



October 16, 2025

Introduction to on-demand and shared mobility



Our lab

Members

Head



Postdoc



PhD



Visiting



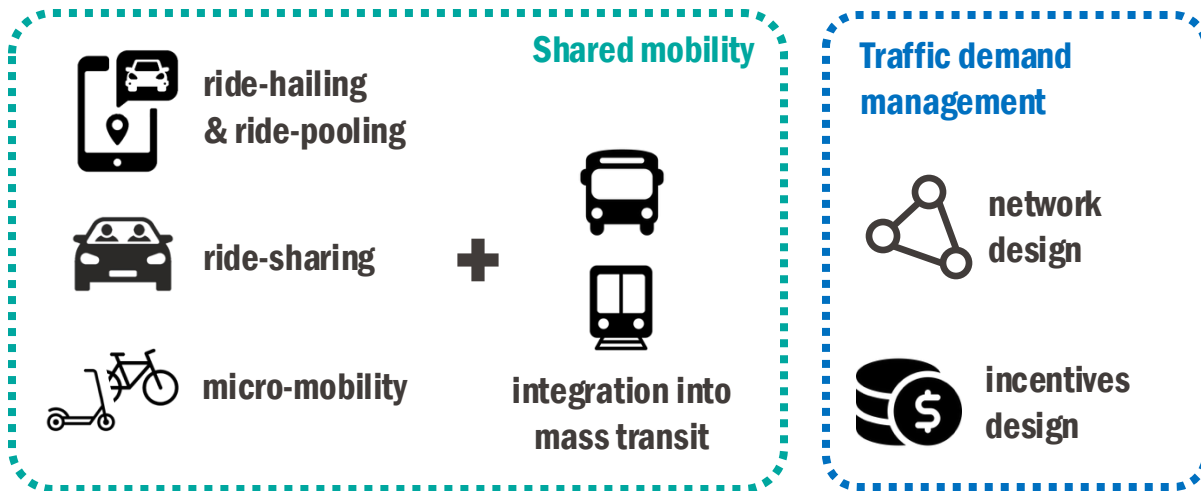
Lab for human-oriented mobility eco-system

At HOMES, we develop human-centric solutions to emerging mobility challenges.





Topics



Methodologies



network modeling



game theory



$\min F(x)$ optimization



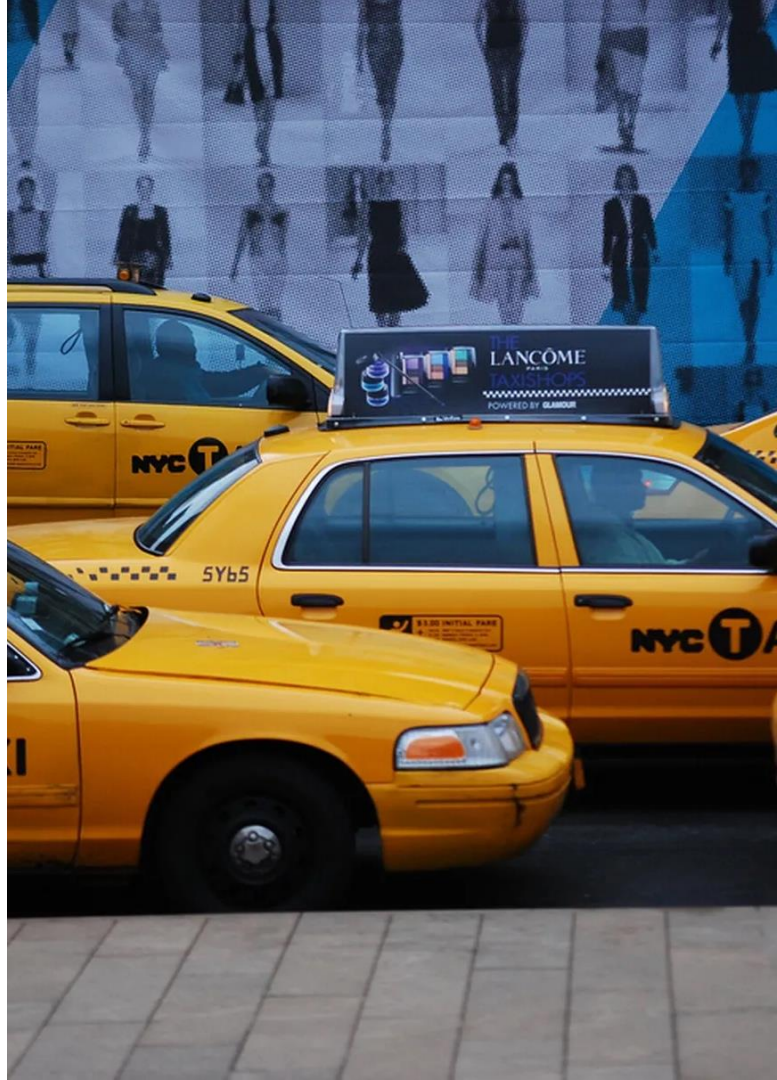
data-driven & learning

- Courses
 - CIVIL 324 (BA6) Urban public transport systems
 - transit system and on-demand service design
 - CIVIL 477 (MA2) Transportation network modeling & analysis
 - traffic assignment and network equilibrium

- Student projects
 - Modeling
 - Route choice, accessibility, shared-mobility
 - Data-driven analyses and machine learning
 - Individual and vehicle trajectories, meal delivery orders, environmental sensing data, transit operation data
 - Simulation
 - On-demand mobility, day-to-day commute



HOMES @ EPFL



Agenda

- Basics
- Matching
- Operations
- Regulations

What is on-demand and shared mobility

- “Shared” means...
 - trips are served by a dedicated fleet of vehicles
 - you share the vehicle with someone, but not necessarily in the same trip



“sharing” in a narrow sense



“sharing” in a broad sense

What is on-demand and shared mobility

- Putting “on-demand” and “shared” together...

ride-hailing

- taxi
- e-hailing
- ride-pooling
- micro-transit



micromobility

- bike-sharing
- scooter-sharing

ride-sharing

- carpooling



(on-demand) car-sharing

Who are the stakeholders?



REGULATOR



OPERATOR



RIDER



DRIVER

Who are the stakeholders?



REGULATOR

Pursuits

- get to my destination asap
- price is reasonable
- trip is comfortable
-



OPERATOR

Costs

- trip fare
- waiting and in-vehicle time
- detours in shared trips
-



RIDER



DRIVER

Who are the stakeholders?



REGULATOR

Pursuits

- high hourly earning
- short search time
- equal dispatch opportunity
-



OPERATOR

Costs

- operation cost
- other job opportunity
- fatigue after long work hours
-



RIDER



DRIVER

Who are the stakeholders?



REGULATOR

Pursuits

- high profit
- dispatch efficiency
- large market share
- reliable supply
-



OPERATOR

Costs

- payment to drivers
- incentives to passengers
- operation cost
-



RIDER



DRIVER

Who are the stakeholders?



REGULATOR

Pursuits

- max social welfare
- min traffic congestion
- equity and accessibility
-



OPERATOR

Instruments

- subsidy and tax
- regulation and incentive mechanism
-

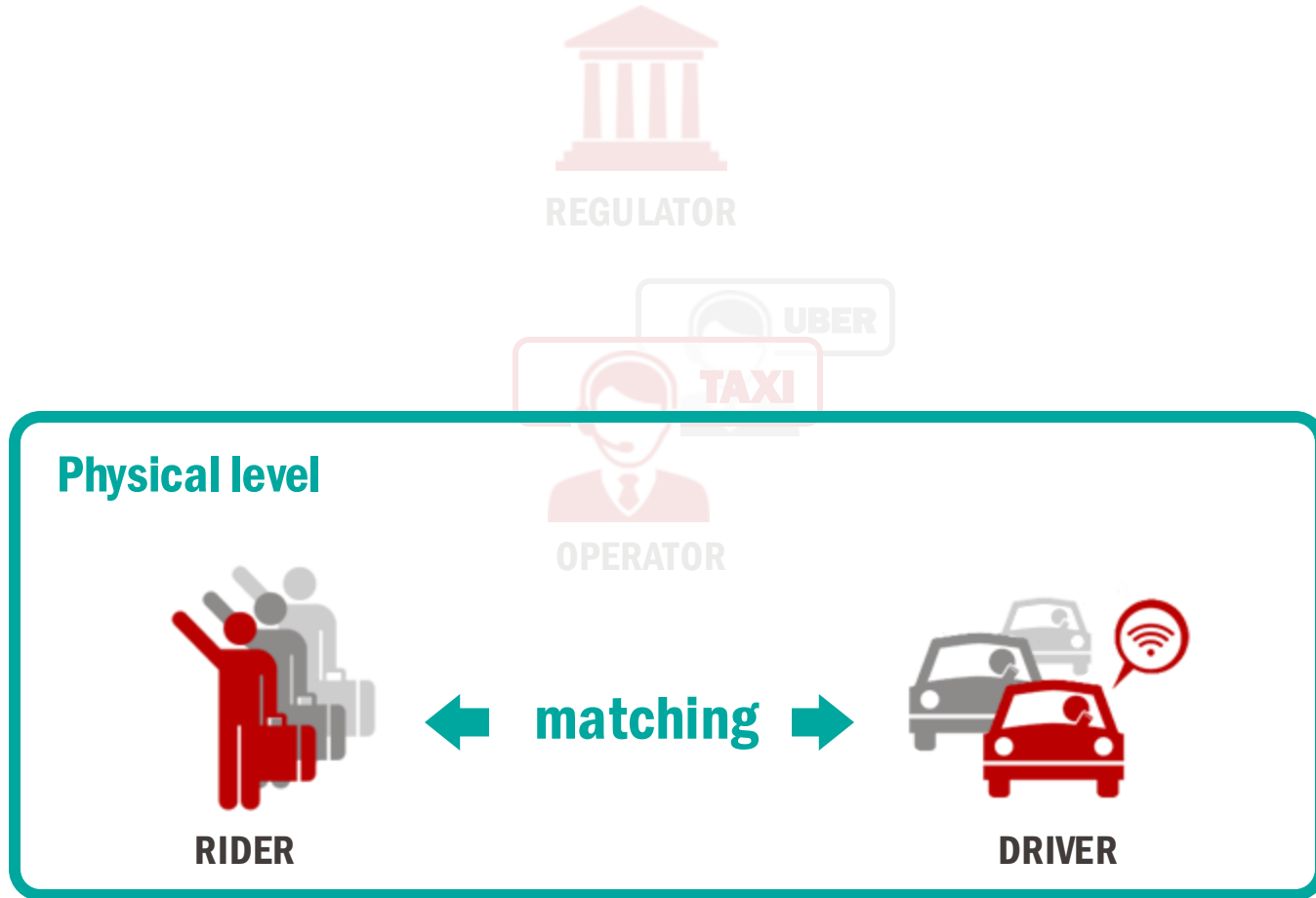


RIDER



DRIVER

What problems do we study?



What problems do we study?



street-hailing



e-hailing

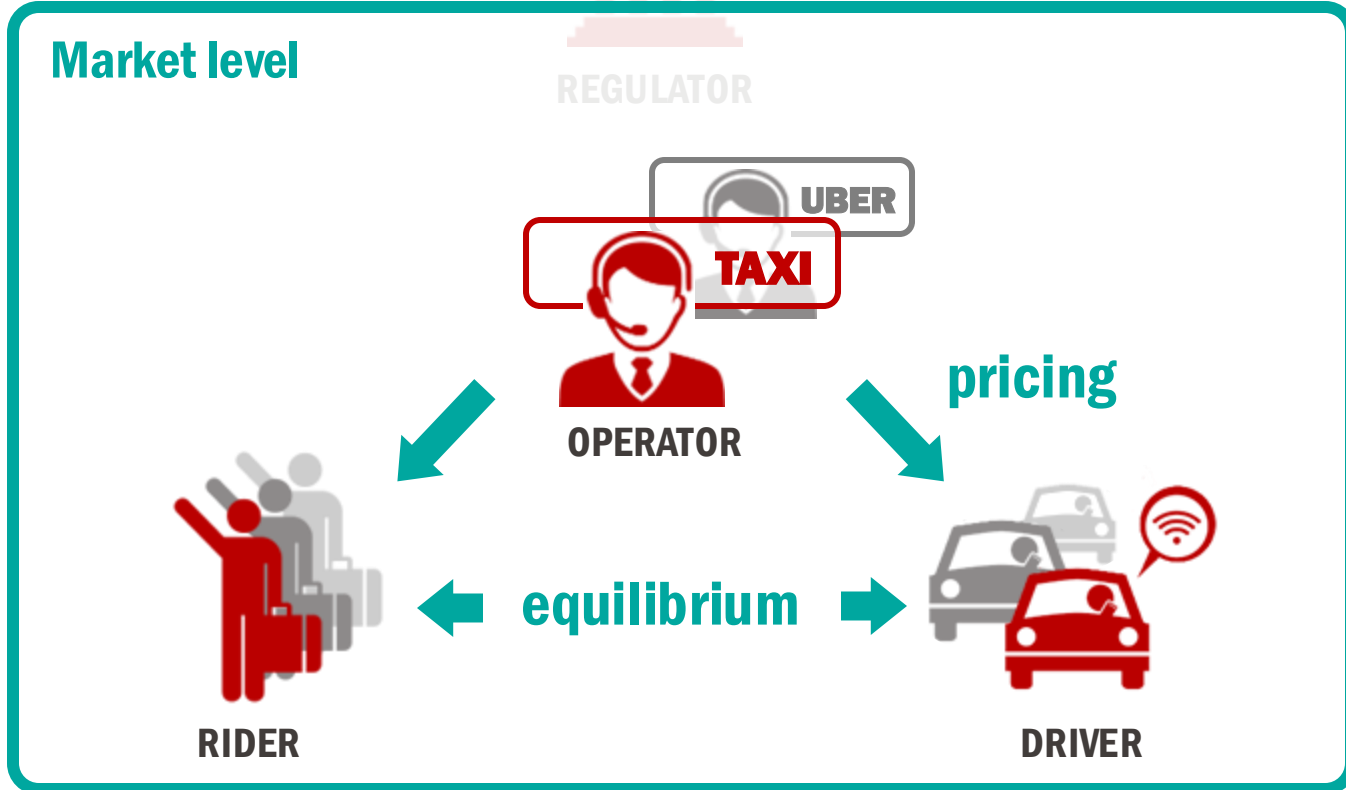


taxi stand



Ride-sharing with meeting point

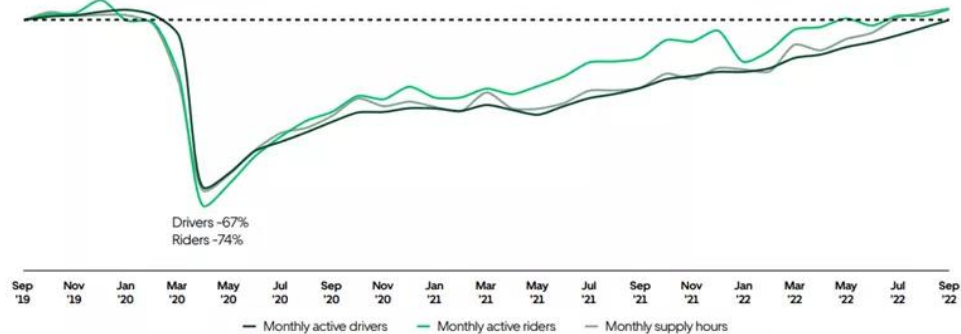
What problems do we study?



What problems do we study?

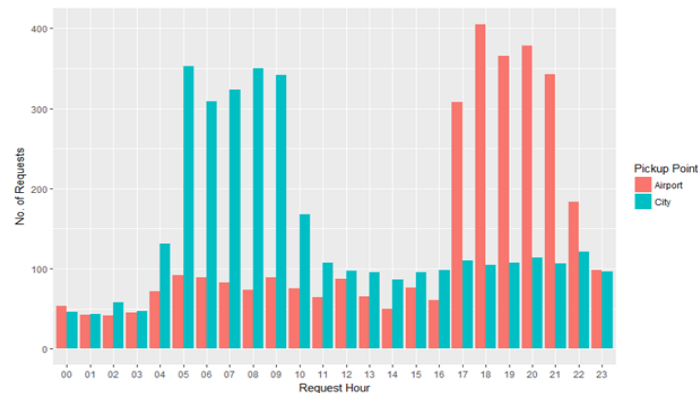
Active riders and drivers

Monthly active drivers at par with 2019 levels, and increasingly active on the platform; Monthly active riders now exceed 2019 levels



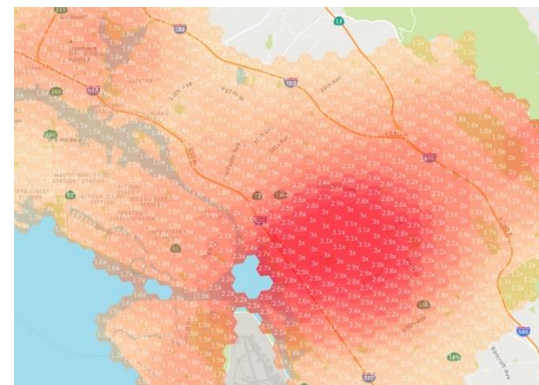
Source: Uber

Temporal demand pattern



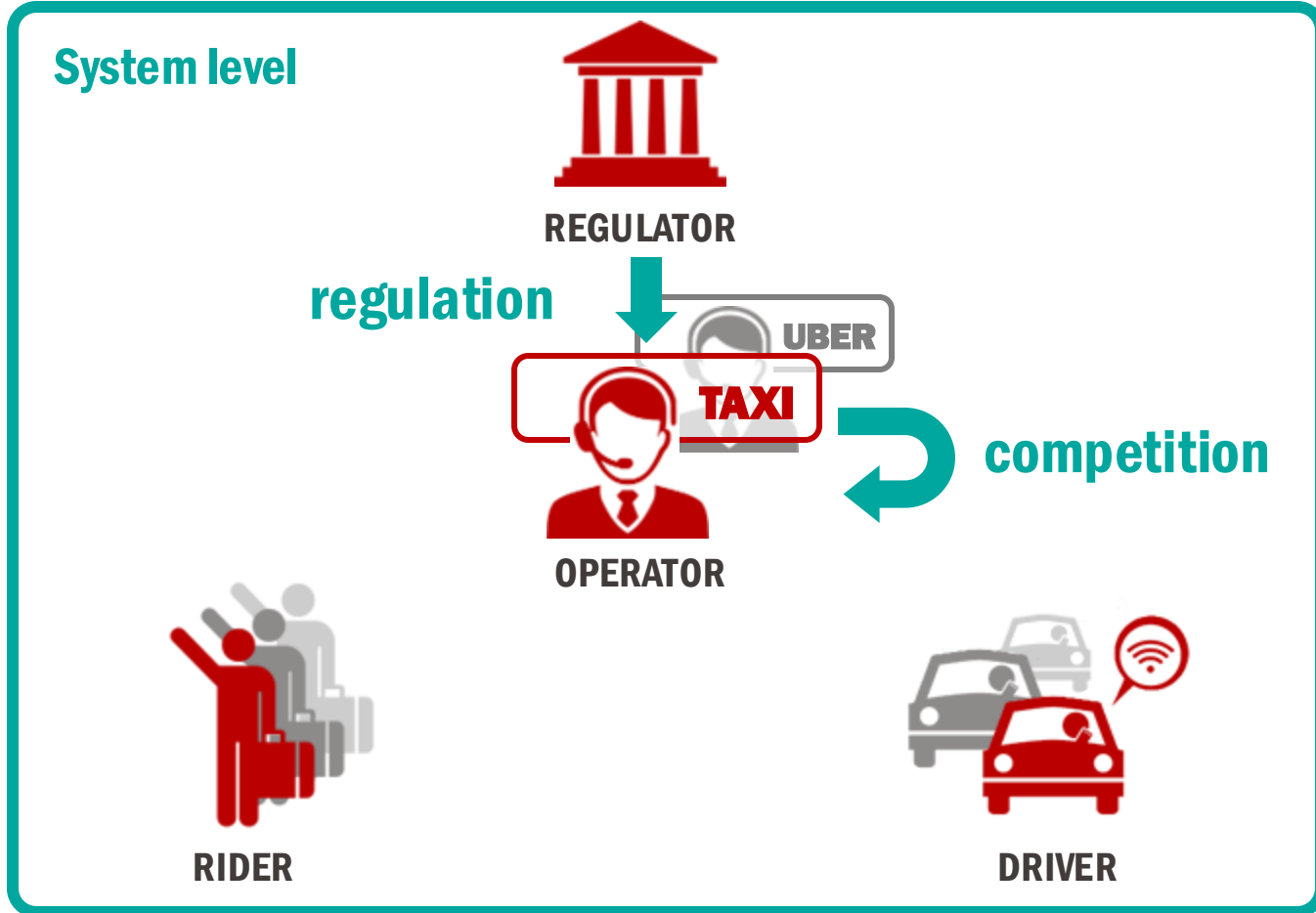
Source: Medium blog

Peak-hour surge pricing

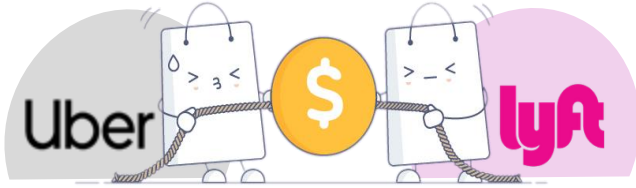


Source: Uber

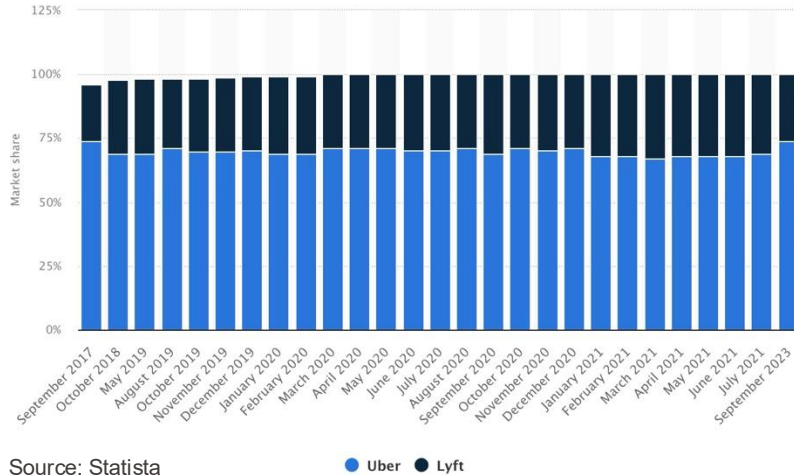
What problems do we study?



What problems do we study?



U.S. market share



Source: Statista

● Uber ● Lyft

Multi-homing drivers



Driver protest for higher payment

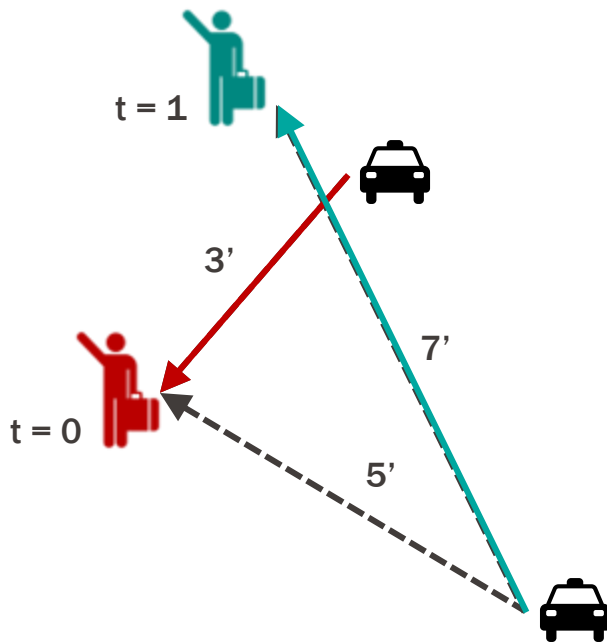


Questions?

Next topic: Matching

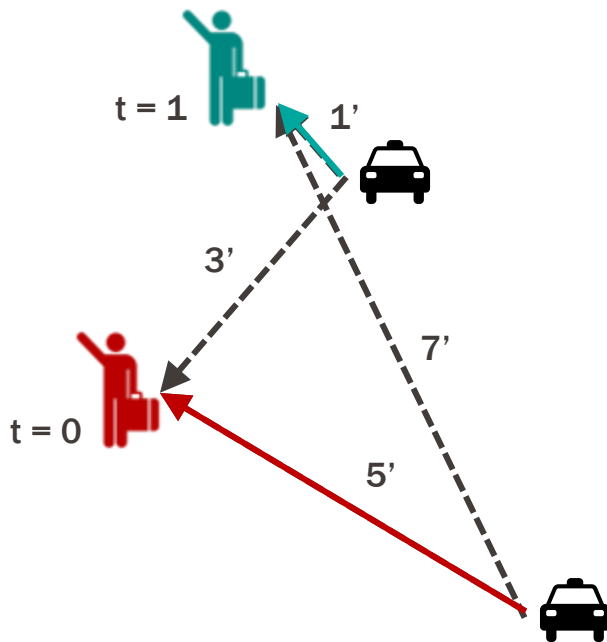
Matching problem

- How to dispatch drivers to pick up riders
 - Policy I: Instant matching
 - riders are first-come-first-served and matched to the closest driver



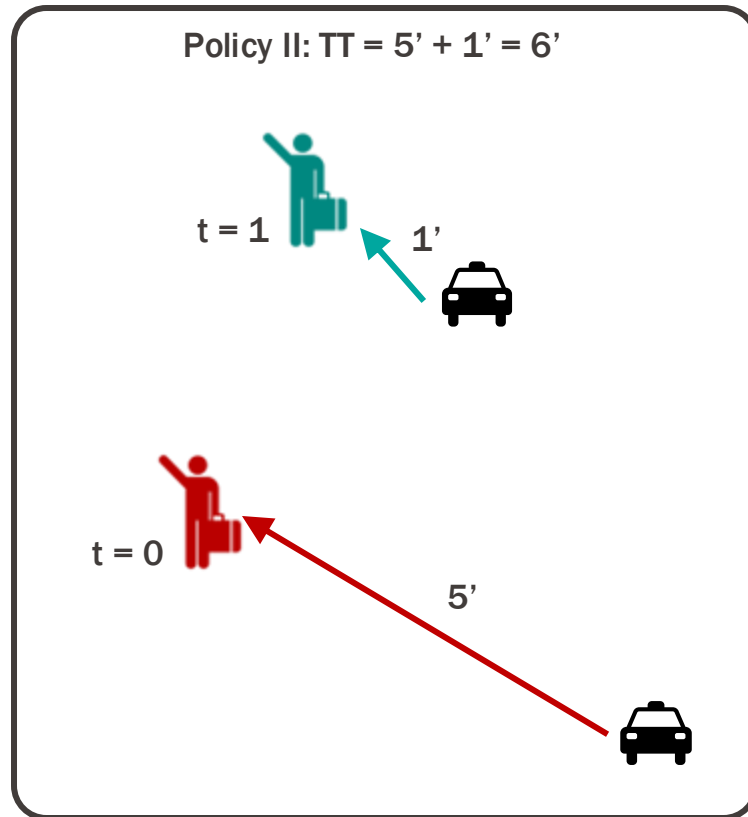
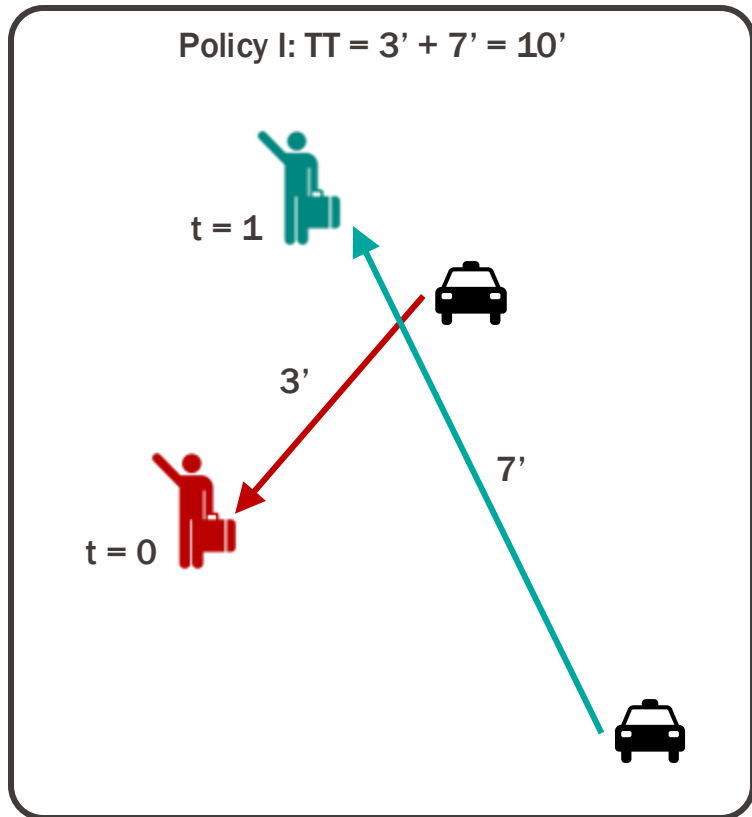
Matching problem

- How to dispatch drivers to pick up riders
 - Policy II: Batch matching
 - consolidate requests over a matching interval into a batch
 - match riders and drivers to minimize the total pickup time



Matching problem

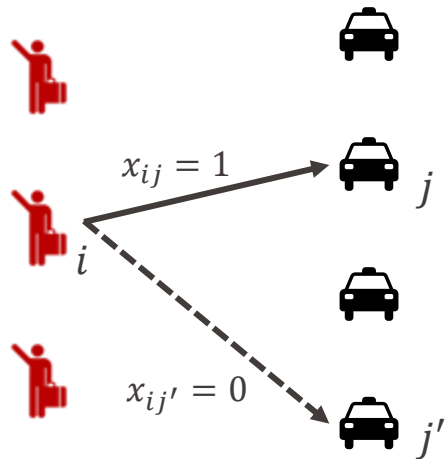
- Advantage of batch matching



Matching problem

- Mathematical formulation of batch matching

- Set of riders: $I = \{1, \dots, N\}$
- Set of drivers: $J = \{1, \dots, M\}, M \geq N$
- Matching indicator: $x_{ij} \in \{0,1\}, i \in I, j \in J$
- Pickup time: $t_{ij} \in R_+, i \in I, j \in J$



Matching problem

- Mathematical formulation of batch matching

- Set of riders: $I = \{1, \dots, N\}$
- Set of drivers: $J = \{1, \dots, M\}, M \geq N$
- Matching indicator: $x_{ij} \in \{0,1\}, i \in I, j \in J$
- Pickup time: $t_{ij} \in R_+, i \in I, j \in J$

$$\min_x \sum_{ij} t_{ij} x_{ij}$$

Objective: min total pickup time

$$s. t. \quad \sum_j x_{ij} = 1,$$

Constraint: all riders are matched

$$\sum_i x_{ij} \leq 1,$$

Constraint: each driver at most serves one rider

$$x_{ij} \in \{0,1\}.$$

Feasibility: binary matching decision

Matching problem

- Mathematical formulation of batch matching

- Set of riders: $I = \{1, \dots, N\}$
- Set of drivers: $J = \{1, \dots, M\}, M \geq N$
- Matching indicator: $x_{ij} \in \{0,1\}, i \in I, j \in J$
- Pickup time: $t_{ij} \in R_+, i \in I, j \in J$

$$\begin{aligned} \min_x \quad & \sum_{ij} t_{ij} x_{ij} \\ \text{s. t.} \quad & \sum_j x_{ij} = 1, \\ & \sum_i x_{ij} \leq 1, \\ & x_{ij} \in \{0,1\}. \end{aligned}$$

linear assignment problem

- a fundamental combinatorial optimization problem
- small instances are easily solved by linear program
- other algorithms have been developed for large instances

Matching problem

- Mathematical formulation of batch matching

- Set of riders: $I = \{1, \dots, N\}$
- Set of drivers: $J = \{1, \dots, M\}, M \geq N$
- Matching indicator: $x_{ij} \in \{0,1\}, i \in I, j \in J$
- Pickup time: $t_{ij} \in R_+, i \in I, j \in J$

$$\begin{aligned} \min_x \quad & \sum_{ij} t_{ij} x_{ij} \\ \text{s. t.} \quad & \sum_j x_{ij} = 1, \\ & \sum_i x_{ij} \leq 1, \\ & x_{ij} \in \{0,1\}. \end{aligned}$$

Extensions

- hold some riders for future match
- ride-pooling, i.e., match one driver to multiple riders
- ensure fairness among drivers in a long term

* Research questions!

Matching model

- Solving the matching problem is good, but ...
 - We need a model to capture the main trade-off in matching
e.g., when # riders remains the same, increasing # drivers can reduce total pickup time

$$w = f(\Pi, \Lambda)$$

matching outputs

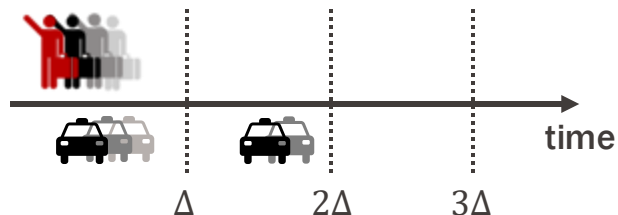
matching inputs

where w : average waiting time (s)
 Π : density of waiting riders (#/km²)
 Λ : density of idle drivers(#/km²)

Matching model

- Model I: Frictionless batch matching

- Matching interval: Δ (s)
- Rider arrival rate: λ (#/s/km²)
- Idle driver arrival rate: μ (#/s/km²)



Matching probability: $p = \min(1, \mu/\lambda)$

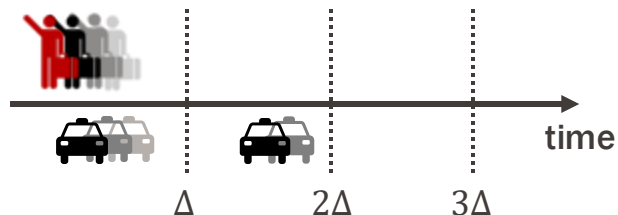
Expected num of matches: $n = 1/p$

Expected waiting time: $w = (n - 1/2)\Delta$

How is each equation derived?

Matching model

- Model I: Frictionless batch matching
 - Matching interval: Δ (s)
 - Rider arrival rate: λ (#/s/km²)
 - Idle driver arrival rate: μ (#/s/km²)



Matching probability: $p = \min(1, \mu/\lambda)$

Expected num of matches: $n = 1/p$

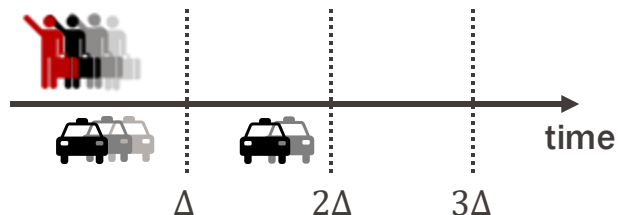
Expected waiting time: $w = (n - 1/2)\Delta$

How is this related to the general model $w = f(\Pi, \Lambda)$?

- Π : density of waiting riders (#/km²)
- Λ : density of idle drivers (#/km²)

Matching model

- Model I: Frictionless batch matching
 - Matching interval: Δ (s)
 - Rider arrival rate: λ (#/s/km²)
 - Idle driver arrival rate: μ (#/s/km²)



Matching probability: $p = \min(1, \mu/\lambda)$

Expected num of matches: $n = 1/p$

Expected waiting time: $w = (n - 1/2)\Delta$

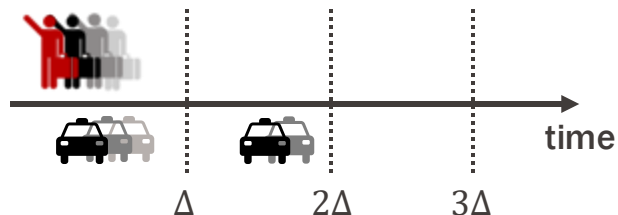
How is this related to the general model $w = f(\Pi, \Lambda)$?

- Π : density of waiting riders (#/km²) $\Pi = \lambda\Delta$
- Λ : density of idle drivers (#/km²) $\Lambda = \mu\Delta$

Matching model

- Model I: Frictionless batch matching

- Matching interval: Δ (s)
- Rider arrival rate: λ (#/s/km²)
- Idle driver arrival rate: μ (#/s/km²)



Matching probability: $p = \min(1, \mu/\lambda)$

Expected num of matches: $n = 1/p$

Expected waiting time: $w = (n - 1/2)\Delta$

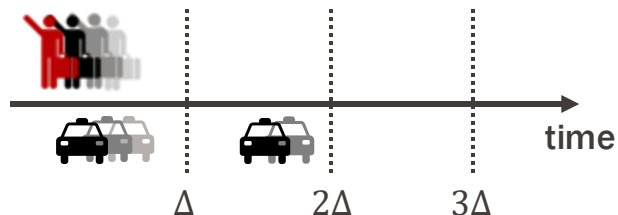
$$w = \left(n - \frac{1}{2}\right) \Delta = \left(\frac{1}{p} - \frac{1}{2}\right) \Delta = \left[\frac{1}{\min\left(1, \frac{\mu}{\lambda}\right)} - \frac{1}{2}\right] \Delta$$

$$= \left[\max\left(1, \frac{\lambda}{\mu}\right) - \frac{1}{2}\right] \Delta = \left[\max\left(1, \frac{\Pi/\Delta}{\Lambda/\Delta}\right) - \frac{1}{2}\right] \Delta$$

Matching model

- Model I: Frictionless batch matching

- Matching interval: Δ (s)
- Rider arrival rate: λ (#/s/km²)
- Idle driver arrival rate: μ (#/s/km²)



Matching probability: $p = \min(1, \mu/\lambda)$

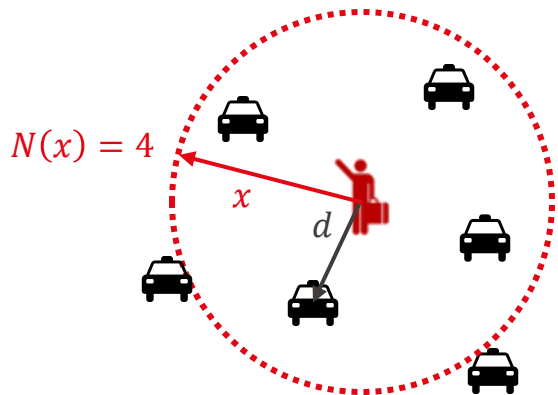
Expected num of matches: $n = 1/p$

Expected waiting time: $w = (n - 1/2)\Delta$

$$w = f(\Pi, \Lambda) = \left[\max\left(1, \frac{\Pi}{\Lambda}\right) - \frac{1}{2} \right] \Delta$$

Matching model

- Model II: Non-congested instant matching
 - Matching radius ∞
 - Vehicle speed v (km/hr)
 - Density of idle drivers Λ (#/km²)



Num of idle drivers within a distance x :

$$N(x) \sim \text{SpatialPP}(\Lambda)$$

Prob of at least one driver within a distance x :

$$1 - \Pr(N(x) = 0) = 1 - \exp(-\pi\Lambda x^2)$$

- equal to prob that the closest driver is within x
- equal to prob that the pickup distance $d \leq x$

Expected waiting time:

$$w = f(\Pi, \Lambda) = \frac{1}{2v\sqrt{\Lambda}}$$

$$w = \frac{\mathbb{E}[d]}{v} = \frac{1}{2v\sqrt{\Lambda}}$$

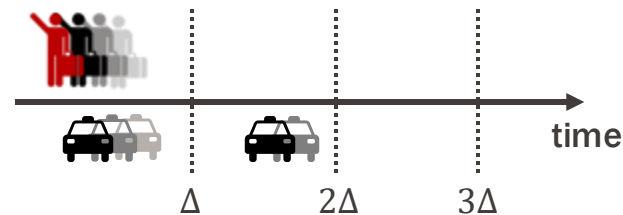
Matching model

- Both make sense but look so different...

* Because they each capture one part of total waiting time

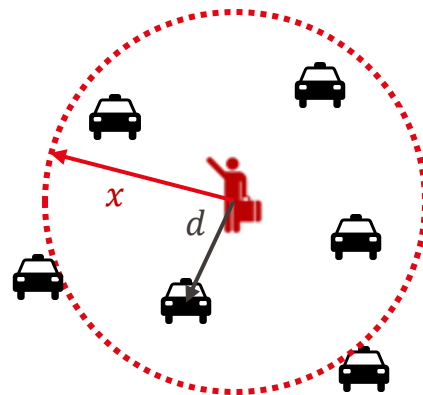
- Model I: $w \approx w_m = \left(\max\left(1, \frac{\Pi}{\Lambda}\right) - \frac{1}{2} \right) \Delta$

* Pickup time is missing

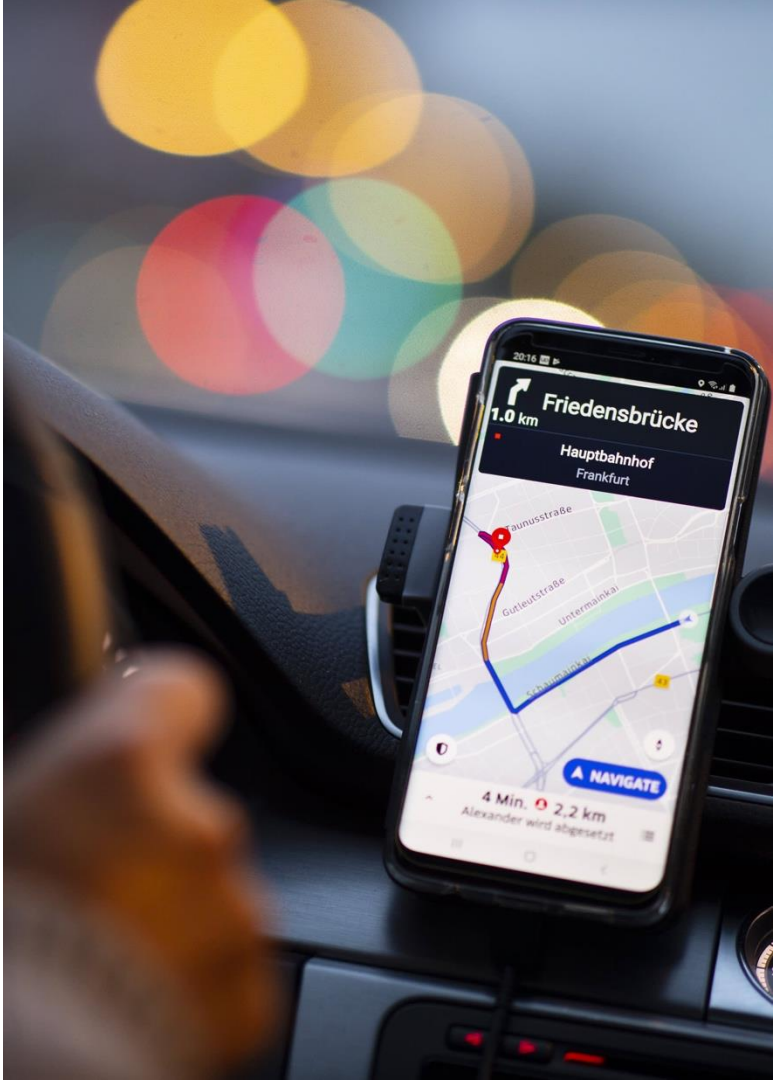


- Model II: $w \approx w_p = \frac{1}{2v\sqrt{\Lambda}}$

* Matching time is missing



* A general should model consider both pickup and matching

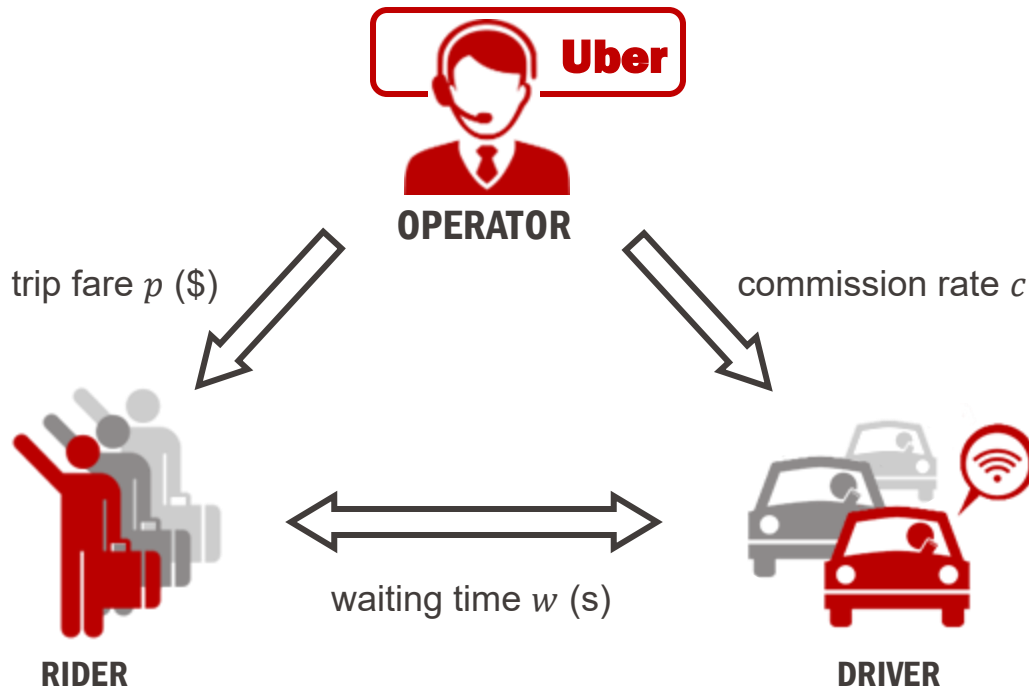


Questions?

Next topic: Operations

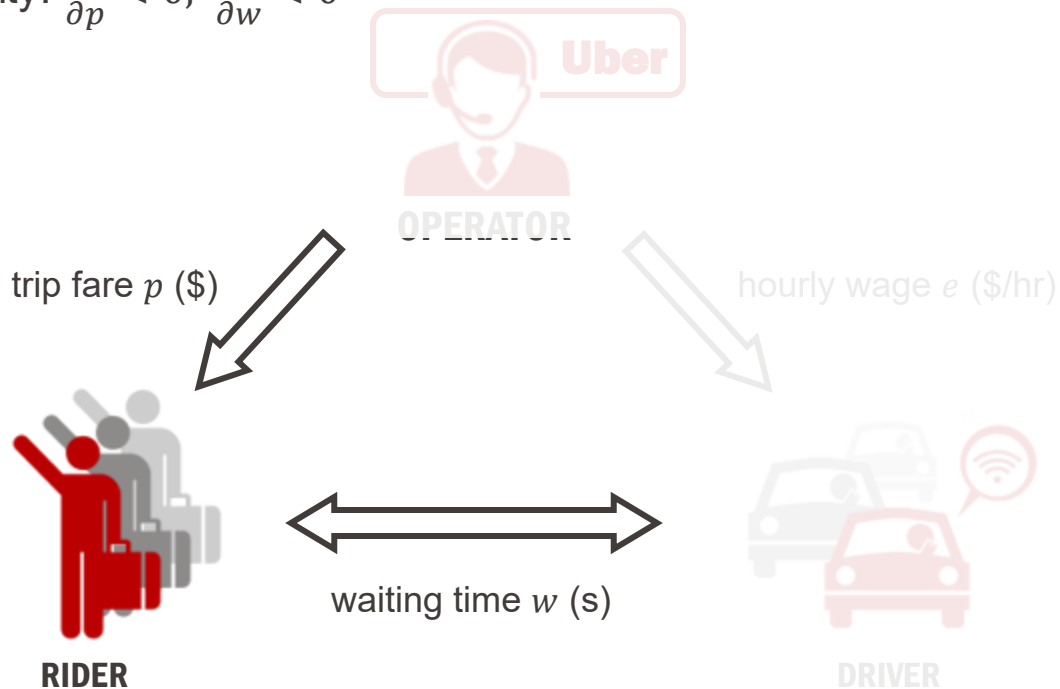
Monopoly market

- A single dominating operator



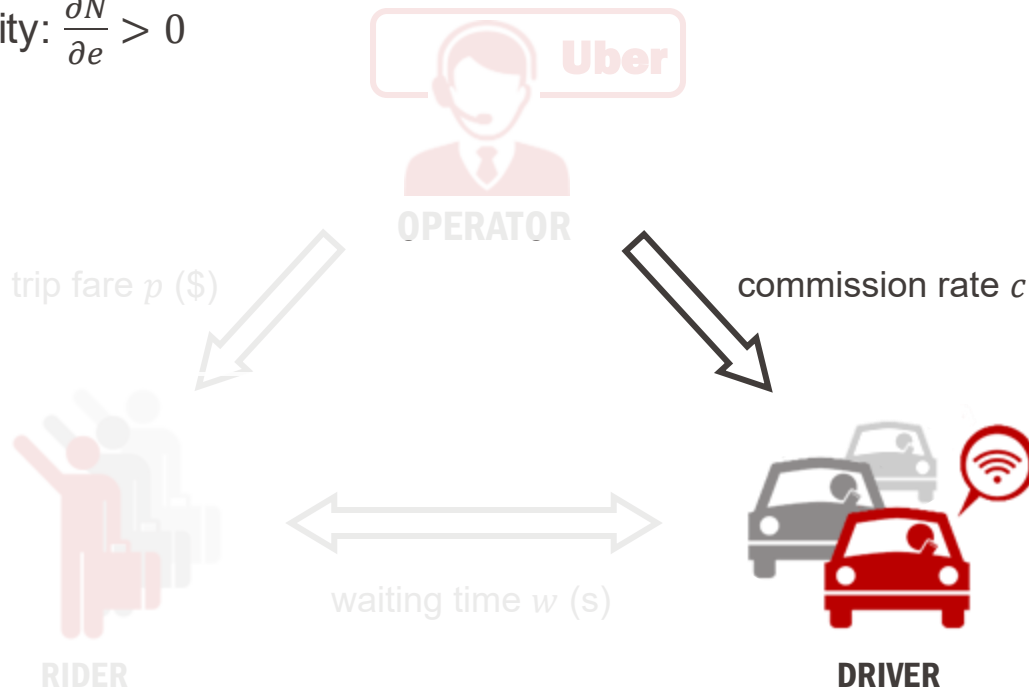
Monopoly market

- Rider demand $Q = D(p, w)$
 - Sensitivity: $\frac{\partial Q}{\partial p} < 0$, $\frac{\partial Q}{\partial w} < 0$



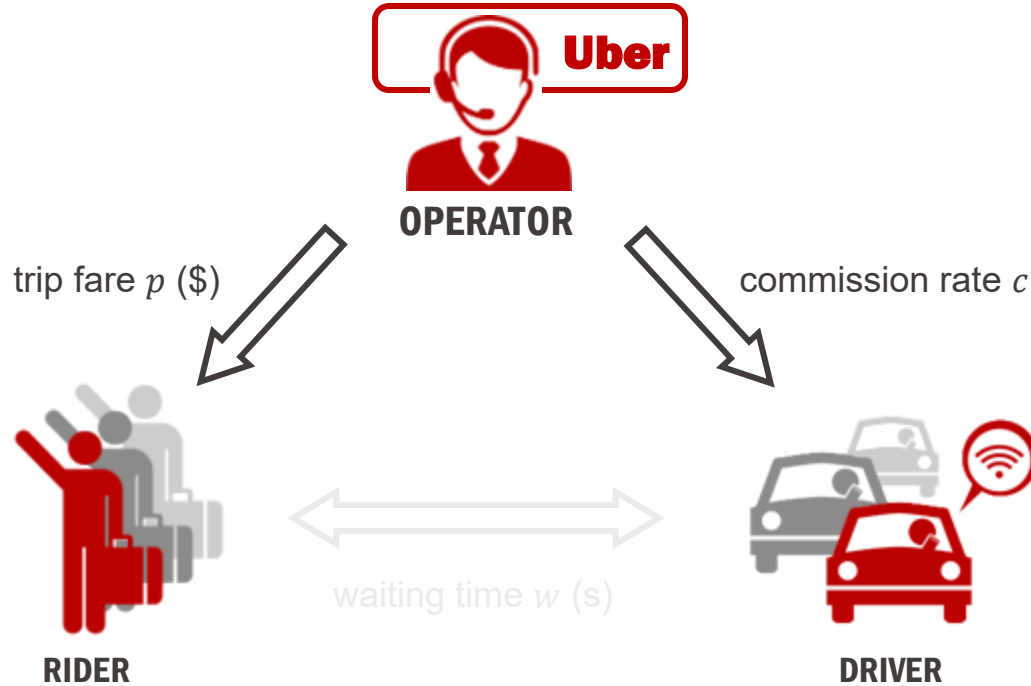
Monopoly market

- Driver supply $N = S(e)$
 - Hourly wage $e = (1 - c)pQ/N$
 - Sensitivity: $\frac{\partial N}{\partial e} > 0$



Monopoly market

- Operator profit $R(p, c) = cQ$



Monopoly market

- Pricing in a monopoly market
 - Assume trips are uniformly distributed with average in-vehicle time τ (s)
 - Decide trip fare p (\$) and commission rate c to max profit

$$\max_{p,c} R(p, c) = cQ$$

s. t.

$$Q = D(p, w),$$

$$N = S(e(p, c, Q)),$$

$$w = f(\Pi, \Lambda),$$

$$N = \Lambda + Q(w + \tau),$$

$$\Pi = Qw.$$

Demand function

Supply function

Matching model

Fleet conservation

Unmatched riders

Market equilibrium constraints

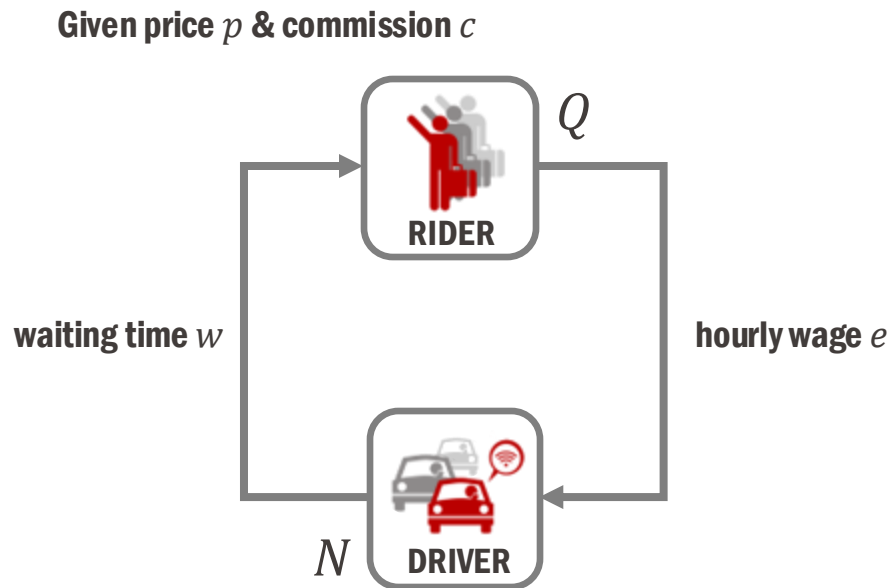
Monopoly market

- How to solve the optimal pricing strategy?
 - Option I: throw the entire problem into solver
 - * **Infeasible when model is highly nonlinear and complicated**

Monopoly market

- How to solve the optimal pricing strategy?
 - Option I: throw the entire problem into solver
 - Option II: solve the equilibrium at each feasible price

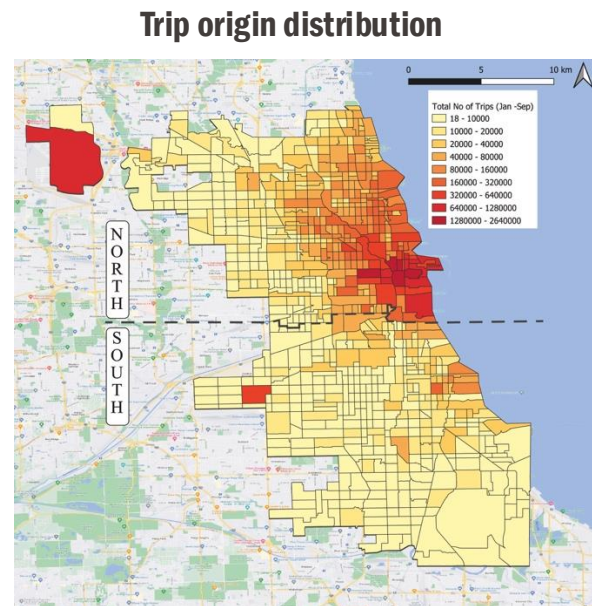
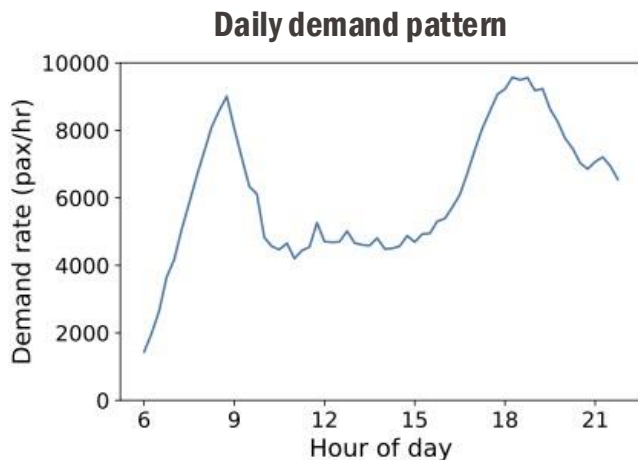
$$\begin{aligned} \max_{p,c} \quad & R(p, c) = cQ \\ \text{s. t.} \quad & Q = D(p, w), \\ & N = S(e(p, c, Q)), \\ & w = f(\Pi, \Lambda), \\ & N = \Lambda + Q(w + \tau), \\ & \Pi = Qw. \end{aligned}$$



*** We will learn how to solve this in CIVIL 324**

Monopoly market

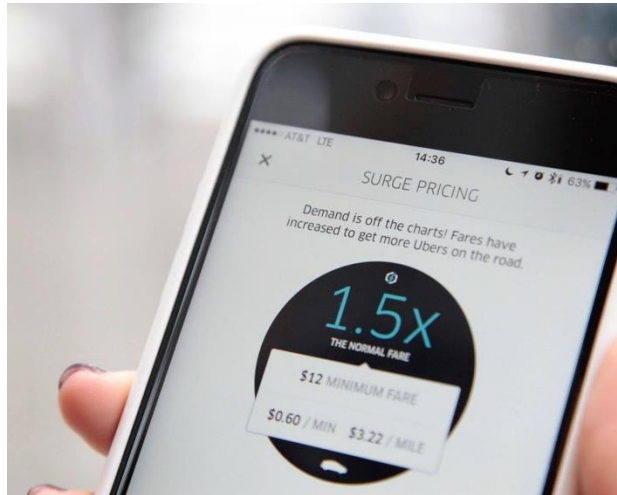
- So far, we consider a uniform market
 - Demand and supply are evenly distributed over time and space
 - Clearly not the case in reality
 - e.g., Chicago ride-hailing trips



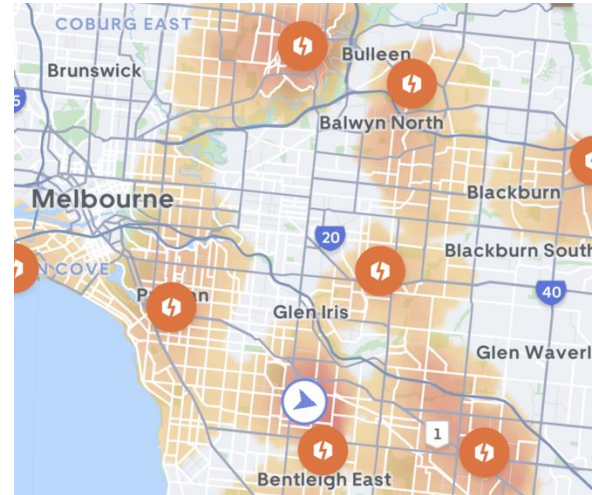
Monopoly market

- Common solutions to address demand-supply imbalance
 - Surge pricing
 - Vehicle rebalancing

Rider side: surge multiplier



Driver side: demand heat map



- Assumption of short-term market

- Demand curve: $Q = D(p, w) = D_0 - p - w$
 - Potential demand D_0 increase during a demand peak

- Supply curve: $Q = S(w) = \frac{N - \Lambda(w)}{w + \tau}$
 - Fleet size N remains the same
 - Density of idle drivers $\Lambda(w)$ as a function of waiting time w

$$w = \frac{1}{2v\sqrt{\Lambda}} \Rightarrow \Lambda = \frac{1}{4v^2w}$$

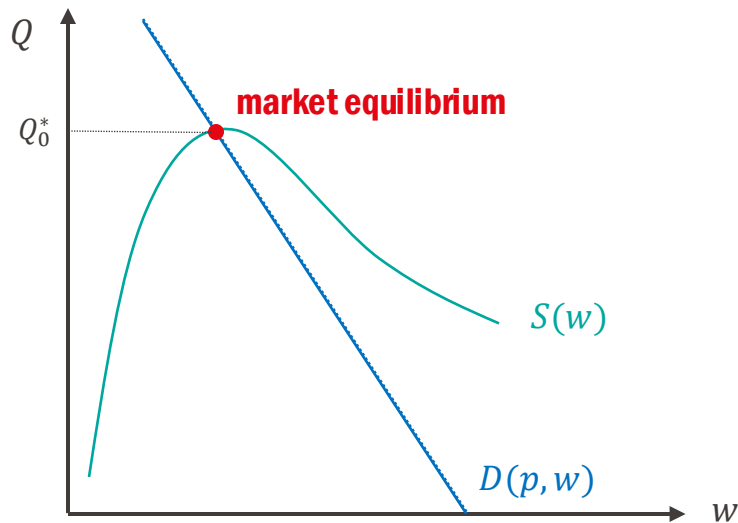
- Trip supply $S(w)$ is derived from flow conservation

$$N = \Lambda + Q(w + \tau),$$
$$\Rightarrow N = \Lambda(w) + S(w)(w + \tau),$$

Surge pricing

- Assumption of short-term market

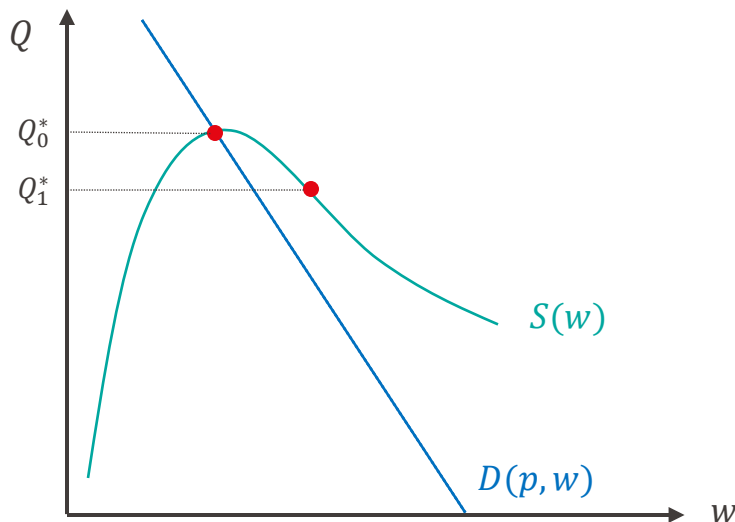
- Demand curve: $Q = D(p, w) = D_0 - p - w$
- Supply curve: $Q = S(w) = \frac{N - \Lambda(w)}{w + \tau}$



Surge pricing

- Assumption of short-term market

- Demand curve: $Q = D(p, w) = D_0 - p - w$
- Supply curve: $Q = S(w) = \frac{N - \Lambda(w)}{w + \tau}$



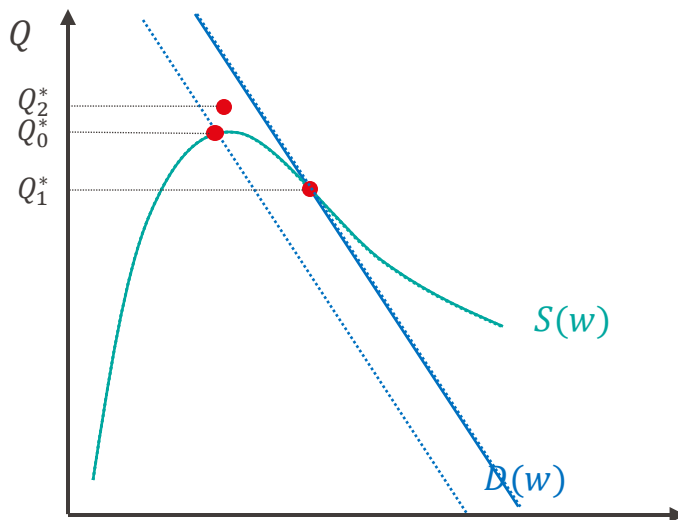
During a demand peak,

- D_0 increase while N remain the same
- Market moves to an inefficient state $Q_0^* \rightarrow Q_1^*$ with fewer served trips

Surge pricing

- Assumption of short-term market

- Demand curve: $Q = D(p, w) = D_0 - p - w$
- Supply curve: $Q = S(w) = \frac{N - \Lambda(w)}{w + \tau}$



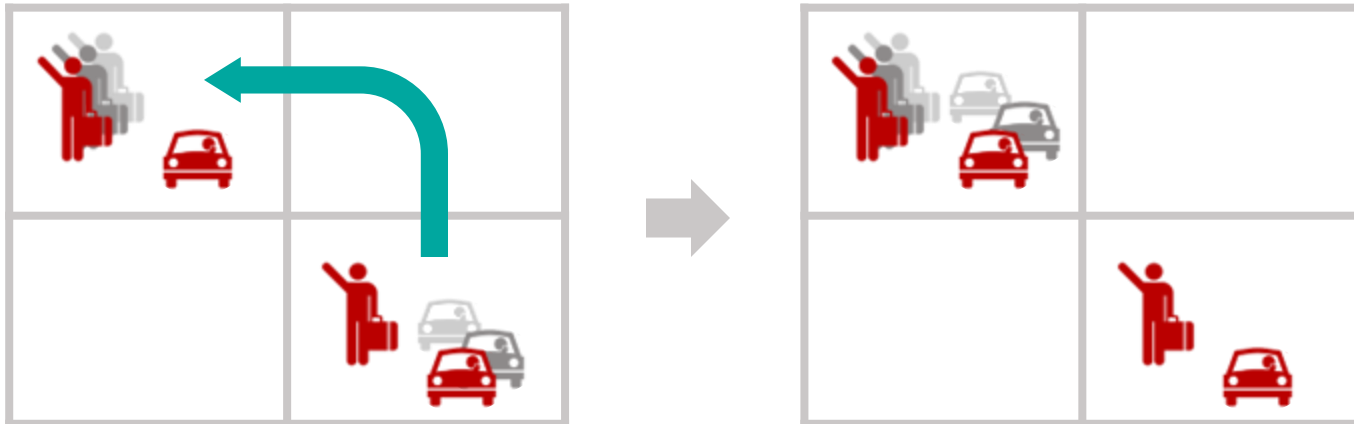
During a demand peak,

- D_0 increase while N remain the same
- Market moves to an inefficient state $Q_0^* \rightarrow Q_1^*$ with fewer served trips
- A surge price pushes demand curve back and possibly induce a larger N
- Market returns to an efficient state $Q_1^* \rightarrow Q_2^*$

Vehicle rebalancing

▪ Motivations

- Trips are not evenly distributed over space
- Drivers' spontaneous search may not be efficient
 - lack of real-time information
 - selfish decisions do not ensure system optimum
- Some centralized control and intervention are beneficial to all



Vehicle rebalancing

- A static model

$$\begin{aligned} \max_{x, x^0} \quad & \sum_{ij} p_{ij} x_{ij} - c_{ij} x_{ij}^0 \\ \text{s. t.} \quad & x_{ij} = \alpha_{ij} g(\lambda_i, \mu_i), \\ & \mu_i = \sum_k (x_{ki} + x_{ki}^0), \\ & \sum_k (x_{ki} + x_{ki}^0) = \sum_j (x_{ij} + x_{ij}^0), \\ & \sum_{ij} (x_{ij} + x_{ij}^0) \tau_{ij} = N, \\ & x_{ij}, x_{ij}^0 \geq 0. \end{aligned}$$

Notations:

- p_{ij} : trip fare
- c_{ij} : relocation cost
- x_{ij} : trip flow
- x_{ij}^0 : relocation flow
- λ_i : rider arrival rate
- μ_i : driver arrival rate
- α_{ij} : OD distribution
- τ_{ij} : trip duration
- N : fleet size

Objective: max total revenue

Trip flow between every two zones

Driver supply in each zone

Inflow equals outflow for each zone

Fleet conservation

Beyond monopoly market

- Cities are rarely dominated by a single operator
- Competition and cooperation often co-exist
 - Uber vs Lyft
 - Uber + taxis
 - Lyft + Metro/Citi Bike

Uber Partners With Yellow Taxi Companies in N.Y.C.

The ride-hailing giant is teaming up with two taxi technology companies in an unlikely alliance.

Share full article | 160



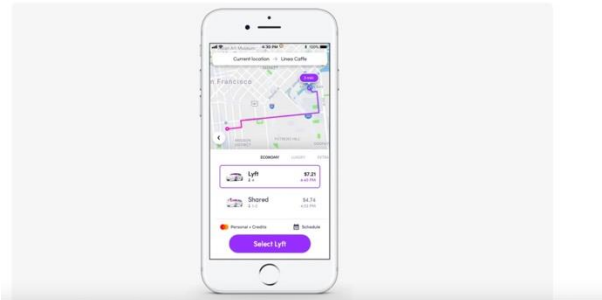
Source: New York Times

Lyft's new app to integrate directly with public transit

Published June 8, 2018

Kristin Musulin
Senior Editor

in f X Print Email



Source: Smart Cities Dive

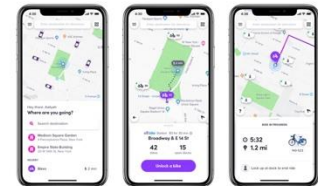
Uber Rewards



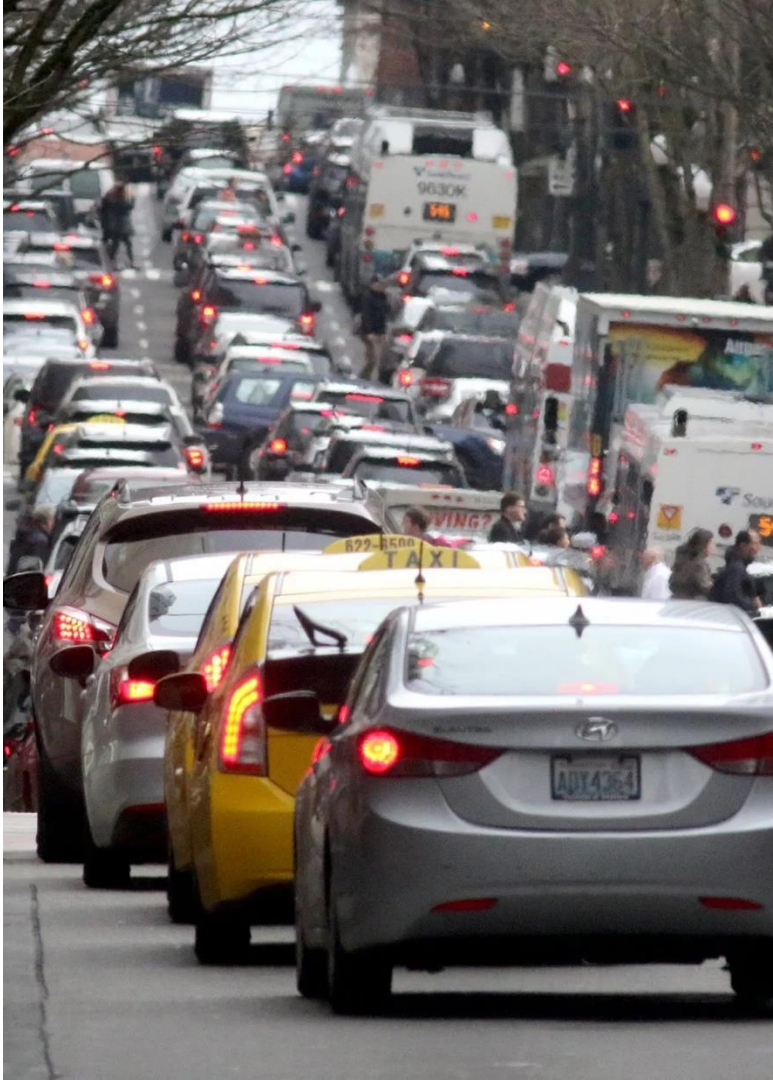
Source: Uber

Citi Bike: Now available in the Lyft app

Update your Lyft app to get riding!



Source: Citi Bike



Questions?

Next topic: Regulations

Why regulation is needed

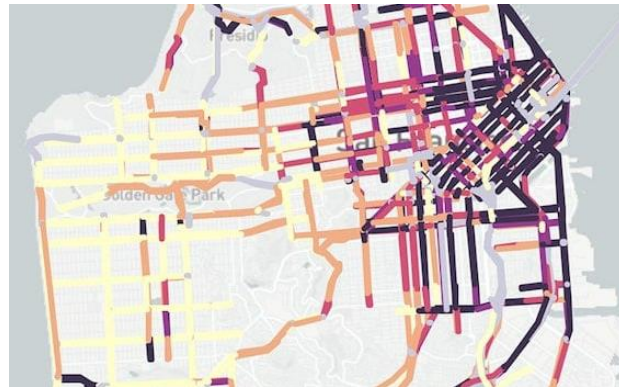
- Incompatible objectives



- Operator:
 - max profit, market share, ...



- Regulator:
 - max social welfare
= travel utility of riders
+ profit of operator
+ earning of drivers
+ ...
 - min congestion and emission
 - ensure mobility accessibility



Ride-hailing vehicles cause more congestion in San Francisco Erhardt et al. (2019)



Oversupply of shared bikes in China

Supply side

- min wage rate
- fleet cap
- contractor vs employee
-

Demand side

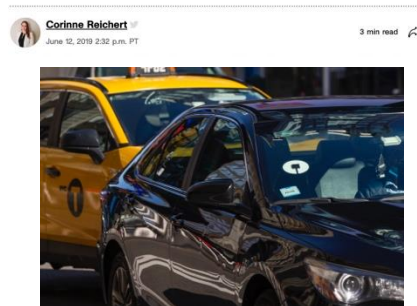
- congestion charge
- mobility credits
-

Operations

- dedicated service region
- data sharing
-

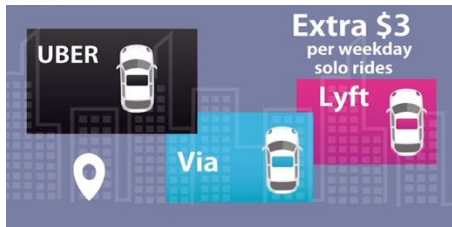
Uber, Lyft must continue to limit size of fleets in New York City

To fight congestion, the city council votes to indefinitely extend a cap on the number of ride-hailing vehicles on New York streets.



Chicago congestion tax on rideshare trips takes effect

By Jessica D'Onofrio and Sarah Schulte
Tuesday, January 7, 2020



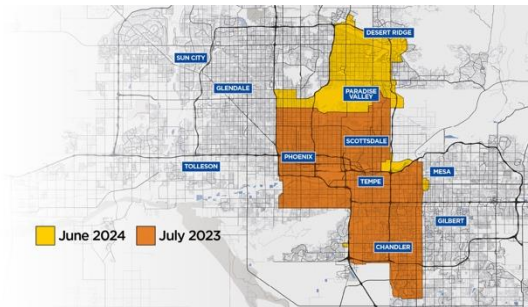
CAPITOL ALERT
 Capitol Alert
California sued over gig economy law. What Uber and Postmates say about AB 5
 BY DALE KASLER
 UPDATED DECEMBER 30, 2019 5:40 PM |



Waymo expands into metro Phoenix, downtown Mesa, Talking Stick

By Lauren Kobley
 Published: Jun. 6, 2024 at 1:08 AM CEST | Updated: Jun. 6, 2024 at 3:39 AM CEST

PHOENIX (AZFamily)—Waymo, the autonomous rideshare program, is expanding its reach in the Valley to include parts of north Scottsdale, north Phoenix, Mesa, and Ahwatukee.



How to analyze a policy

- Additional constraints in the operator's problem
 - e.g., max fleet size \bar{N} and min wage range \underline{e}

$$\max_{p,c} R(p,c) = cQ$$

s. t.

$$Q = D(p, w),$$

$$N = S(e(p, c, Q)),$$

$$w = f(\Pi, \Lambda),$$

$$N = \Lambda + Q(w + \tau),$$

$$\Pi = Qw.$$

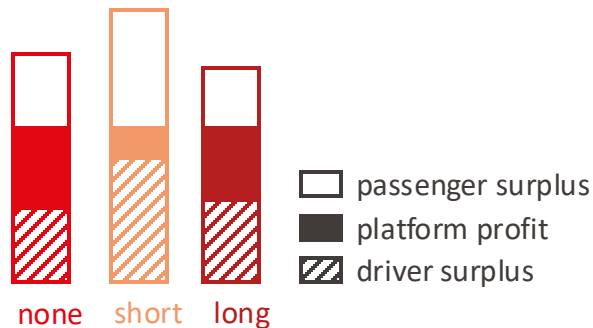
market equilibrium

$$N \leq \bar{N}, e \geq \underline{e},$$

regulatory constraints

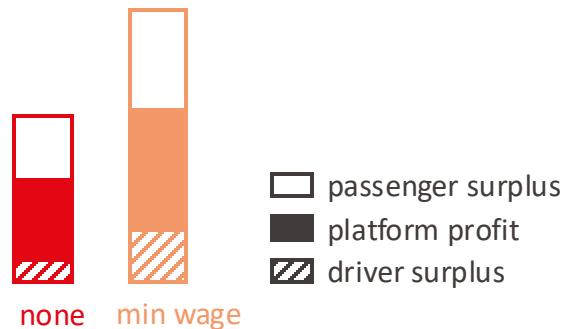
How to analyze a policy

- Be careful of potential “pitfalls”
 - Short-term vs long-term



- min wage helps improve social welfare in short-term by sacrificing platform profit
- in a long run, it could even be harmful to social welfare

- Market structure



- min wage does benefit a duopoly market with multi-homing by maintaining a sufficient supply

Summary

- What we've discussed today
 - Matching in a solo ride-hailing trip
 - Pricing strategy of a monopoly operator
 - Surge pricing and vehicle relocation
 - Overview of regulations

- What are other interesting topics
 - Matching in micromobility and pooling trips
 - Competition and cooperation among operators
 - Introduction of autonomous vehicles
 - Integration of mobility-on-demand into transit system
 - Issue of equity and fairness
 -



Thanks!
Q & A



HOMES @ EPFL