EPFL

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Advanced composites in engineering structures

GRoMeC

Composite Mechanics Group

Exercise 1

Given material elastic properties calculate lamina reduced stiffness and compliance matrix.

Material properties
E_1 = 140 GPa
E_2 = 10 GPa
ν_{12} = 0.3
G_{12} = 5 GPa

Theoretical background

Constitutive equation

$\varepsilon = S\,\sigma$

$\sigma = Q\,\varepsilon$

$Q = S^{-1}$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{bmatrix}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{bmatrix}$$

Solution

Compliance components

$S_{11} = 1 / E_1$

$S_{22} = 1 / E_2$

$S_{12} = -\nu_{12} / E_1$

$S_{21} = -\nu_{21} / E_2$

$S_{66} = 1 / G_{12}$

Simmetry

$S_{12} = S_{21}$

$\nu_{21} = \nu_{12} * E_2 / E_1$

$$S = \begin{bmatrix} 0.00714 & -0.00214 & 0 \\ -0.00214 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} (GPa^{-1})$$

Stiffness components

$Q_{11} = E_1 / \beta$

$Q_{11} = E_2 / \beta$ $\quad \beta = 1 / (1 - \nu_{12} * \nu_{21})$

$Q_{12} = -E_{1*}\ \nu_{12} / \beta$

$Q_{66} = G_{12}$

Simmetry

$Q_{12} = Q_{21}$

$$Q = \begin{bmatrix} 140.9 & 3.02 & 0 \\ 3.02 & 10.07 & 0 \\ 0 & 0 & 5 \end{bmatrix} (GPa)$$

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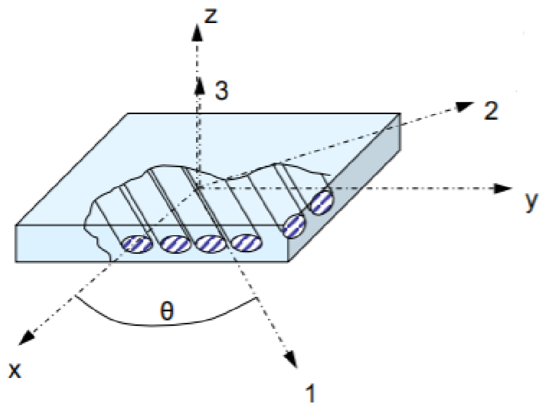
Exercise 2

Given material elastic properties calculate lamina reduced compliance and stiffness matrix in a rotated reference system.



$\theta = 30°$

$m = \cos\theta$

$n = \sin\theta$

Material properties

$E_1 = 140$ GPa

$E_2 = 10$ GPa

$\nu_{12} = 0.3$

$G_{12} = 5$ GPa

Theoretical background

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{bmatrix} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xs} \\ S_{xy} & S_{yy} & S_{ys} \\ S_{xs} & S_{ys} & S_{ss} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{bmatrix} \qquad \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{xy} & Q_{yy} & Q_{ys} \\ Q_{xs} & Q_{ys} & Q_{ss} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{bmatrix}$$

Terms in positon 1-3 and 2-3 appears, now shear and tensile stresses and strains are coupled

Compliance matrix solution

Rotation matrix solution

$$\begin{bmatrix} S_{xx} & S_{xy} & S_{xs} \\ S_{xy} & S_{yy} & S_{ys} \\ S_{xs} & S_{ys} & S_{ss} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & -mn \\ n^2 & m^2 & mn \\ 2mn & -2mn & m^2-n^2 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} m^2 & n^2 & -mn \\ n^2 & m^2 & mn \\ 2mn & -2mn & m^2-n^2 \end{bmatrix}$$

Hard coded solution

$$S_{xx} = S_{11}m^4 + m^2n^2 (2S_{12} + S_{66}) + S_{22}n^4$$

$$S_{xy} = m^2n^2 (S_{11} + S_{22} - S_{66}) + S_{12}(m^4 + n^4)$$

$$S_{yy} = S_{11}n^4 + m^2n^2 (2S_{12} + S_{66}) + S_{22}m^4$$

$$S_{ys} = mn[2S_{11}n^2 - 2S_{22}m^2 + (2S_{12} + S_{66}) (m^2 - n^2)]$$

$$S_{xs} = mn[2S_{11}m^2 - 2S_{22}n^2 - (2S_{12} + S_{66}) (m^2 - n^2)]$$

$$S_{ss} = 2(2S_{11} + 2S_{22} - 4S_{12}) m^2n^2 + S_{66}(m^2 - n^2)^2$$

$$S = \begin{bmatrix} 0.0470 & -0.00214 & -0.0594 \\ -0.0188 & 0.0934 & -0.0210 \\ -0.0594 & -0.0210 & 0.1336 \end{bmatrix} (GPa^{-1})$$

Stiffness matrix solution

Rotation matrix solution

$$\begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{xy} & Q_{yy} & Q_{ys} \\ Q_{xs} & Q_{ys} & Q_{ss} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2-n^2 \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2-n^2 \end{bmatrix}$$

Hard coded solution

$$Q_{xx} = Q_{11}m^4 + 2\,(Q_{12} + 2Q_{66})\, m^2n^2 + Q_{22}n^4$$

$$Q_{xy} = (Q_{11} + Q_{22} - 4Q_{66})\, m^2n^2 + Q_{12}m^4 + n^4$$

$$Q_{yy} = Q_{11}n^4 + 2\,(Q_{12} + 2Q_{66})\, m^2n^2 + Q_{22}m^4$$

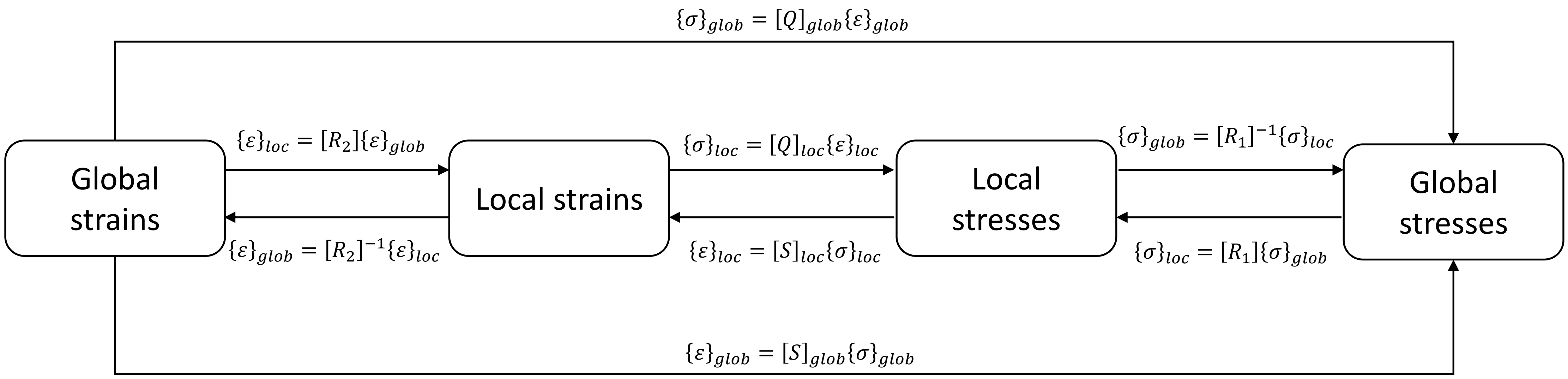
$$Q_{xs} = (Q_{11} - Q_{12} - 2Q_{66})\, nm^3 + (Q_{22} - Q_{12} + 2Q_{66})\, n^3m$$

$$Q_{ys} = (Q_{11} - Q_{12} - 2Q_{66})\, mn^3 + (Q_{22} - Q_{12} + 2Q_{66})\, m^3n$$

$$Q_{ss} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\, n^2m^2 + Q_{66}n^4 + m^4$$

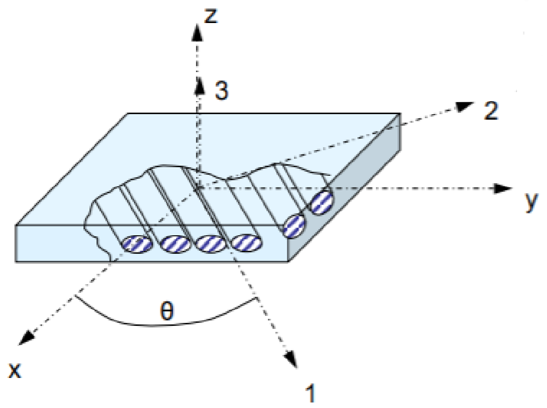
$$Q = \begin{bmatrix} 84.77 & 26.44 & 41.85 \\ 26.44 & 19.35 & 14.80 \\ 41.85 & 14.80 & 28.42 \end{bmatrix} \text{(GPa)}$$

Stress and strain from global to local and viceversa workflow



Exercise 3

Given strain vector characterizing a lamina oriented at 60° calculate lamina local and global stresses.



$\theta = 60°$
$m = \cos\theta$
$n = \sin\theta$

Material properties
$E_1 = 140$ GPa
$E_2 = 10$ GPa
$\nu_{12} = 0.3$
$G_{12} = 5$ GPa

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{bmatrix} = \begin{bmatrix} -5050\ \mu\varepsilon \\ 388\ \mu\varepsilon \\ 0\ \mu\varepsilon \end{bmatrix}$$

Calculate local strains with rotation matrix

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ 2mn & -2mn & m^2-n^2 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{bmatrix} \Rightarrow \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} -971\ \mu\varepsilon \\ -3690\ \mu\varepsilon \\ 4709\ \mu\varepsilon \end{bmatrix}$$

Calculate local stresses with constitutive equation

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{bmatrix} \Rightarrow \begin{bmatrix} -148 \\ -40 \\ 23.5 \end{bmatrix} (\text{MPa}) = \begin{bmatrix} 140.9 & 3.02 & 0 \\ 3.02 & 10.07 & 0 \\ 0 & 0 & 5 \end{bmatrix} (\text{GPa}) \begin{bmatrix} -971\ \mu\varepsilon \\ -3690\ \mu\varepsilon \\ 4709\ \mu\varepsilon \end{bmatrix}$$

Calculate global stresses with rotation matrix

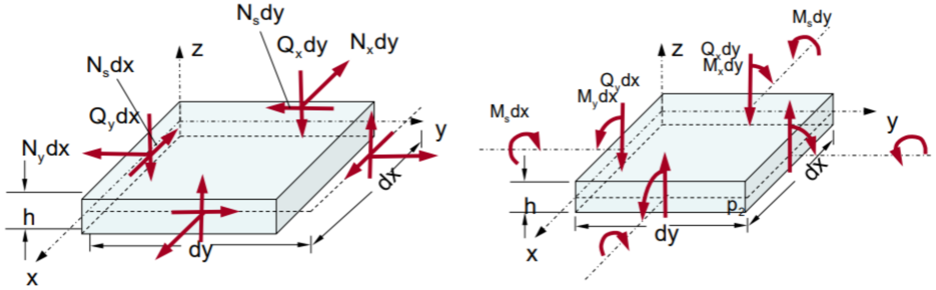
$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2-n^2 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{bmatrix} \Rightarrow \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{bmatrix} = \begin{bmatrix} -87.5 \\ -100.6 \\ -58.5 \end{bmatrix} \text{(MPa)}$$

Alternative solution

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{xy} & Q_{yy} & Q_{ys} \\ Q_{xs} & Q_{ys} & Q_{ss} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{bmatrix}$$

Stresses in local and global reference frame can be significantly different, be sure to be in the local one befor evaluating failure criterion

CLT basic formulas



$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon_0 \\ \kappa \end{bmatrix}$$

$$\varepsilon = \varepsilon_0 + z\,k$$

- N = force resultants per unite length
- M = moment resultants per unit length
- ε_0 = midplane strains
- κ = midplane curvature

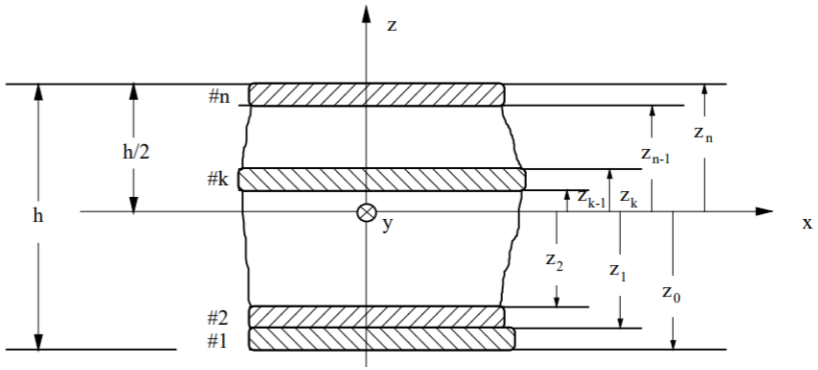
$$\begin{bmatrix} N_x \\ N_y \\ N_s \\ M_x \\ M_y \\ M_s \end{bmatrix} = \begin{bmatrix} Axx & Axz & Axs & Bxx & Bxy & Bxs \\ Axy & Ayy & Ays & Bxy & Byy & Bys \\ Axs & Ays & Ass & Bxs & Bys & Bss \\ Bxx & Bxy & Bxs & Dxx & Dxy & Dxs \\ Bxy & Byy & Bys & Dxy & Dyy & Dys \\ Bxs & Bys & Bss & Dxs & Dys & Dss \end{bmatrix} \begin{bmatrix} \varepsilon_{x0} \\ \varepsilon_{y0} \\ \varepsilon_{s0} \\ \kappa_x \\ \kappa_y \\ \kappa_s \end{bmatrix}$$

Exercise 4

Given a laminate with a stacking sequence of $[80/50]_s$, lamina elastic properties are given. Calculate A, B, D matrices.

Material properties
E_1 = 140 GPa
E_2 = 10 GPa
ν_{12} = 0.3
G_{12} = 5 Gpa
t = 0.125 mm

Stacking sequence definition



Layer	Start	End	Angle
Layer 4	z_3 = 0.125	z_4 = 0.250	80
Layer 3	z_2 = 0.0	z_3 = 0.125	50
Layer 2	z_1 = -0.125	z_2 = 0.0	50
Layer 1	z_0 = -0.250	z_1 = -0.125	80

Rotate reduced stiffness matrix of different orientations

$$Q_{80} = \begin{bmatrix} 10.4 & 6.7 & 1.15 \\ 6.7 & 133.3 & 21.2 \\ 1.15 & 21.2 & 8.65 \end{bmatrix} \text{(GPa)}$$

$$Q_{50} = \begin{bmatrix} 33.8 & 33.3 & 26.9 \\ 33.3 & 56.5 & 37.5 \\ 26.9 & 37.5 & 35.3 \end{bmatrix} \text{(GPa)}$$

ABD matrix construction

$$A = \sum_{k=0}^{n} Q^k (z_k - z_{k+1}) \quad \text{(F/L)} \qquad B = \frac{1}{2}\sum_{k=0}^{n} Q^k (z^2_k - z^2_{k+1}) \quad \text{(F)} \qquad D = \frac{1}{3}\sum_{k=0}^{n} Q^k (z^3_k - z^3_{k+1}) \quad \text{(FL)}$$

$$\begin{bmatrix} A & B \\ B & D \end{bmatrix} = \begin{bmatrix} 11.05 & 10 & 7 & 0 & 0 & 0 \\ 10 & 47.4 & 14.7 & 0 & 0 & 0 \\ 7 & 14.7 & 11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.14 & 0.10 & 0.05 \\ 0 & 0 & 0 & 0.10 & 133.3 & 0.24 \\ 0 & 0 & 0 & 0.05 & 21.2 & 0.12 \end{bmatrix}$$

The laminate we are considering is symmetric → **Coupling matrix is zero! The laminate is`*decoupled*`**

Exercise 5

Given a laminate with a stacking sequence of [45/-45/45/-45] . Lamina elastic properties are given. Calculate A, B, D matrices.

Material properties
E_1 = 140 GPa
E_2 = 10 GPa
ν_{12} = 0.3
G_{12} = 5 Gpa
t = 0.125 mm

Stacking sequence definition

Layer	Start	End	Angle
Layer 4	$z_3 = 0.125$	$z_4 = 0.250$	-45
Layer 3	$z_2 = 0.0$	$z_3 = 0.125$	45
Layer 2	$z_1 = -0.125$	$z_2 = 0.0$	-45
Layer 1	$z_0 = -0.250$	$z_1 = -0.125$	45

Rotate reduced stiffness matrix of different orientations

$$Q_{45} = \begin{bmatrix} 44.2 & 34.2 & 32.7 \\ 34.2 & 44.2 & 32.7 \\ 32.7 & 32.7 & 36.2 \end{bmatrix} \text{(GPa)}$$

$$Q_{-45} = \begin{bmatrix} 44.2 & 34.2 & 32.7 \\ 34.2 & 44.2 & 32.7 \\ 32.7 & 32.7 & 36.2 \end{bmatrix} \text{(GPa)}$$

- Q_{xx} and Q_{yy} are the same for both orientations
- Q_{ss} is maximized at 45 degrees
- Q_{xy} is the same for both orientations
- Q_{xz} and Q_{yz} are the same for both orientations
- Q_{xz} and Q_{yz} differ for only a minus comparing the two angles

ABD matrix construction

$$\begin{bmatrix} A & B \\ B & D \end{bmatrix} = \begin{bmatrix} 22.1 & 17.1 & 0 & 0 & 0 & -1.0 \\ 17.1 & 22.1 & 0 & 0 & 0 & -1.0 \\ 0 & 0 & 18.1 & -1.0 & -1.0 & 0 \\ 0 & 0 & -1.0 & 0.46 & 0.36 & 0 \\ 0 & 0 & -1.0 & 0.36 & 0.46 & 0 \\ -1.0 & -1.0 & 0 & 0 & 0 & 0.38 \end{bmatrix}$$

The laminate we are considering is not only balanced but also antisymmetric. → **Axs, Ays, Dxs, Dys = 0**

Exercise 6

Consider a laminate with a stacking sequence of [0/90] with lamina made of T300/5208 and subjected to a temperature change of -75°. Calculate the residual stresses at the top face of the 90° layer.

Material properties
E_1 = 181 GPa
E_2 = 10.3 GPa
ν_{12} = 0.28
G_{12} = 7.17 Gpa
t = 0.5 mm

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_6 \end{bmatrix} = \begin{bmatrix} 0.2 \times 10^{-7} \\ 0.225 \times 10^{-4} \\ 0 \end{bmatrix} \quad (1/°C)$$

Stacking sequence definition

Layer	Start	End	Angle
Layer 2	$z_1 = 0.0$	$z_2 = 0.5$	90
Layer 1	$z_0 = -0.5$	$z_1 = 0.0$	0

Theoretical background

$$N^T = \sum_{k=0}^{n} Q^k (z_k - z_{k+1}) \alpha^k$$

$$M^T = \frac{1}{2} \sum_{k=0}^{n} Q^k (z^2_k - z^2_{k+1}) \alpha^k$$

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_s \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2-n^2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_6 \end{bmatrix}$$

Thermal loads calculation

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_s^T \end{bmatrix} = \begin{bmatrix} -1.31 \times 10^{-7} \\ -1.31 \times 10^{-7} \\ 0 \end{bmatrix} \text{(N/m)}$$

$$\begin{bmatrix} M_x^T \\ M_y^T \\ M_s^T \end{bmatrix} = \begin{bmatrix} -1.54 \times 10^{2} \\ -1.54 \times 10^{2} \\ 0 \end{bmatrix} \text{(N)}$$

CLT solution

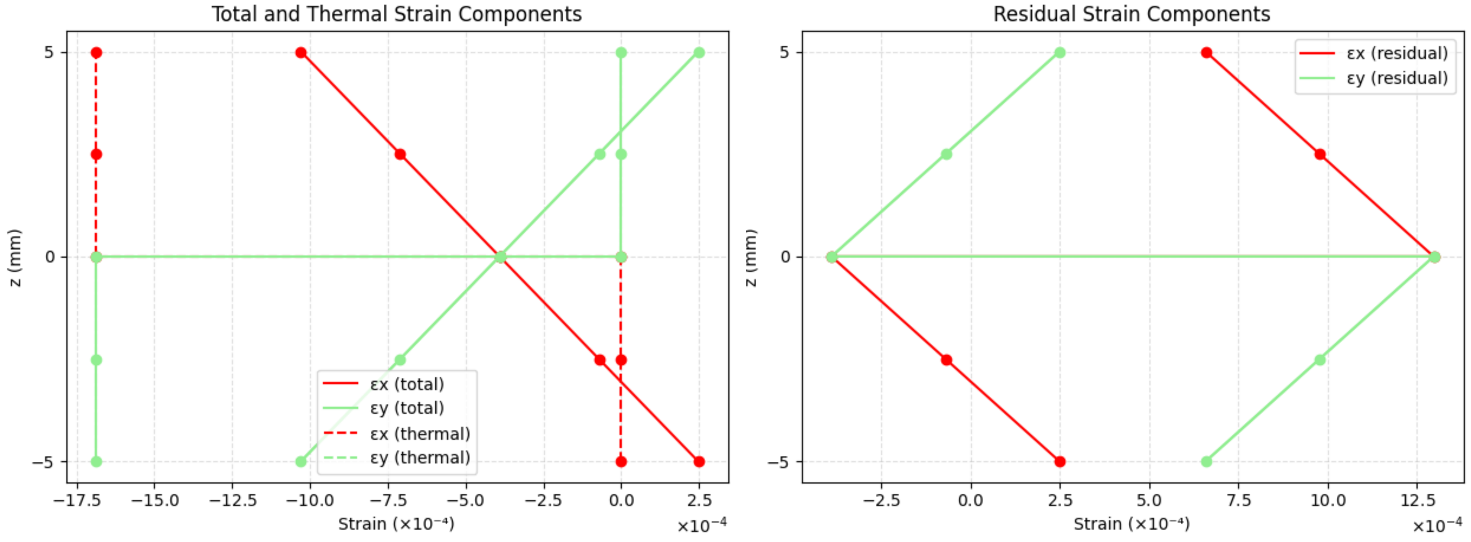
$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon_0 \\ \kappa \end{bmatrix} \qquad \begin{bmatrix} \varepsilon_{x0} \\ \varepsilon_{y0} \\ \varepsilon_{s0} \end{bmatrix} = \begin{bmatrix} -3.91 \times 10^{-4} \\ -3.91 \times 10^{-4} \\ 0 \end{bmatrix} \qquad \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_s \end{bmatrix} = \begin{bmatrix} -1.28 \times 10^{-1} \\ 1.28 \times 10^{-1} \\ 0 \end{bmatrix} \ (1/m)$$

Theoretical background and results

$$\varepsilon = \varepsilon_0 + z\,k$$

$$\begin{bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \varepsilon_s^T \end{bmatrix} = \Delta T \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_s \end{bmatrix}$$

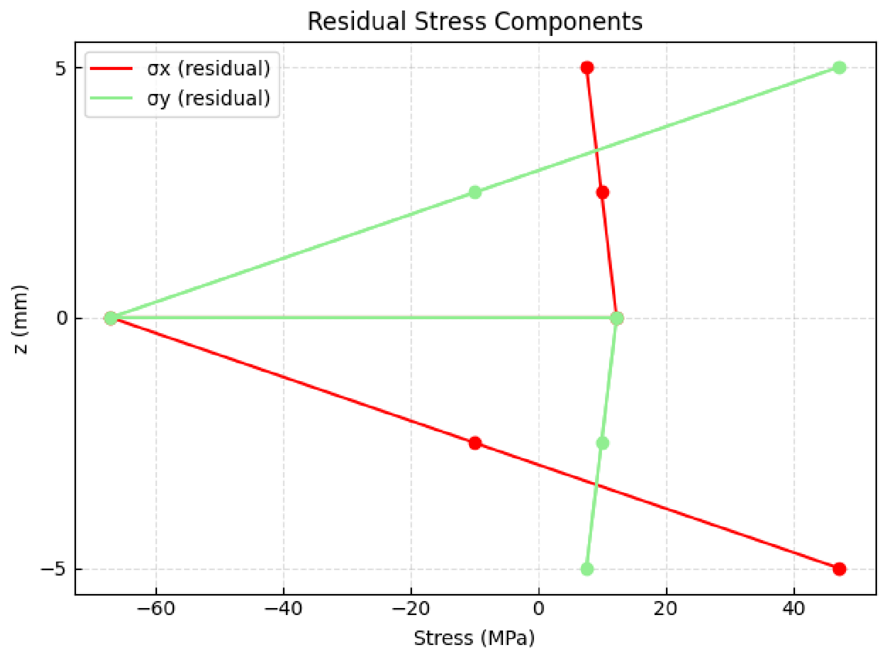
$$\begin{bmatrix} \varepsilon_x^r \\ \varepsilon_y^r \\ \varepsilon_s^r \end{bmatrix} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{bmatrix} - \begin{bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \varepsilon_s^T \end{bmatrix}$$



Residual stresses calculation

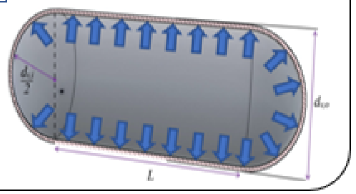
$$\begin{bmatrix} \sigma_x^r \\ \sigma_y^r \\ \sigma_s^r \end{bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{xy} & Q_{yy} & Q_{ys} \\ Q_{xs} & Q_{ys} & Q_{ss} \end{bmatrix} \begin{bmatrix} \varepsilon_x^r \\ \varepsilon_y^r \\ \varepsilon_s^r \end{bmatrix}$$

$$\begin{bmatrix} \sigma_x^r \\ \sigma_y^r \\ \sigma_s^r \end{bmatrix}_{90°,\ top} = \begin{bmatrix} 7.5 \\ 47.2 \\ 0 \end{bmatrix} \text{(Mpa)}$$



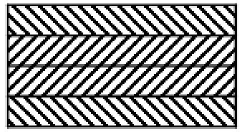
Problem definition

- Geometrical and load definition
- Preliminary load analysis



Material choice

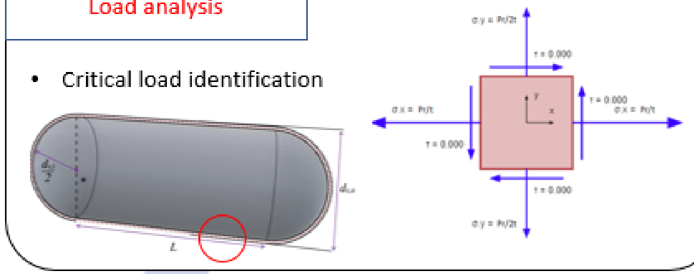
- Material definition/selection
- Stacking sequence definition

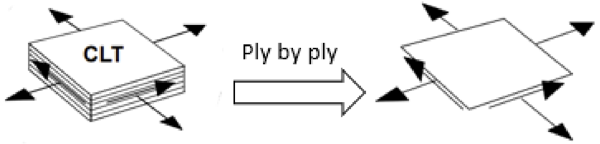


T300/5208, $[0/90]_S$

Load analysis

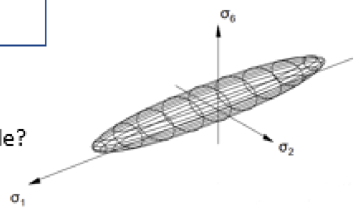
- Critical load identification





Failure criteria

- Serviseability and strength criteria
- Is the design feasible?
- Is it optimized?



No

Yes

Conclusion

Design process conclusion

Report writing and presentation

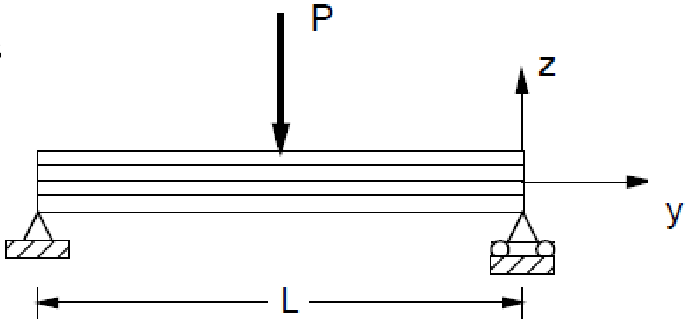
Exercise 7

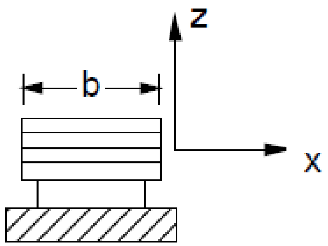
Consider the idealized model of a bridge proposed in the picture. Calculate stresses in the global reference frame for each ply and the maximum deflection in the load direction. Compare the results obtained with two different stacking sequences.

Stacking sequences

$[0_4/90_4]_s$ $[(0/90)_4]_s$

Material properties
E_1 = 181 GPa
E_2 = 10.3 GPa
ν_{12} = 0.28
G_{12} = 7.17 Gpa
t = 0.125 mm



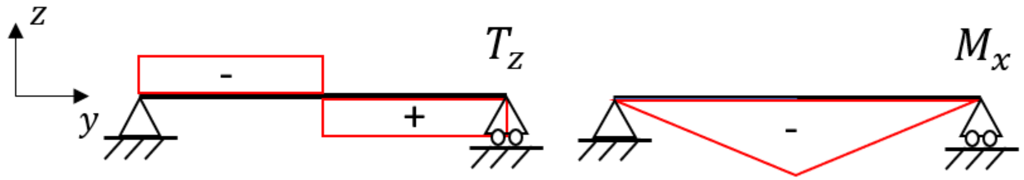


Property	Value
P	100 N
L	0.1 m
b	0.01 m

Stacking sequence definition

Layer	Start	End	Angle [1]	Angle [2]
Layer 16	z_{15} = 0.875	z_{16} = 1.000	0	0
Layer 15	z_{14} = 0.750	z_{15} = 0.875	0	90
Layer 14	z_{13} = 0.625	z_{14} = 0.750	0	0
Layer 13	z_{12} = 0.500	z_{13} = 0.625	0	90
Layer 12	z_{11} = 0.375	z_{12} = 0.500	90	0
Layer 11	z_{10} = 0.250	z_{11} = 0.375	90	90
Layer 10	z_9 = 0.125	z_{10} = 0.250	90	0
Layer 9	z_8 = 0.0	z_9 = 0.125	90	90
SYMMETRIC				

Load analysis



The critical point is the middle section of the composite beam,
At that point both bending moment and vertical displacement reaches their maximum

From load to CLT

$$M = -\frac{PL}{4} \quad \Rightarrow \quad \begin{bmatrix} M_x \\ M_y \\ M_s \end{bmatrix} = \begin{bmatrix} M/b \\ 0 \\ 0 \end{bmatrix}$$

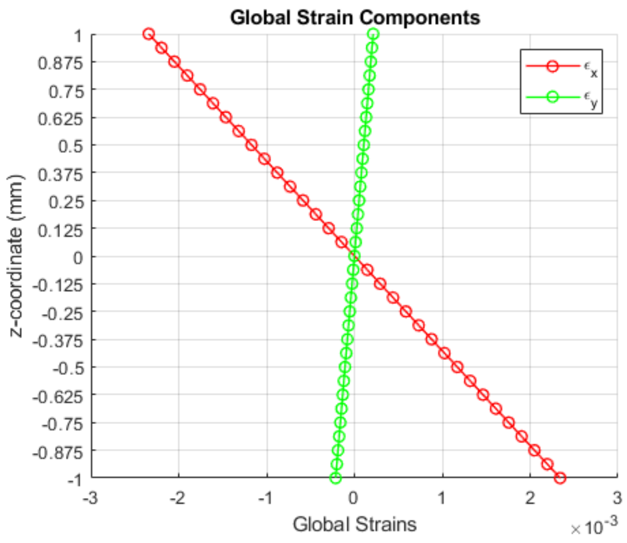
$$\begin{Bmatrix} \cancel{N} \\ M \end{Bmatrix} = \begin{bmatrix} A & \cancel{B} \\ \cancel{B} & D \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \kappa \end{Bmatrix} \quad \Rightarrow \quad \begin{matrix} \{M\} = [D]\{\kappa\} \\ \{\kappa\} = [D']\{M\} \end{matrix}$$

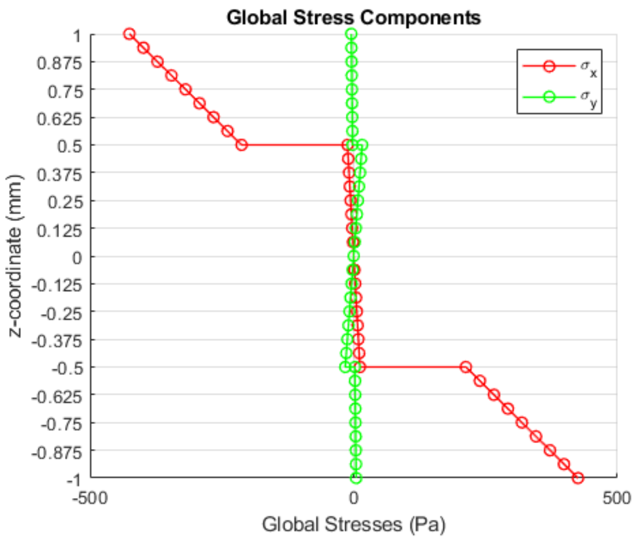
Do you expect any particular property from the D matrix of the two considered laminate?

Laminate 1

$$D = \begin{bmatrix} 106.92 & 1.93 & 0 \\ 1.93 & 21.19 & 0 \\ 0 & 0 & 4.78 \end{bmatrix} \text{(Nm)}$$

$$D' = \begin{bmatrix} 9.4 \times 10^{-3} & -8.54 \times 10^{-4} & 0 \\ -8.54 \times 10^{-4} & 4.7 \times 10^{-2} & 0 \\ 0 & 0 & 2.1 \times 10^{-1} \end{bmatrix} \text{(1/Nm)}$$

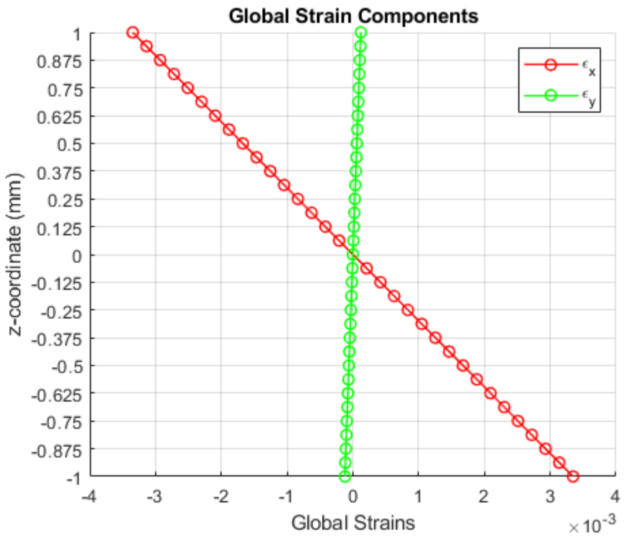


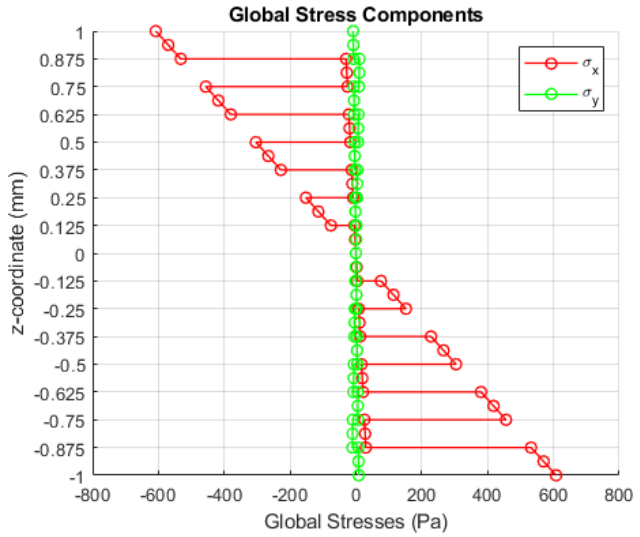


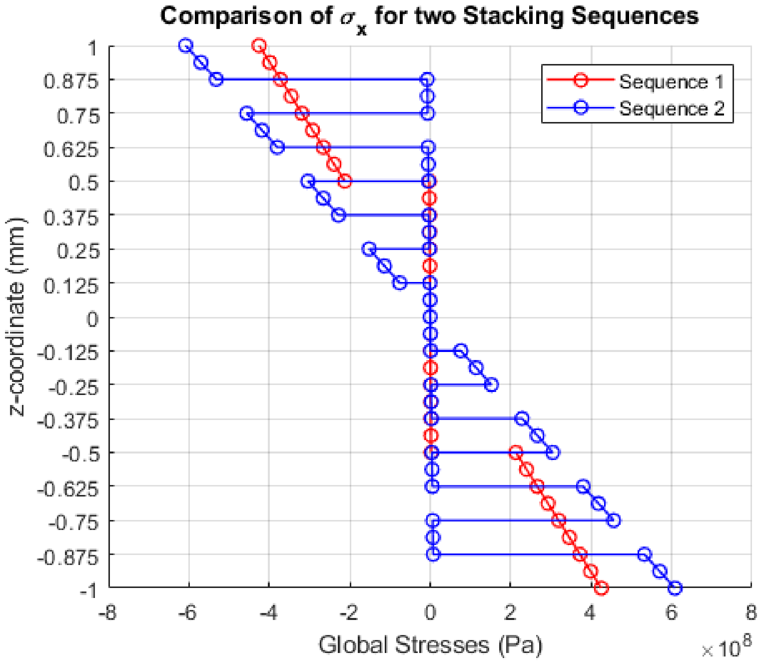
Laminate 2

$$D = \begin{bmatrix} 74.77 & 1.93 & 0 \\ 1.93 & 53.3 & 0 \\ 0 & 0 & 4.78 \end{bmatrix} \text{(Nm)}$$

$$D' = \begin{bmatrix} 1.3 \times 10^{-2} & -4.8 \times 10^{-4} & 0 \\ -4.8 \times 10^{-4} & 1.9 \times 10^{-2} & 0 \\ 0 & 0 & 2.1 \times 10^{-1} \end{bmatrix} \text{(1/Nm)}$$







Maximum displacement, theoretical background

$$\delta_{max} = \frac{PL^3}{48ExJ}$$

$$J = \frac{bh^3}{12}$$ with h = total thickness of the laminate

$$E_{xf} = \frac{12}{D_{11'}h^3}$$

Maximum displacement, calculations

$$E_{xf1} = 1280.9\ GPa$$

$$\delta_{max1} = 0.02\ m$$

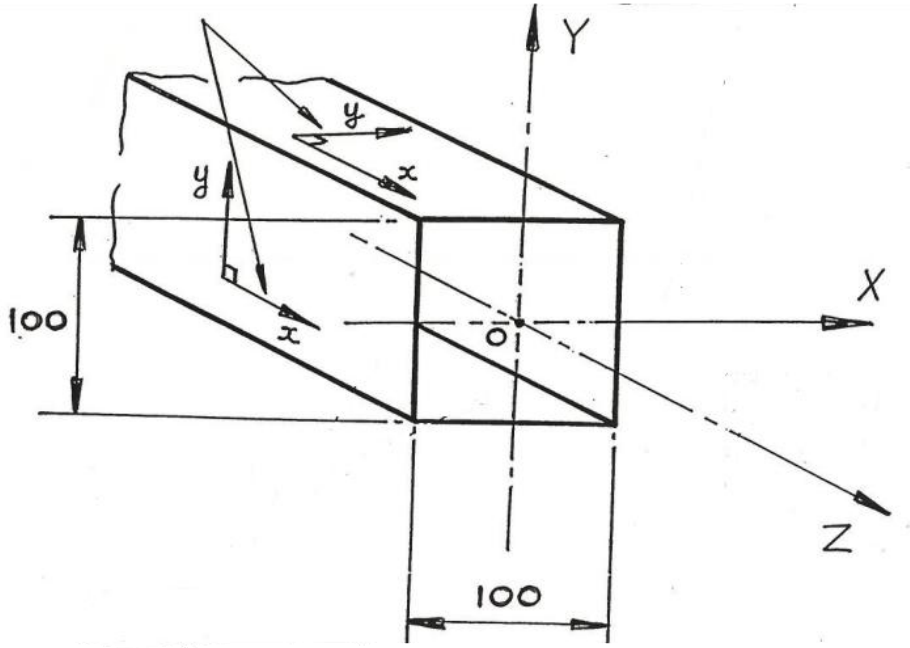
$$E_{xf2} = 896.4\ GPa$$

$$\delta_{max2} = 0.028\ m$$

Laminate 1 give a lower maximum displacement value. Why?

Exercise 8

A thin-walled composite box beam is subjected to a torque T and a bending moment M_x. The walls of the composite box are of symmetric lay-up configuration of $[0/90/+45/-45]_s$. Find the most stressed ply using Tsai-Wu failure index.



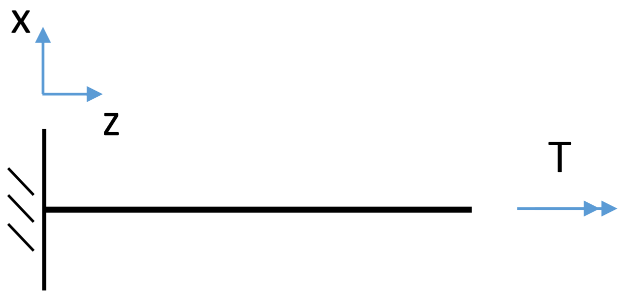
Material properties
E_1 = 140 GPa
E_2 = 10 GPa
ν_{12} = 0.3
G_{12} = 5 Gpa
X_t = 1500 MPa
Y_t = 1500 MPa
X_c = 40 MPa
Y_c = 246 MPa
S = 68 MPa

Loads & geometry	Value
T	10 kNmm
M_x	15 kNmm
L	100mm

Stacking sequence definition

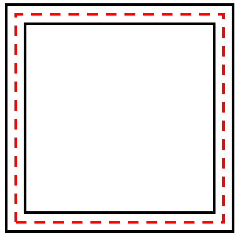
Layer	Start	End	Angle
Layer 8	$z_7 = 0.375$	$z_8 = 0.5$	0
Layer 7	$z_6 = 0.25$	$z_7 = 0.375$	90
Layer 6	$z_5 = 0.125$	$z_6 = 0.25$	45
Layer 5	$z_0 = 0.0$	$z_5 = 0.125$	-45
Symmetric			

Load analysis, torsion problem



Bredt theory

$$\tau = \frac{T}{2Ah}$$



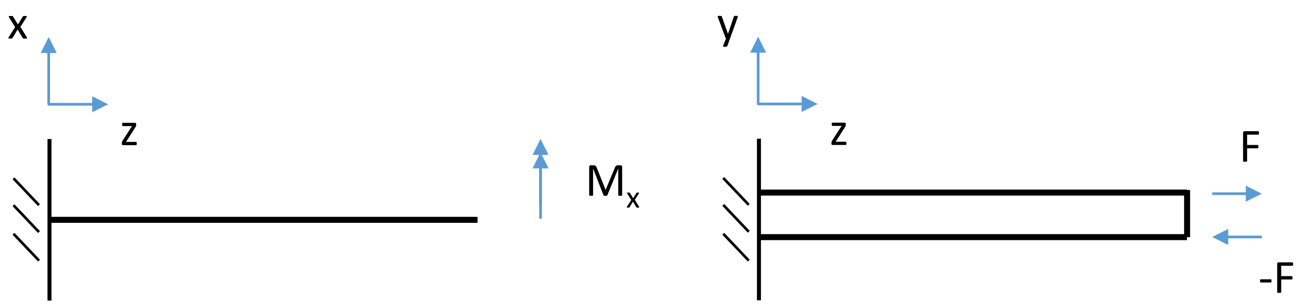
T = torque
A = area inside the mean line
h = laminate thickness

$$\begin{bmatrix} N_x \\ N_y \\ N_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tau * h \end{bmatrix}$$

Load analysis, bending problem

The bending moment is decomposed in 2 forces that give the same effect

$$M_x = F * L$$



Upper ply

$$\begin{bmatrix} N_x \\ N_y \\ N_s \end{bmatrix} = \begin{bmatrix} F/h \\ 0 \\ \tau * h \end{bmatrix}$$

Lower ply

$$\begin{bmatrix} N_x \\ N_y \\ N_s \end{bmatrix} = \begin{bmatrix} -F/h \\ 0 \\ \tau * h \end{bmatrix}$$

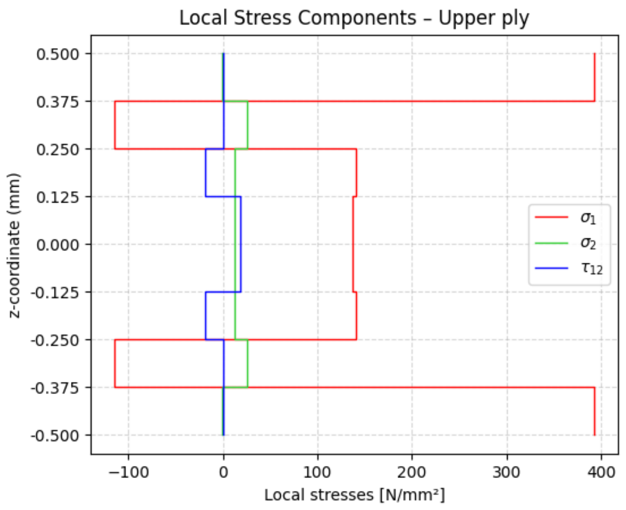
Forget about torsion now, do you have alternative design solutions? The simplifications we have made is conservative or not?

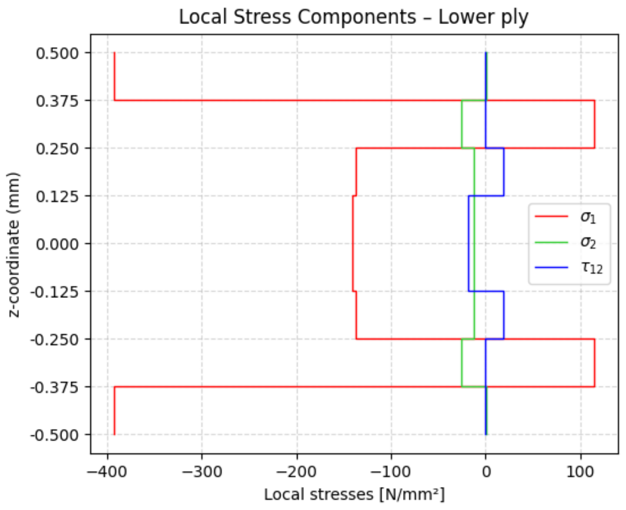
CLT analysis

$$\begin{Bmatrix} N \\ \cancel{M} \end{Bmatrix} = \begin{bmatrix} A & \cancel{B} \\ \cancel{B} & D \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \kappa \end{Bmatrix} \rightarrow \begin{matrix} \{N\} = [A]\{\varepsilon_0\} \\ \{\varepsilon_0\} = [A']\{N\} \end{matrix}$$

→ **I expect a step evolution of both local and global stresses and strain components ply by ply**

Stress analysis





The most stressed ply are the ones oriented at 0°, which are the external ones, in the upper part of the beam. This was predictable since are the one oriented as the load!

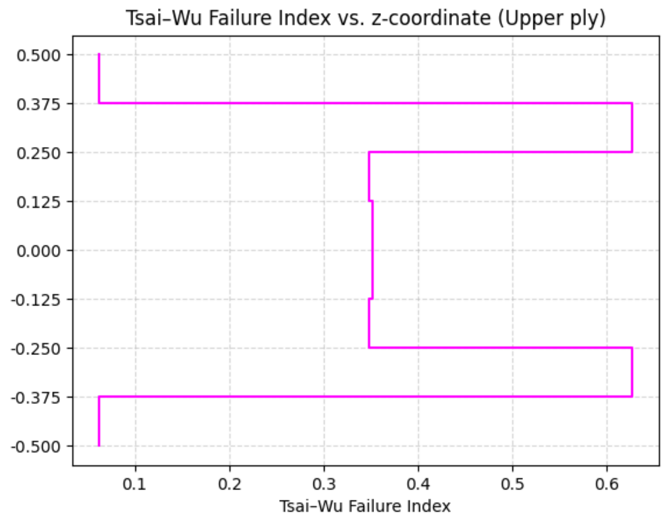
Tsai-Wu criterion

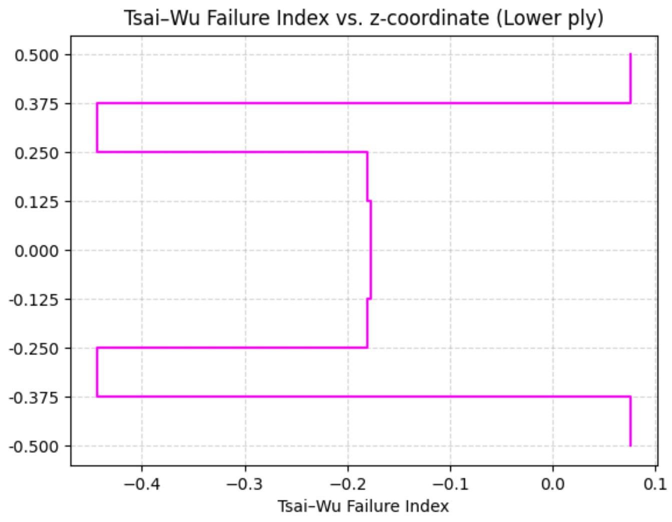
$$F_{11}\,\sigma_1^{\,2} + F_{22}\,\sigma_2^{\,2} + 2F_{12}\,\sigma_1\sigma_2 + F_{66}\,\sigma_6^{\,2} + F_1\sigma_1 + F_2\sigma_2 < 1$$

$$F_{11} = \frac{1}{X_t X_c} \qquad F_1 = \frac{1}{X_t} - \frac{1}{X_c} \qquad F_{66} = \frac{1}{S^2}$$

$$F_{22} = \frac{1}{Y_t Y_c} \qquad F_2 = \frac{1}{Y_t} - \frac{1}{Y_c} \qquad F_{12} = \frac{1}{2}\sqrt{F_{11}F_{22}}$$

Tsai-Wu index analysis





1) The graphs are not symmetric, why?
2) In this model has the order of the plies has made any effect?
3) If I have decided to put inside CLT directly Mx the order of the plies would have been relevant?

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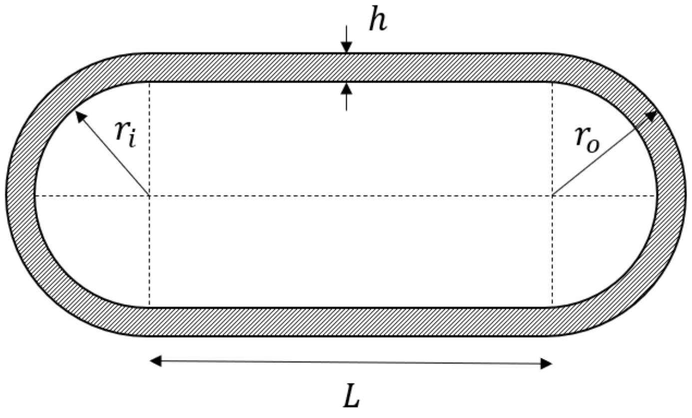
EPFL

Exercise 9

Design a composite pressure vessel for automotive applications maintaining minimal weight to enhance vehicle performance.



Material	E(GPa)	ν	G(GPa)	ρ (kg/m³)	V. fraction
Fiber	230	0.20	15	1760	0.65
Matrix	3.6	0.35	1.34	1200	0.35
Layer thickness = 0.2 mm					

Material strength properties (MPa)				
X_t	X_c	Y_t	Y_c	S
1500	1500	40	246	68

Loads and geometry	Value
P	10 MPa
L	100 mm
d_0	100 mm
Safety factor (SF)	1.5

GOAL: reduce of 50% the weight comparing to a steel solution

Preliminary load analysis

Cylindrical section

$$\sigma_h = \frac{Pr}{h} \qquad \sigma_a = \frac{Pr}{2h}$$

Emispherical section

$$\sigma_h = \frac{Pr}{2h} \qquad \sigma_a = \frac{Pr}{2h}$$

Material properties calculation via mixtures rule

$$E_1 = V_f E_f + V_m E_m = (0.65 \times 230) + (0.35 \times 3.6) = 150.7\ GPa$$

$$E_2 = \left(\frac{V_f}{E_f} + \frac{V_m}{E_m}\right)^{-1} = 10\ GPa$$

$$\nu_{21} = \frac{E_2 \nu_{12}}{E_1} = 0.0167$$

$$\nu_{12} = (V_f \nu_f) + (V_m \nu_m) = 0.2525$$

$$G_{12} = G_m \frac{(1 + \xi \chi V_f)}{(1 - \chi V_f)} = 4.53\ GPa$$

$\xi = 1$ shear factor

$$\chi = \frac{(G_f - G_m)}{(G_f + \xi G_m)} = 0.836$$ Halpin-Tsai factor

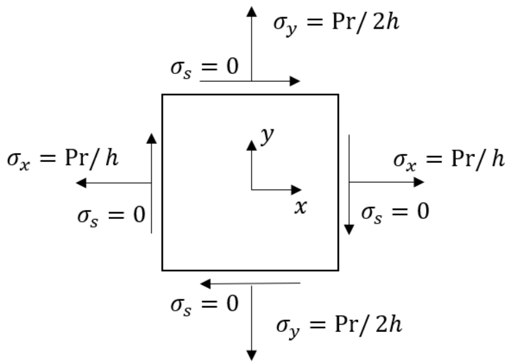
$$Q = \begin{bmatrix} 151.40 & 2.53 & 0 \\ 2.53 & 10.04 & 0 \\ 0 & 0 & 4.53 \end{bmatrix} \text{(GPa)}$$

First laminate definition

Stacking sequences: $[0_4/90_2]_s$

Layer	Start	End	Angle
Layer 12	z_{11} = 1.0	z_{12} = 1.2	0
Layer 11	z_{10} = 0.8	z_{11} = 1.0	0
Layer 10	z_9 = 0.6	z_{10} = 0.8	0
Layer 9	z_8 = 0.4	z_9 = 0.6	0
Layer 8	z_7 = 0.2	z_8 = 0.4	90
Layer 7	z_6 = 0.0	z_7 = 0.2	90
SYMMETRIC			

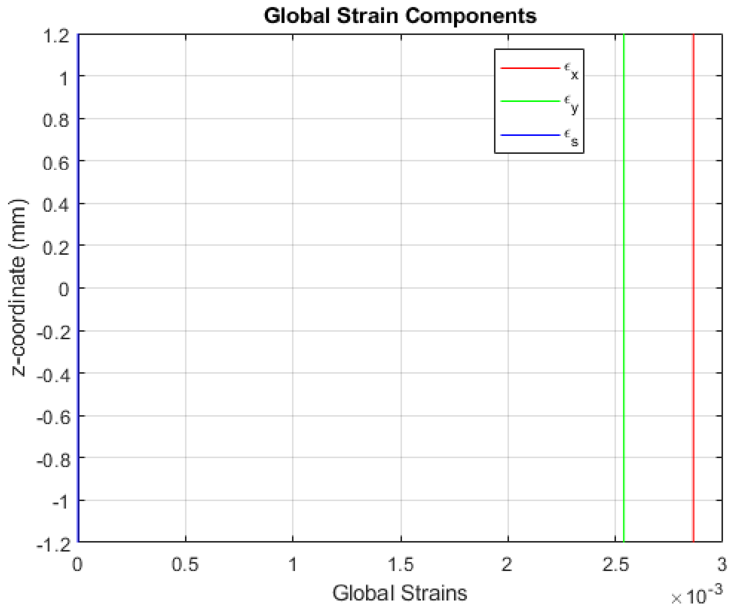
Load analysis

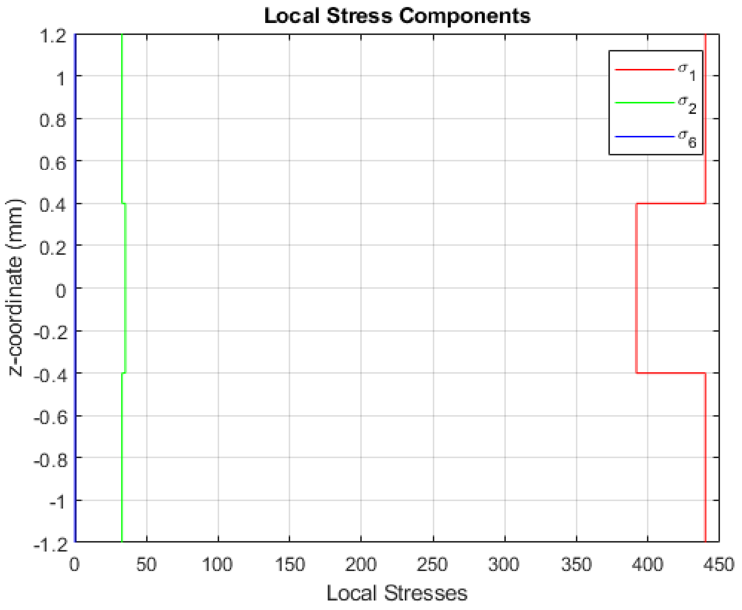


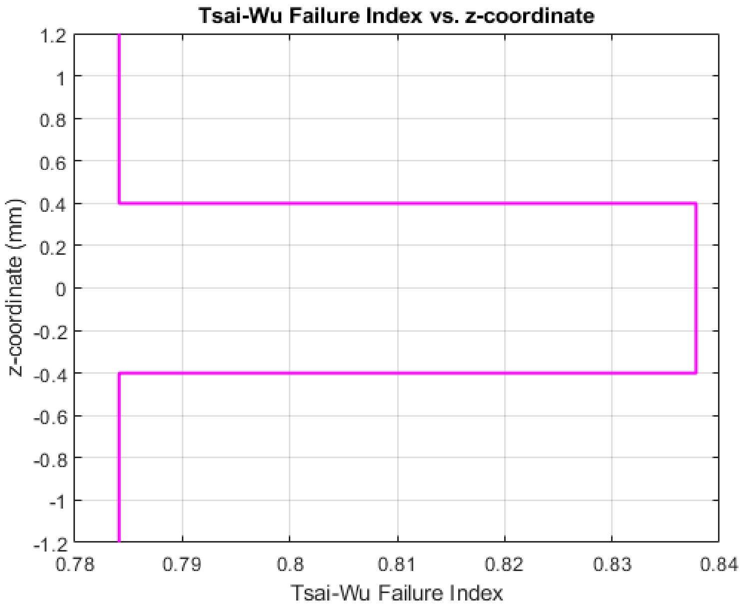
$$\begin{bmatrix} N_x \\ N_y \\ N_s \end{bmatrix} = \begin{bmatrix} SF * \sigma_h * h \\ SF * \sigma_a * h \\ 0 \end{bmatrix} \qquad \begin{bmatrix} N \\ \cancel{M} \end{bmatrix} = \begin{bmatrix} A & \cancel{B} \\ \cancel{B} & D \end{bmatrix} \begin{bmatrix} \varepsilon_0 \\ \kappa \end{bmatrix}$$

We are interested only on the A matrix, which doesn't depend on the plies order.

First iteration results







It's always a good idea in looking at the literature for looking at already existing solution for the considered problems.
For composite pressure vessels has been demonstrated that the optimal fiber orientation is 35.3° with antisymmetric and balanced stacking sequences. Considering that, several design cycles has been done until obtaining the final design presented hereafter. The goal was to obtain a TW index between 0.8 and 0.9!

Final laminate definition

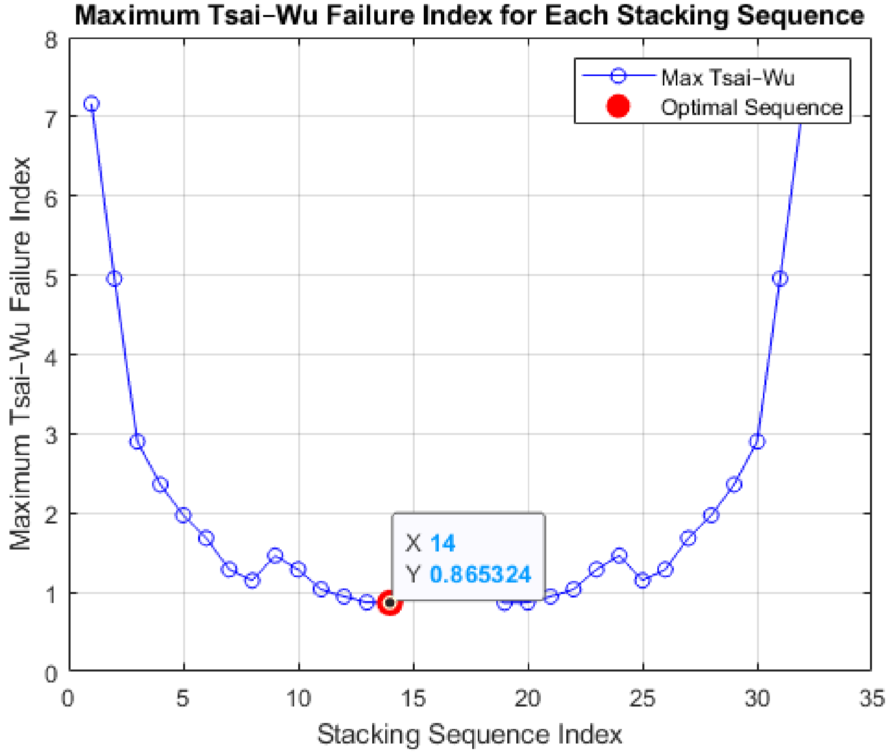
Stacking sequences: [-35.3/35.3$_2$/-35.3/35.3/-35.3/35.3/-35.3$_2$/35.3]

Layer	Start	End	Angle
Layer 10	$z_9 = 0.8$	$z_{10} = 1.0$	35.3
Layer 9	$z_8 = 0.6$	$z_9 = 0.8$	-35.3
Layer 8	$z_7 = 0.4$	$z_8 = 0.6$	-35.3
Layer 7	$z_6 = 0.2$	$z_7 = 0.4$	35.3
Layer 6	$z_5 = 0.0$	$z_6 = 0.2$	-35.3
SYMMETRIC			

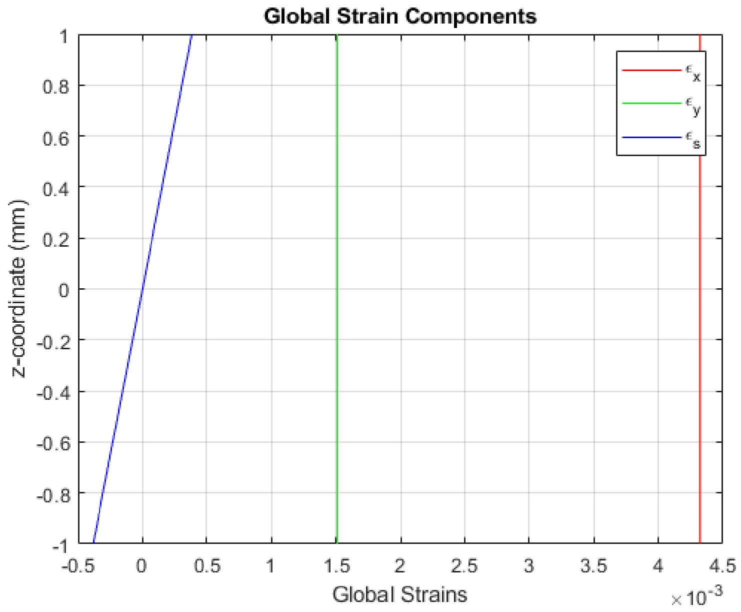
CLT analysis

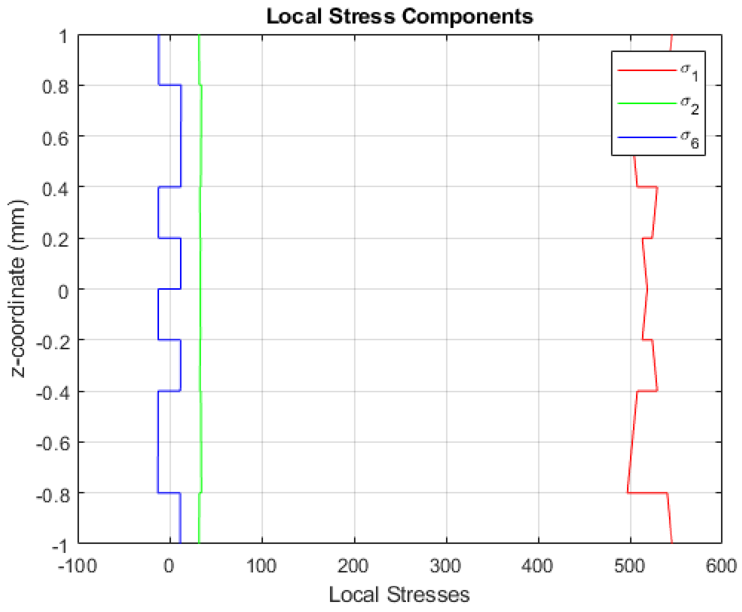
$$\begin{Bmatrix} N \\ \cancel{M} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \kappa \end{Bmatrix}$$

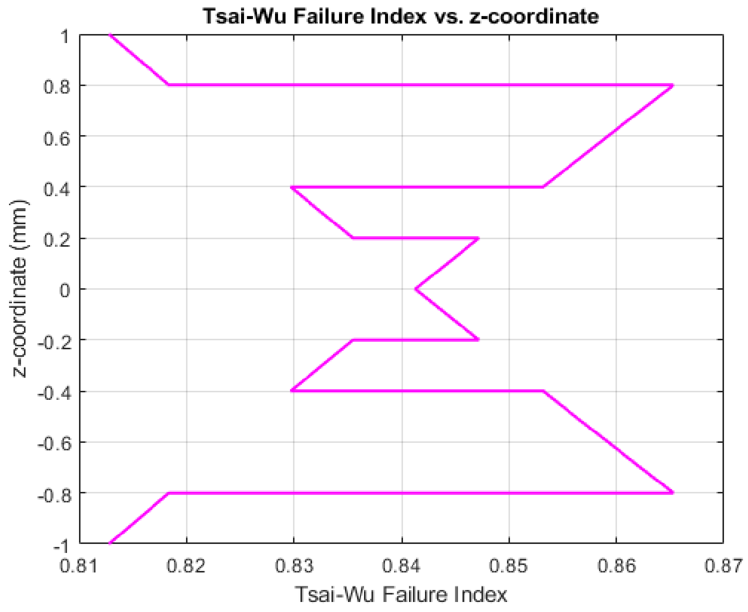
In this case we were interested in both A and B matrix so the order counted. How did I choose this order?



Design assessment







With an antisymmetric stacking sequence also the local stresses and Tsai-Wu graph are antysimmetric! In addition we have reached a maximum index of 0.865 and that's enough.

Final report

Description	Data
Cylindrical Length	$L = 0.100$ m
Outer Diameter	$D_o = 0.100$ m
Inner Diameter	$D_i = 0.096$ m
Laminate Thickness	$t = 0.002$ m
Internal Pressure	$P = 10$ MPa
Load in x-direction	$N_x = 735.0$ kN/m
Load in y-direction	$N_y = 367.5$ kN/m
Lamina Thickness	$th = 200\,\mu$m
Number of Lamina	10
Failure Criterion	Tsai-Wu
Safety Factor	$SF = 1.5$
Stacking sequence	$[-35.3, 35.3_2, -35.3, 35.3, -35.3, 35.3, -35.3_2, 35.3]$
Maximum Tsai-Wu index	0.865

Property	CFRP	STEEL
Volume Capacity	1.2 L	1.2 L
Maximum Pressure	10 MPa	10 MPa
Safety Factor	1.5	1.5
Thickness	2.0 mm	3.5 mm
Density	1564 kg/m^3	7800 kg/m^3
Mass	0.19 kg	1.63 kg

This weight reduction is 88%, which is crucial for a highly competitive racing application.