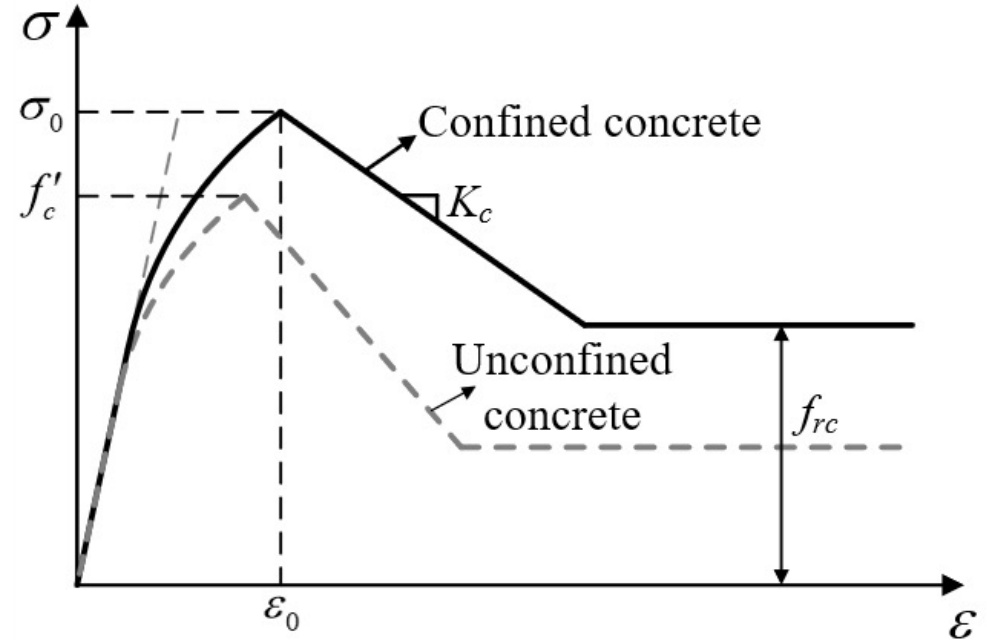
# Constitutive models for construction materials

Structural steel



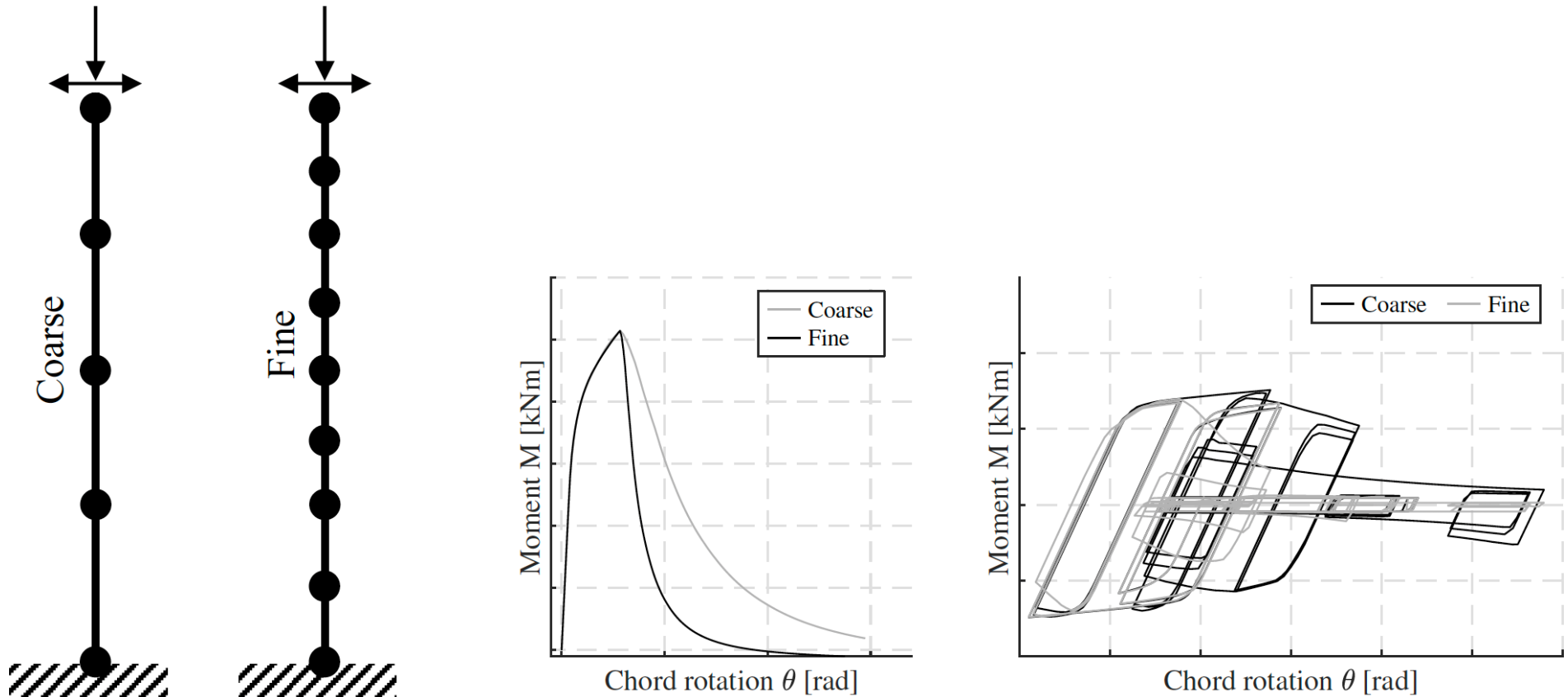
Source Hartloper et al. (2021)

Concrete (reinforced or unreinforced)



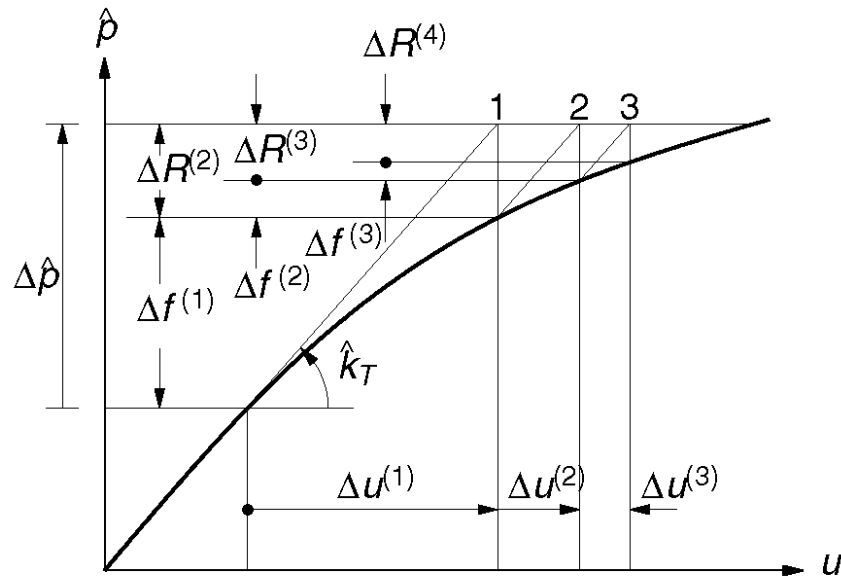
Source Mander (1988)

# Mesh dependency in the presence of softening

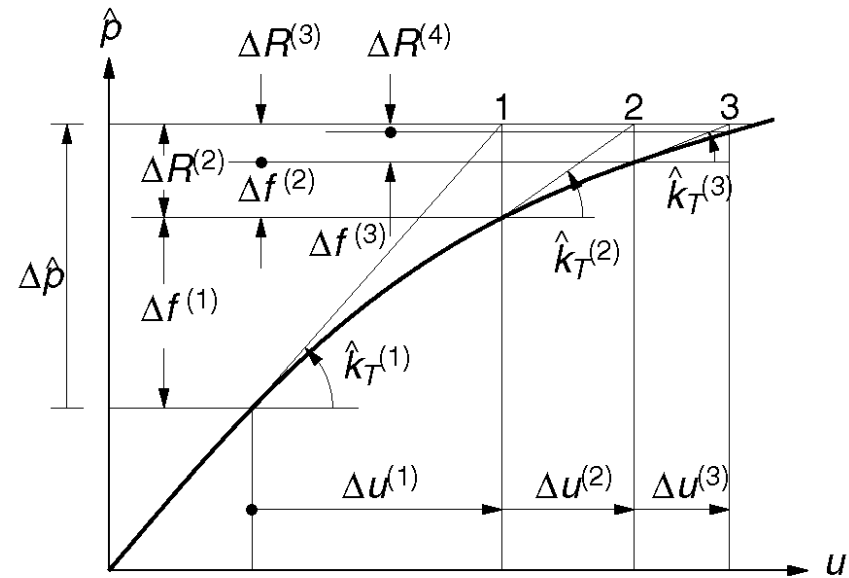


Source: Heredia, de Castro e Sousa and Lignos (2024)

# EPFL Numerical solution techniques



(a)



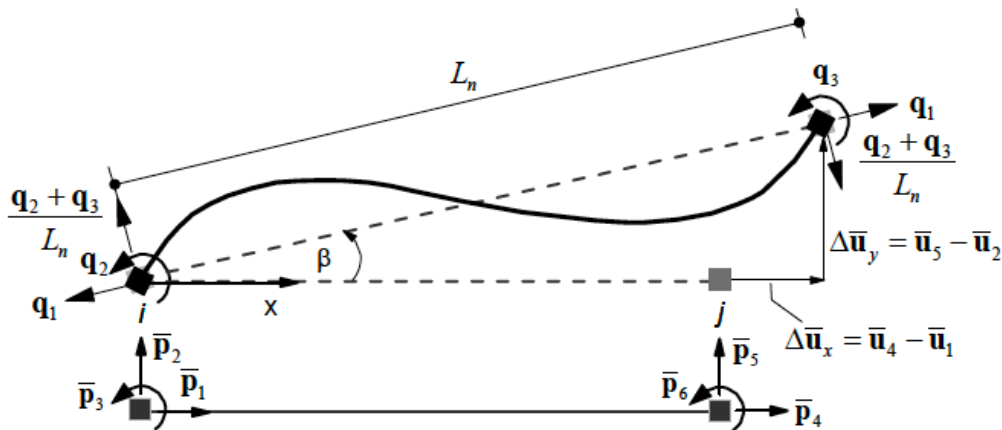
(b)

$$(\hat{k})_T \Delta u^{(1)} = \Delta \hat{p}$$

$$\Delta R^{(2)} = \Delta \hat{p} - \Delta f^{(1)}$$

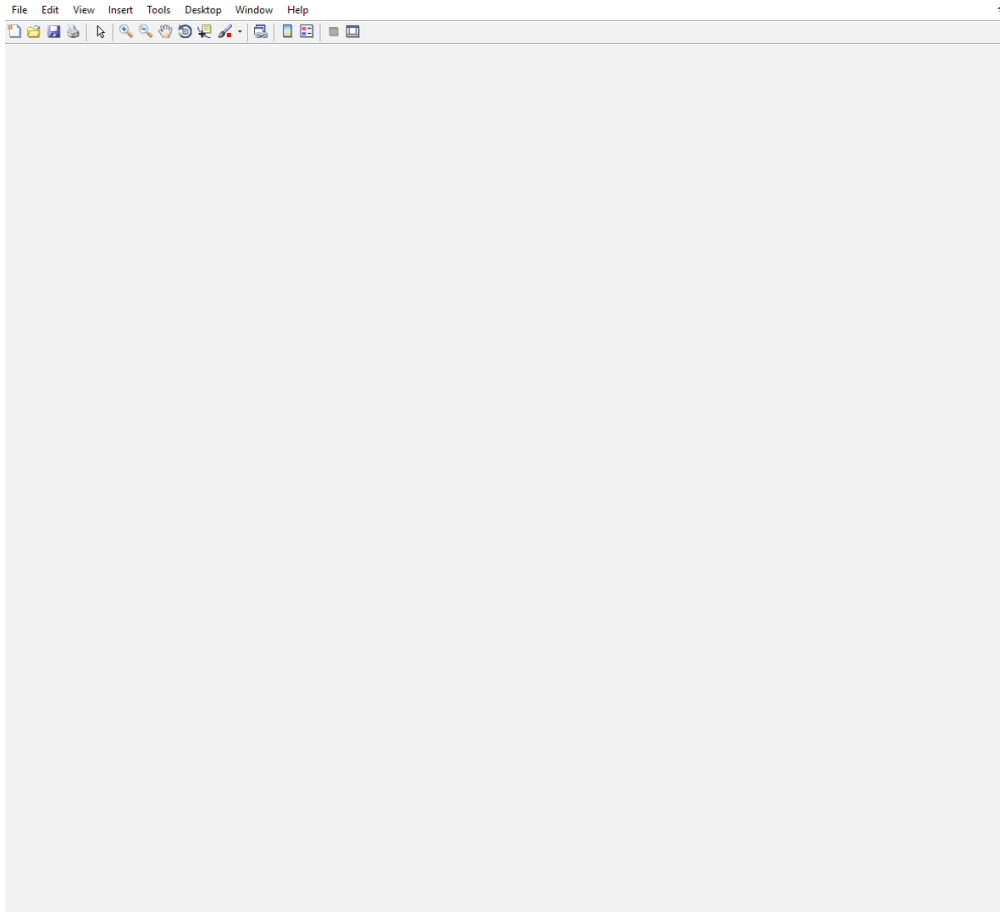
$$(\hat{k})_T \Delta u^{(2)} = \Delta R^{(2)} = \Delta \hat{p} - \Delta f^{(1)}$$

# EPFL Nonlinear geometric transformations

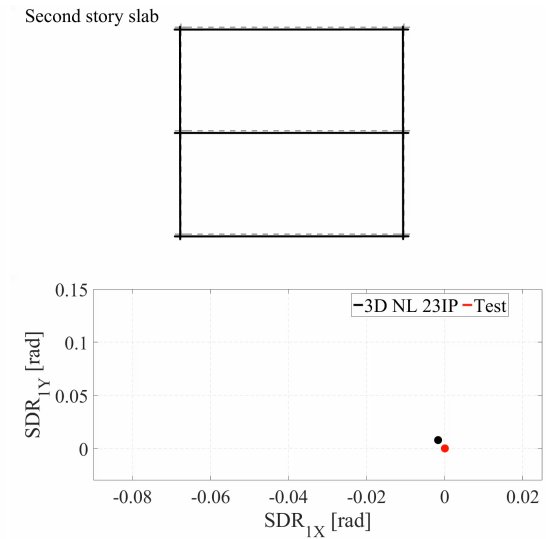
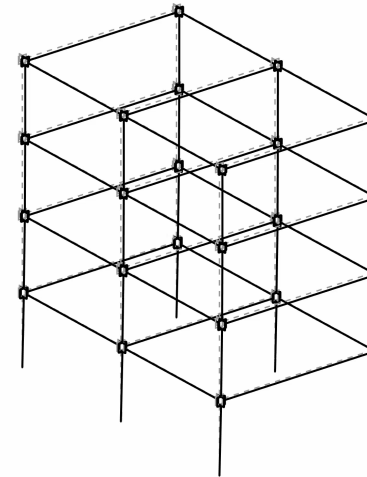


$$\begin{Bmatrix} \bar{p}_1 \\ \bar{p}_2 \\ \bar{p}_3 \\ \bar{p}_4 \\ \bar{p}_5 \\ \bar{p}_6 \end{Bmatrix} = \begin{bmatrix} \frac{L + \Delta \bar{u}_x}{L_n} & -\frac{\Delta \bar{u}_y}{L_n^2} & -\frac{\Delta \bar{u}_y}{L_n^2} \\ -\frac{\Delta \bar{u}_y}{L_n} & \frac{L + \Delta \bar{u}_x}{L_n^2} & \frac{L + \Delta \bar{u}_x}{L_n^2} \\ 0 & 1 & 0 \\ \frac{L + \Delta \bar{u}_x}{L_n} & \frac{\Delta \bar{u}_y}{L_n^2} & \frac{\Delta \bar{u}_y}{L_n^2} \\ \frac{\Delta \bar{u}_y}{L_n} & -\frac{L + \Delta \bar{u}_x}{L_n^2} & -\frac{L + \Delta \bar{u}_x}{L_n^2} \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

# EPFL Nonlinear static and dynamic analysis



Source Elkady and Lignos (2018)



Source Heredia and Lignos (2024)

# EPFL Case studies



Lignos et al. (2013)



Vanin et al. (2020)

Week	Person	Course	Assignments
1	08/09/25	Motivation; course organization and evaluation.	
	10/09/25	Linear versus nonlinear systems / static versus dynamic loading	
2	15/09/25	Revision on matrix structural analysis / static and dynamic equilibrium equations	Assignment #1
	17/09/25		
3	22/09/25 (holiday)	Truss and frame elements	
	24/09/25		
4	29/09/25	Linear and nonlinear geometric transformations	Submission of Assignment #1 / Assignment #2
	01/10/25		
5	06/10/25	Material response and uniaxial constitutive formulations	
	08/10/25		
6	13/10/25	Solution methods in nonlinear analysis	
	15/10/25		
7	20/10/25	Study break	
	22/10/25		
8	27/10/25	Concentrated plasticity	Submission of Assignment #2 / Assignment #3
	29/10/25		
9	03/11/25	Displacement-based beam-column elements	
	05/11/25		
10	10/11/25	Integration methods	
	12/11/25		
11	17/11/25	Flexibility-based beam-column elements / Multipoint constraints	Submission of Assignment #3
	19/11/25		
12	24/11/25	Constitutive models based on plasticity	Assignment #4
	26/11/25		
13	01/12/25	Constitutive models based on continuum damage mechanics	
	03/12/25		
14	08/12/25	Smeared crack models / mesh dependency / Application examples of constitutive models for structural analysis	
	10/12/25		
15	15/12/25	Examples of nonlinear analysis of buildings – Discussion of choice of modelling approaches and their effect on the results	Submission of Assignment #4
	17/12/25		

## EPFL Simulation tools

Programming tools



Finite element analysis



[opensees.berkeley.edu](http://opensees.berkeley.edu)