

A HALF CENTURY OF ARCH DAMS DESIGN WITH TRIAL LOAD METHOD

G. Tarbox¹, L. Nuss², J. Salamon³,

ABSTRACT

The era of modern arch dam design in the USA spans over 150 years with the first masonry dams built in California by the early settlers in the 1880s. The analysis and design techniques have dramatically evolved since then with a better understanding of the behavior of concrete materials, development of new laboratory testing methods, advancements in mathematical structural analysis (in particularly the Trial-Load Method and Finite Element Method), and the collection of data on performance of dams. Although the sophistication and accuracy of the 21st-century analyses are based on numerical methods that are superior when compared with the methods used in the previous century, ingenuity and sound engineering judgment still serve as a solid foundation in the design practices of today.

This paper presents a historical overview of the Trial-Load Method and its application to the design of arch dams. A significant part of the paper is dedicated to verification and validation activities that accompanied the development of the method, such as testing physical models of dams in the laboratory and investigations conducted using an experimental dam. The influences of the Trial-Load Method on the evolution of the practice of arch dam layout and design are summarized.

INTRODUCTION

There are about 2,300 arch dams built in the World. All these dams are of different shapes ranging from very thin arches to thick curved gravity dams. They were built using various construction materials, including rubble masonry, cyclopean masonry, cyclopean concrete, homogeneous mass concrete, and roller compacted concrete. Over about a 150-year era, various structural analysis techniques have been implemented in the design of arch dams, starting with simple cylinder formulas in the 19th century to advanced finite element (FE) analyses used today. The Trial-Load Method was the dominating structural analysis approach in the design of arch dams in the USA for over half of the 20th century.

AN HISTORICAL PERSPECTIVE OF ANALYSIS METHODS FOR ARCH DAMS

The first curved dams were constructed in the United States in the late nineteenth century in California: Bear Valley Dam (1884), Sweetwater Dam (1888), LaGrange Dam (1893), Lake Hemet Dam (1896) or Upper Otay Dam (1901). However, it is worth mentioning the construction of a masonry Jones Falls arch dam in 1832 at the Kingston Canal in Ontario. The shapes and designs of the arch dams of that time were imposed by the available

¹Senior Vice President, Emeritus, Stantec, Glenn.Tarbox@Stantec.com.

²Senior Structural Engineer, Nuss Engineering, LLC, Larry.K.Nuss@NussEngineering.com

³Technical Specialist, Waterways & Concrete Dams Group, Bureau of Reclamation, JSalamon@USBR.gov

material for construction and limited understanding of the structural behavior of such structures. Very often, the rubble masonry, cyclopean masonry, and cyclopean concrete dams were designed as curved gravity dams to increase their stability. The maximum compressive stress calculated by the cylinder formula for uniform water load was limited to 200-300 psi. In the 1920s, the use of homogeneous concrete was almost universal, and the analysis and design methods for arch dams were advanced and widely implemented at several projects [4].

Cylinder Formula

At the turn of the century, dam shapes and their thickness were determined using a thin-cylinder formula developed by Louis Navier (1826). In such an analysis, the hydrostatic load was distributed along horizontal isolated dam arches of a unit width (Figure 1). The arches would carry the load as long as a calculated average compressive stress did not exceed the defined conservative allowable strength value. For the rubble masonry dams, the allowable average strength was initially assumed in a range between 150 to 300 psi, and the value was increased up to 1,000 psi as the designer's confidence in the material strength raised over time. Several prominent dams have been designed using the Navier's formula including the 370-foot-high Pacoima arch dam (1928).

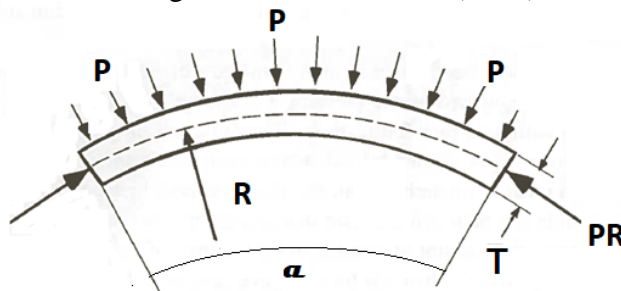


Figure 1. Cylinder formula

$$T = \frac{P R}{f_{all}} \quad [Eq.1]$$

- where: T – is the thickness of the arch
- P – is the water pressure
- R – is the arch radius
- f_{all} – is the allowable average compressive strength

In 1884, Frank Brown completed the construction of Bear Valley Dam, a slender masonry arch dam in a remote location of the San Bernardino Valley in Southern California. Designed by the cylinder formula, the 64-foot-high dam was only 3 feet thick at the crest and 8.5 feet thick at 48 feet below the crest. The base of the structure had an offset of about 2 feet on each side, making the dam base 13 feet wide. By completion of Bear Valley Dam, Brown demonstrated that the mathematical formulas could develop a very economical dam design.

Arch and Cantilever

An important event for the advancement of arch dam design technology was a conference that the Reclamation Service (previous name of the Bureau of Reclamation (Reclamation)), held in 1903, where George Y. Wisner called for a need to investigate the structural behavior of high masonry and concrete dams to maintain safety and achieve minimum costs. Considering the planned Pathfinder (Figure 2) and Shoshone (Buffalo Bill) Dam construction in Wyoming, the Reclamation Service appointed Wisner and E.T. Wheeler to develop a new stress analysis. They developed an Arch and Cantilever method assuming that the water load would be divided between arch and cantilever elements to produce equal arch and cantilever deflections in the radial direction at the crown cantilever. In the approach, the distribution of loads along the arch was assumed to be constant, and only one vertical element was considered at the crown section. In the design of Pathfinder Dam, Wisner and Wheeler demonstrated that the water load could be taken by a series of arches with compressive stress held under 200 psi for stone masonry whose compressive strength was well above 2,000 psi. Thermal load conditions used a similar analysis.

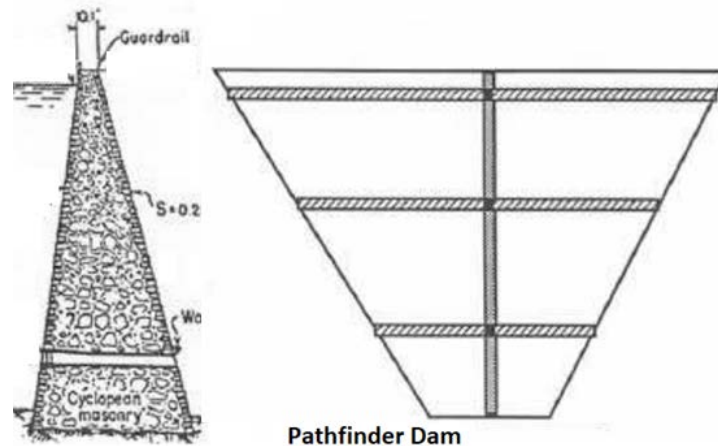


Figure 2. Cross-section of Pathfinder Dam (left) and concept of the Arch and Cantilever model used by Wisner and Wheeler in the analysis of the dam.

In the 1921 publication [5], Fred Noetzli made an overview of the practice of arch dams and applied his semi-graphical approach to Pathfinder Dam and concluded that his “distribution of load between cantilever and arches compared very favorably with that obtained analytically by Mr. Wheeler.” Noetzli’s paper was followed by eight major articles published by the American Society of Civil Engineers Transactions in 1921 and 1922. William Cain, a Mathematics Professor at the University of North Carolina, presented a significant article [10]. Cain developed complex mathematical derivation and equations complemented by designer-friendly tables. In the approach, a judgment by the designer was necessary to choose the rigid or hinged foundation, and abutments support conditions [6].

TRIAL-LOAD METHOD

Early Trial-Load Method Developments

The Trial-Load method is essentially an advancement of the Arch and Cantilever concept that has been intensively developed by Reclamation for over a half-century, starting in 1923 [1] [2]. The Trial-Load method distributes water load between the number of vertical cantilever and horizontal arch elements. The division of load does not need to be uniform, but the final division is one that results in equal deflections in radial, tangential and rotational directions for all points of intersection between the cantilever and arch elements (instead of an agreement between the arches and just the single crown cantilever in the Arch and Cantilever method).

The method is an iterative process such that loads are adjusted between all arch and cantilever intersections until the specified criterion is met, thus the name: “Trial Load.” Note that although the concept of the trial load method was used in the design of several dams for many years, the actual name “Trial-Load Method” was first publicly used by Howell and Jaquith in their 1929 paper [7]. Each arch and cantilever is assumed to move independently of all others, but at the conclusion of the analysis, each arch and cantilever element must remain in equilibrium and geometrical continuity at all intersection points. The sum of the arch and cantilever loads must equal the external load (weight, reservoir, tailwater, temperature, silt, and ice) at each intersection point. These external loads are divided between arches and cantilevers to get satisfactory radial deflection agreement. To complete the deformation agreement without changing the external loads, internal pairs of equal and opposite tangential and twist loads are applied to arches and cantilevers. Computations of deflections of arch elements are made by the elementary theory of flexure for curved beams with the effects of rib-shortening, transverse shear, and yielding abutments. The trial load method produces:

- cantilever stresses vertically along all cantilevers,
- arch stresses at multiple locations along all arches,
- principal stresses in the correct orientation not just horizontal along the arches,
- and displacements and rotations at all cantilever and arch intersections.

Advances of the Trial-Load Method at Reclamation

Trial-Load Method was computerized in the 1960s by Reclamation. The Arch Dam Stress Analysis System (ADSAS) computer program [12] greatly advanced the state-of-the-art arch dam analysis, sped up the design process, and helped justify the use of engineering mainframe computers. ADSAS changed the way the Concrete Dam Group at Reclamation operated because more load combinations and geometrical shapes could be investigated in minutes rather than weeks. Output from ADSAS was originally in a paper form and was about 1 to 2 inches thick. Designers would thumb through the large volume of the paper output, make some hand calculations, propose changes to the dam geometry, and have the younger engineers run ADSAS and bring back the paper output. Despite the advances that came with ADSAS, it was not appropriate for dynamic analysis. Also, the ADSAS program and user’s manual were developed for internal use; there was a machine-dependent computer code specifically for a Cyber 70-74/28, and the program was more than 39,000

cards long with over 240 subroutines. This caused problems for others while converting the program to their computers, as well as using the program.

The computerization of the Trial-Load method greatly helped in the development of doubly curved arch dams because many analyses could be quickly performed. In the 1950s before ADSAS, one complete trial load analysis took 6 to 12 months for one senior to do the calculations and one junior engineer to carry out a simultaneous check with. With the speed of ADSAS, the final arch shape for Morrow Point Dam (Figure 3) was layout number 60 with maximum tension less than 100 psi, maximum compression less than 1000 psi, and the upstream face in compression for gravity, reservoir, silt, and winter temperatures.

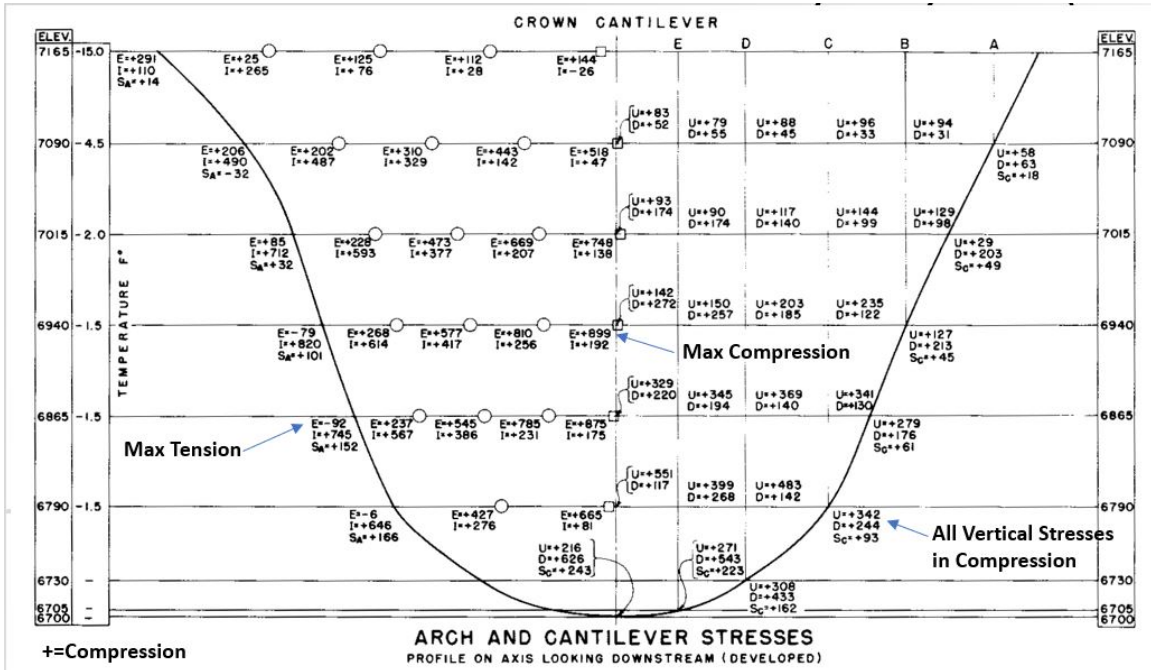


Figure 3. ADSAS output displays arch and cantilever stresses for a thin arch dam at the upstream and downstream faces.

VALIDATION OF THE ANALYSIS METHODS

Many of the early constructed arch dams lacked a rational method of design. The designers used mostly their engineering intuition, experience, or simplified analysis formulas in developing a dam layout. Some of the dams were very thick, and others very thin. The thick dams were inefficient with unnecessary volume of material and some very thin dams encroached on safety limits. But none of them have failed for reasons related to structural weakness. During the 1920's it was recognized that determination of stresses in arch dams is a complex problem and verification and validation of the analysis method was a necessity. Validation of the accuracy of the analysis methods used in the design of arch dams was a primary goal of the engineering community.

Experimental Dam

A Committee on Arch Dams Investigation of the American Societies of Civil, Mining and Metallurgical Mechanical and Electrical Engineers, with the financial support of the New York-based Engineering Foundation, built a large-scale concrete arch dam, expressly for research purposes. A 60-ft high Stevenson Creek test dam was located on Stevenson Creek, about 60 miles east of Fresno, and was tested in spring of 1926 (Figure 4). The dam thickness was 24 inches for the top 30 feet, and it tapered to 7.5 feet at the bottom. The primary goal of the test was to determine the distribution of loads in the structure for thermal and hydrostatic load conditions. The dam was equipped with numerous instruments, some of the special design, to obtain accurate information on strain and deformation.

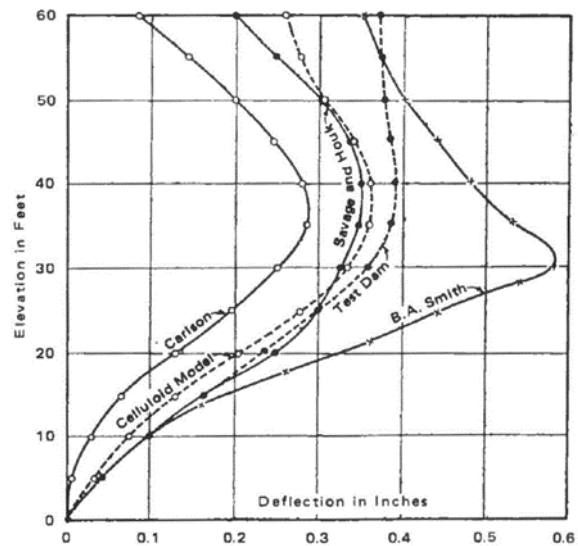
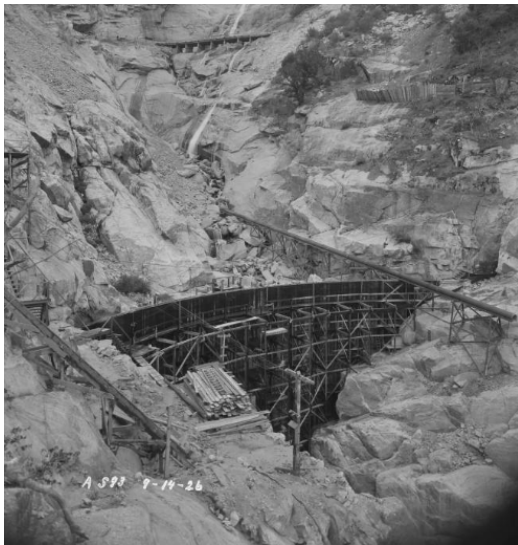


Figure 4. Stevenson Creek Test Dam (left) and comparison of test data with Trial-Load method analysis results (right) [9].

The effort was made to compute the loads carried by arches and cantilevers, stresses, moment, and shear forces by differentiation of deflection curves. A comparison between the measurements on the Stevenson Creek Dam, laboratory models, and the trial load analysis results by Savage and Houk was in a good agreement (Figure 4). The Arch and Cantilever method used by Smith and Carlson produced larger variations in displacement results (Figure 4).

Laboratory Model Testing

Following up the Stevenson Creek Dam testing, Reclamation conducted a series of experiments on a one-twelfth scale physical model of Stevenson Creek Dam and on a 1 to 68 scale Gibson Dam model (Figure 5). The tests were completed at the University of Colorado at Boulder in 1928. Concrete mixes for the models were made from the same aggregate as in the dams, and reservoir loads were simulated using mercury.

The behavior of both models was in good agreement with the measured deflections of the prototypes and with those calculated by the trial load method. The outcome of these investigations demonstrated that the trial load method of analysis produces accurate results

for both thin and thick arch dams and proved satisfactory adequacy for analyzing arch dams. Several refinements were introduced to the trial load method, including the effects of tangential shear and twist, as a result of the model testing.

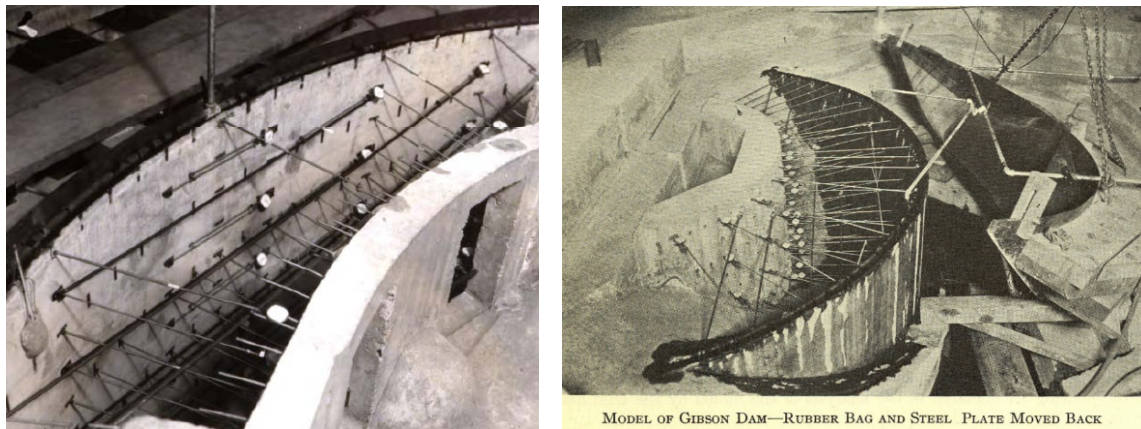


Figure 5. Laboratory models of Stevenson Creek Dam (left) and Gibson Dam (right).

Monitoring of Dams

Parallel to testing activities at Stevenson Creek Dam, a comprehensive field investigation program was implemented at several dams. Measurements of strains, deflection, temperatures changes were conducted at: Clear Creek Dam, Gerber Dam, Emigrant Creek Dam and Dam No. 6 of Big Creek Development and then later at Gibson Dam, Bull Run Storage Dam, and Coolidge Dam were taken during construction and operation of these dams [8]. This complementary field data was in good agreement with the results of Stevenson Creek Dam and laboratory testing results; this greatly contributed to better engineering understanding of arch dam behavior. Also, interpretation of some measured field data was easier to understand by the engineers by using the verified analytical tool. An interesting fact was noted, and it was related to the measurement data obtained from Emigrant Creek Dam in 1924 when an arch deflection at the crest was determined in the upstream direction as the reservoir level rose. The initially difficult to understand dam behavior was verified by the analysis using Trial-Load method and confirmed by the laboratory tests.

FROM STATE-OF-THE ART TO STATE-OF-THE PRACTICE

Design Criteria

The Trial Load method and validation programs allowed Reclamation to establish an analytical approach and the criteria for design of arch dams based on the factor of safety and permissible stresses. The criteria that Reclamation used in the past to successfully design 27 arch dams are summarized in the Engineering Monograph 19 [13]. As such, the following Reclamation design criteria have been adopted by other US federal agencies and have become a standard design practice for analysis, design, and construction of arch dams in the USA.

In the design of arch dams, three load categories are considered: Usual (normal operating conditions), Unusual (maximum design flood), and Extreme (any usual load combination

plus the maximum credible earthquake). Loading combinations included dead load due to gravity, reservoir, tailwater, silt, ice, and temperatures. Uplift was not considered in the analysis of thin arch dams because of its small effect (reported 5% change in allowable stress), but as safeguard to address any potential influence of uplift, drains were constructed in the foundation and dam body. In the 1960s, seismic loads were considered as pseudo-static loads or response spectra analysis as time history analysis was just being developed.

Typical concrete material properties used in the initial design were assumed to be: density of 150 pcf; of 3000 to 5000 psi (f'_c); tensile strength of 5 to 6% of compressive strength; cohesion 10% of compressive strength; 45-degree friction angle; 0.2 Poisson's ratio; 5,000,000 psi instantaneous modulus; 3,000,000 psi sustained modulus; and 0.000005 in/in/°F coefficient of thermal expansion.

Factors of safety in the dam (and foundation) were developed as 3.0 (4.0) for usual, 2.0 (2.7) for unusual, and 1.0 (1.3) for extreme. Allowable stresses were obtained by applying these factors of safety to the design material strength properties. However, the compressive stresses could not exceed 1,500 psi for usual loads, 2,250 psi for unusual loads. The tensile stress could not exceed 150 psi for usual loads and 225 psi for unusual loads. For seismic loads, cracking was assumed to occur when the tensile strengths were exceeded.

The maximum degree of horizontal curvature at the crest was defined by a vertical reference plane having a circular radius equal to approximately 60% of the chord length at the dam crest elevation across the valley with a central angle between 90° and 110° degrees [3]. The angle of incidence of a tangent to the intrados and competent rock was not less than 30°. The arches had abutments radial to the axis center. Post-seismic criteria called for reanalysis considering the depth of cracking and considering the effects of both horizontal and opening of vertical contraction joints. The Trial Load method was capable of performing analyses with cracked cantilevers.

Design Practice

Merlin Copen in 1963 [11] reported that arch dams were being designed by 1) small scale models, 2) thin cylinder theory, 3) relaxation methods, 4) shell theory, and 5) trial-load analysis. Reclamation considered the trial-load method to be completely satisfactory and unexcelled in the design of arch dams until the 1970s.

POST TRIAL LOAD METHOD ANALYSIS TIME

In 1969, the FE method was introduced to Reclamation when Glenn Tarbox and others attended a 3-week lecture and laboratory series at the University of California at Berkeley. They returned with source codes for uniaxial heat flow and flow in porous media, and 2- and 3-dimensional SAP structural FE software.

In 1973, Nambe Falls Dam was designed using the Trial Load method. Tarbox used SAP for the first time at Reclamation to analyze a critical aspect of the dam: the composite arch dam/thrust block structure. The structure included a novel Freyssinet flat jack system to prestress the dam.

Expanded computing power in the 1970s allowed the implementation of the finite element method in dam engineering. In 1977, Reclamation used the finite element method for the first time in the design of Auburn Dam [14]. Trial Load was used to carry out numerous preliminary designs. SAPIV was used for the final static and dynamic analyses and incorporated the foundation. SAPIV results were compared to ADSAS results and validated with data obtained from laboratory model tests at the Laboratório Nacional de Engenharia Civil (LNEC) in Lisbon. The results for both linear analysis and laboratory tests showed relatively good agreement for the static loads. As noted, there were some differences in stresses at the base of the dam (Figure 6).

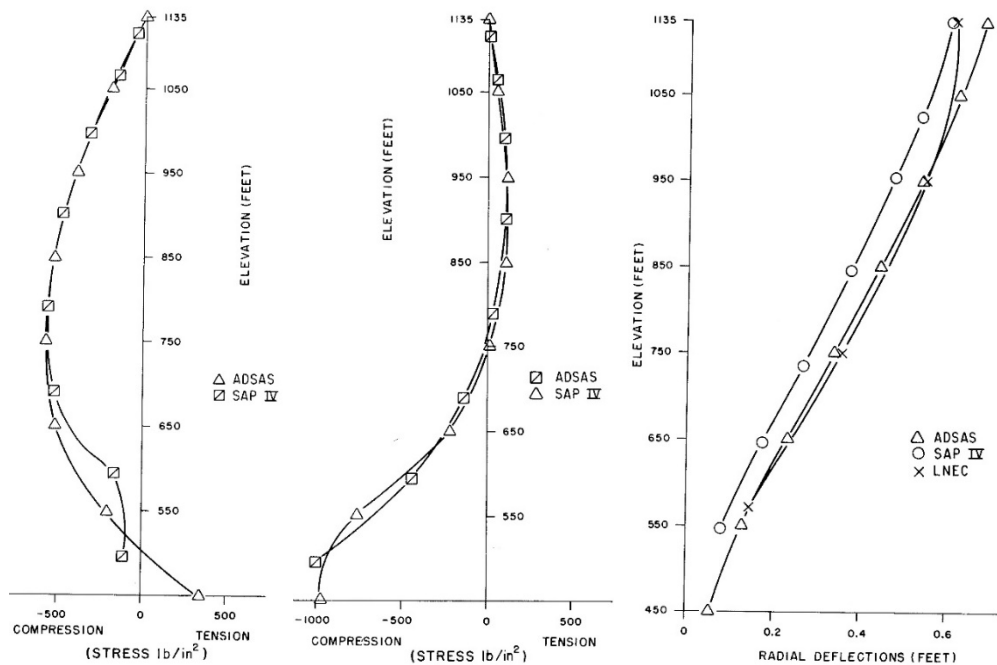


Figure 6. Comparison of SAPIV and ADSAS results with laboratory data for Auburn Dam (crown cantilever stresses: d/s face (left), u/s face (middle) and cantilever deflections (right) [13].

Over the last forty years, the numerical methods have further developed. Complex linear and non-linear analyses in frequency and time domains are now conducted in structural assessments of existing and new concrete dams. Complexity and advancement of the analyzed engineering and mathematical problems can introduce high levels of uncertainty of the obtained results. Several issues related to the accuracy of the advanced structural analyses were pointed out and discussed during the USSD workshops and the ICOLD Benchmark Workshop studies over the last several years. The outcome of all these investigations shows an urgent need for verifications and validations of these advanced computation methods in the analyses of concrete dams, in a similar way that the Trial Load Method was validated in the 1920s and 1930s before it became a part of the engineering practice.

FINAL REMARKS AND CONCLUSIONS

The approach initiated in 1920s in the structural analysis of arch dams using the Trial-Load method was verified with other analytical methods and validated by tests conducted on an experimental dam and laboratory models as well as long term observations of dam performance and monitoring. This comprehensive approach provided confidence to the designer and has been successfully implemented at several arch dam projects for half of the 20th century until the FE method was introduced and became a common computation tool in the engineering practice since the 1970s.

The advanced structural analysis models, currently used in the structural assessment of existing dams and design of new dams these days, require evaluation of efficiency, reliability, and validity. Engineering judgments and proper interpretation of the results is an important component in the structural assessment of concrete dams.

ACKNOWLEDGMENTS

This paper is dedicated to the memory of three great arch dam designers at the Bureau of Reclamation: Merlin Copen (Glen Canyon and Flaming Gorge Dams), Howard Boggs (Morrow Point and East Canyon Dams), and David A. Dollar (Portuguese Dam).

REFERENCES

- [1] U.S. Bureau of Reclamation, *Trial Load Method of Analyzing Arch Dams*, Boulder Canyon Project, Final Report Part V – Technical Investigations, Bulletin 1, Denver, Colorado 1938.
- [2] U.S. Bureau of Reclamation, *Design of Arch Dams*, Design Manual for Concrete Arch Dams, Denver, Colorado 1977.
- [3] U.S. Bureau of Reclamation, *Guide for Preliminary Design of Arch Dams*, Engineering Monograph No. 36, Denver, Colorado 1977.
- [4] Kollgaard E.B., Chadwick W.L., *Development of Dam Engineering in the United States*, The United States Committee of The International Commission on Large Dams, 1988.
- [5] Noetzli F.A., *Gravity and Arch Action in Curved Dams*, American Society of Civil Engineers, Transactions, Paper No. 1463, 1921.
- [6] Jorgensen L.R., *The Constant-Angle Arch Dams*, American Society of Civil Engineers, Transactions, Paper No. 1322, 1914.
- [7] Howell C.H., *Analysis of Arch Dams by the Trial Load Method*, American Society of Civil Engineers, Transactions, Vol. 93, Paper No. 1712, 1929.
- [8] Savage J.L., Houk I.E., Gilkey H.J., Vogt F., *Test of Models of Arch Dams and Auxiliary Concrete Tests Conducted y the Bureau of Reclamation at the University of Colorado*, Engineering Foundation, Committee on Arch Dam Investigation, 1931.
- [9] Slater W.A., *Report of Test on Stevenson Creek Dam*, Bureau of Standards, 1927.

- [10] William Cain, *The Circular Arch Under Normal Loads*, American Society of Civil Engineers, Transactions, Vol. LXXXV, Paper No. 1483, pp. 223, 1922.
- [11] Merlin Copen, “*Design of Arch Dams by Trial Load Method of Analysis*,” American Society of Civil Engineers, Power Division, August 1960.
- [12] ADSAS Arch Dam Stress Analysis System, *User’s Manual*, Bureau of Reclamation, Denver, Colorado.
- [13] U.S. Bureau of Reclamation, *Design Criteria for Concrete Arch and Gravity Dams*, Engineering Manual No.19, Denver, Colorado 1977.
- [14] U.S. Bureau of Reclamation, “*Design and Analysis of Auburn Dam*,” Design Report, Denver, 1977.