GEOMECHANICS

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FORMULAE – LECTURE 5

Hardening Elasto-Plasticity

Basic ingredients:

1. Strain decomposition $$\delta\varepsilon_{ij} = \delta\varepsilon_{ij}^{e} + \delta\varepsilon_{ij}^{p}$$
2. Elastic relationship $$\sigma'_{ij} = D^{e}_{ijhk}\varepsilon^{e}_{hk}$$
3. Yield function $$F = F(\sigma_{ij}, p_k) \quad p_k = p_k(\varepsilon_{ij}^{p}) \longrightarrow F = F(\sigma_{ij}, \varepsilon_{ij}^{p})$$
4. Plastic potential function $$g(\sigma_{ij}, r_k) \qquad \delta\varepsilon_{ij}^{p} = \mu\frac{\partial g}{\partial \sigma_{ij}}$$

Consistency condition
$$dF = \left.\frac{\partial F}{\partial \sigma_i}\right|_{p_k}\delta\sigma_i + \left.\frac{\partial F}{\partial p_k}\right|_{\sigma_i}\frac{\partial p_k}{\partial \varepsilon_j^{p}}\delta\varepsilon_i^{p} =$$
$$= \left.\frac{\partial F}{\partial \sigma_i}\right|_{p_k}\delta\sigma_i + \mu\left.\frac{\partial F}{\partial p_k}\right|_{\sigma_i}\frac{\partial p_k}{\partial \varepsilon_j^{p}}\frac{\partial g}{\partial \sigma_j} = 0$$

Plastic multiplier
$$\mu = -\frac{1}{\frac{\delta F}{\delta p_k}\frac{\delta p_k}{\delta \varepsilon_j^{p}}\frac{\delta g}{\delta \sigma_j}}\frac{\partial F}{\partial \sigma_i}\delta\sigma_i = \frac{1}{H}\frac{\partial F}{\partial \sigma_i}\delta\sigma_i$$

Hardening modulus
$$H = -\frac{\delta F}{\delta p_k}\frac{\delta p_k}{\delta \varepsilon_j^{p}}\frac{\delta g}{\delta \sigma_j}$$

General incremental stress-strain relationship for plastic strain isotropic hardening elasto-plasticity
$$\delta\sigma_i = \underbrace{\left(D_{ij}^{e} - \frac{D_{ik}^{e}\frac{\partial g}{\partial \sigma_l}\frac{\partial F}{\partial \sigma_k}D_{kl}^{e}}{H + \frac{\partial F}{\partial \sigma_i}D_{ij}^{e}\frac{\partial g}{\partial \sigma_j}}\right)}_{D_{ij}^{ep}}\delta\varepsilon_j$$

Extended Mohr-Coulomb model

Elastic component
$$\begin{bmatrix}\delta\varepsilon_v^e\\ \delta\varepsilon_d^e\end{bmatrix} = \begin{bmatrix}1/K & 0\\ 0 & 1/3G\end{bmatrix}\begin{bmatrix}\delta p'\\ \delta q\end{bmatrix}$$

Yield function
$$F = F(\sigma_{ij},\, p_k) = q - \eta_y p'$$

Plastic potential
$$g = q - Mp'\, ln\frac{p_r'}{p'} = 0$$

Volumetric plastic strain
$$\delta\varepsilon_v^p = \mu\frac{\partial g}{\partial p'} = \mu(M - \eta_y)$$

Deviatoric plastic strain
$$\delta\varepsilon_d^p = \mu\frac{\partial g}{\partial q} = \mu$$

Flow rule
$$\frac{\delta\varepsilon_v^p}{\delta\varepsilon_d^p} = M - \frac{q}{p'} = M - \eta_y$$

Hardening rule
$$\frac{\eta_y}{\eta_p} = \frac{\varepsilon_d^p}{a+\varepsilon_d^p}$$

$$\begin{pmatrix}\partial\eta_y/\partial\varepsilon_v^p\\ \partial\eta_y/\partial\varepsilon_d^p\end{pmatrix} = \begin{pmatrix}0\\ (\eta_p-\eta_y)^2/a\eta_p\end{pmatrix}$$