GEOMECHANICS

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FORMULAE – LECTURE 4

Perfect plasticity model – Basic ingredients

1. Strain decomposition $\delta\varepsilon_{ij} = \delta\varepsilon_{ij}^{e} + \delta\varepsilon_{ij}^{p}$
2. Elastic relationship $\sigma'_{ij} = D^{e}_{ijhk}\varepsilon^{e}_{hk}$
3. Yield function $F = F(\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23}, p_k)$
4. Plastic potential function $g(\sigma_{ij}, r_k)$ $\delta\varepsilon_{ij}^{p} = \mu\dfrac{\partial g}{\partial \sigma_{ij}}$

Von Mises

$$F = J_{2D} - k^2$$

Drucker-Prager

$$F = \sqrt{J_{2D}} - \alpha \cdot J_1 - k \qquad \alpha = \frac{2\cdot\sin\phi}{\sqrt{3}\cdot(3-\sin\phi)} \qquad k = \frac{6\cdot c\cdot\cos\phi}{\sqrt{3}\cdot(3-\sin\phi)}$$

Mohr-Coulomb

$$\tau_f = c' + \sigma_n{}' \tan\varphi'$$

$$F = -\frac{\sigma_1 - \sigma_3}{2} + \frac{\sigma_1 + \sigma_3}{2}\cdot\sin\phi + c\cdot\cos\phi = o$$

$$F = J_1\cdot\sin\phi + \sqrt{J_{2D}}\cdot\cos\theta - \frac{\sqrt{J_{2D}}}{3}\cdot\sin\phi\cdot\sin\theta - c\cdot\cos\phi = 0$$

$$\theta = -\frac{1}{3}\cdot\sin^{-1}\left(-\frac{3\cdot\sqrt{3}}{2}\cdot\frac{J_{3D}}{J_{2D}^{3/2}}\right)$$

$$q_f = \frac{6\sin\varphi'}{3-\sin\varphi'}p'_f + \frac{6\cos\varphi'}{3-\sin\varphi'}c' = Mp'_f + a'$$

$$q_f = -\frac{6\sin\varphi'}{3+\sin\varphi'}p'_f - \frac{6\cos\varphi'}{3+\sin\varphi'}c' = M_{ext}p'_f + a'_{ext}$$

Mohr-Coulomb elasto-perfectly plastic model

1. Strain decomposition $\delta\varepsilon_{ij} = \delta\varepsilon_{ij}^{e} + \delta\varepsilon_{ij}^{p}$
2. Elastic relationship $\begin{pmatrix} \delta p' \\ \delta q \end{pmatrix} = \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{pmatrix} \delta\varepsilon_{vol}^{e} \\ \delta\varepsilon_{q}^{e} \end{pmatrix}$
3. Yield function $F(p', q, p_k) = q - Mp'$
4. Plastic potential function $g(p', q, r_k) = q - M^{*}p' + k$

$$\begin{pmatrix} \delta p' \\ \delta q \end{pmatrix} = \frac{3GK}{MKM^{*} + 3G} \begin{bmatrix} 1 & M^{*} \\ M & MM^{*} \end{bmatrix} \begin{pmatrix} \delta\varepsilon_{v} \\ \delta\varepsilon_{d} \end{pmatrix}$$