Traffic Engineering (CIVIL-349)
Nikolas Geroliminis
Exercise 2 - Solution
Traffic Stream Characteristics

Fall 2025

École Polytechnique Fédérale de Lausanne
School of Architecture, Civil & Environmental Engineering

Problem 1

a. Since the stationary observer sees a fraction of p buses over the total sum of vehicles, this means that:

$$p = \frac{n_b}{n_b + n_c} = \frac{\frac{n_b}{T}}{\frac{n_b+n_c}{T}} = \frac{q_b}{q_b + q_c} \tag{1}$$

where n_b and n_c denote the number of buses and cars respectively, that the stationary observer sees during an observation period T.

The fraction of vehicles that are seen in an aerial photograph in a specific time point (assuming homogeneous traffic conditions) can be calculated as:

$$p' = \frac{n'_b}{n'_b + n'_c} \tag{2}$$

where n'_b and n'_c correspond to the number of buses and cars respectively, in an aerial photograph of the road. If we divide nominator and denominator of the fraction of Eq. 2 by the total length L of the road, we have:

$$p' = \frac{\frac{n'_b}{L}}{\frac{n'_b+n'_c}{L}} = \frac{k_b}{k_b + k_c} \tag{3}$$

Then, by using the fundamental relation of traffic flow $q = kv$ and the relation of Eq. 1, we transform Eq. 3 as follows:

$$p' = \frac{k_b}{k_b + k_c} = \frac{\frac{q_b}{v'}}{\frac{q_b}{v'} + \frac{q_c}{v}} = \frac{\frac{q_b}{q_b+q_c}\frac{1}{v'}}{\frac{q_b}{q_b+q_c}\frac{1}{v'} + \frac{q_c}{q_b+q_c}\frac{1}{v}} = \frac{\frac{p}{v'}}{\frac{p}{v'} + \frac{1-p}{v}} = \frac{pv}{pv + (1-p)v'} \tag{4}$$

Equation 4 provides us with the answer for question **a**.

b. Since we refer to the stationary observer, the flow of vehicles can be used to calculate the average vehicle occupancy, which we denote as o_{obs}. It can be found as:

$$o_{obs} = \frac{\text{Total \# of passengers passing}}{\text{Total \# of vehicles passing}} = \frac{q_b n' + q_c n}{q_b + q_c} = pn' + (1-p)n \tag{5}$$

c. Since now we refer to an aerial photograph, the density can be used to calculate the average vehicle occupancy, which we denote as o_{ph}. It can be found as follows:

$$o_{ph} = \frac{\text{Total \# of passengers on photo}}{\text{Total \# of vehicles on photo}} = \frac{k_b n' + k_c n}{k_b + k_c} = p'n' + (1-p')n \tag{6}$$

For $v' = 50$ km/h, $v = 80$ km/h, $n' = 20$ passengers/bus $n = 1$ passengers/car and $p = 0.2$, using the relations 4 and 6, we find:

$$o_{obs} = 4.8 \ \text{pax/veh}, \ \ o_{ph} = 6.43 \ \text{pax/veh}, \ \ p' = 0.29$$

d. Pollution is estimated per vehicle and per time unit of travel. One car produces α units of pollutants per time unit while one bus produces 2.5α units of pollutants per time unit when they travel by speeds v and v', respectively.

Therefore, in order to find the proportion of the pollutants produced by cars in a road segment of length L during a time-period T over the pollutants produced in total (by cars and buses), we first calculate the amount of pollutants that cars produce. This can be found as the number of cars inside the road segment $k_c \times L$ (which remains constant over time as we assume homogeneous conditions) multiplied by the product of the travel time T spent inside the road times the units of pollutants α produced per time unit. Therefore, we use the total Vehicle Hours Travelled (VHT) for each vehicle type in order to calculate the amount of pollutants produced, as it is defined per unit of travel time[1]. In the same way, we can calculate the amount of pollutants produced by buses. Then the contribution of cars η_c to the total pollution produced during time T along distance L of the road[2] can be found as the ratio:

$$\eta_c = \frac{VHT_c\alpha}{VHT_c\alpha + VHT_b 2.5\alpha} = \frac{k_c LT\alpha}{k_c LT\alpha + k_b LT 2.5\alpha} = \frac{k_c}{k_c + 2.5k_b} =$$

$$= \frac{\frac{k_c}{k_c+k_b}}{\frac{k_c}{k_c+k_b} + 2.5\frac{k_b}{k_c+k_b}} = \frac{1-p'}{1-p'+2.5p'} = \frac{0.714}{0.714 + 2.5 \times 0.286} = 0.5 \tag{7}$$

[1] Note that the product $k_c LT = VHT_c$ represents the total Vehicle Hours Travelled of cars in the road segment based on the Generalized Definitions of flow and density of Edie (see slide of Video 1.1) where $K = \frac{VHT}{LT}$

[2] This can be seen as the rectangle of area $L \times T$ in the time-space diagram

Problem 2

a. The fundamental relationship between flow, density and speed dictates that $q = ku$. By substituting u in Eq. 1 we get:

$$\frac{q}{k} = u_f \exp\left(-\alpha\left(\frac{k}{k_{jam}}\right)^2\right) \Leftrightarrow q = ku_f \exp\left(-\alpha\left(\frac{k}{k_{jam}}\right)^2\right) \quad (8)$$

By substituting the values of u_f and k_{jam} we get:

$$q = 30k \exp\left(-\alpha\left(\frac{k}{0.15}\right)^2\right) (veh/sec) \quad (9)$$

where k is in veh/m.

b. The critical density, k_{crit}, corresponds to the maximum flow q_{crit}. In order to find the maximum flow, we set $\frac{dq}{dk} = 0$ and solve the equation for k:

$$\frac{dq}{dk} = \frac{d\left(u_f\ k\exp\left(-\alpha\left(\frac{k}{k_{jam}}\right)^2\right)\right)}{dk} = 0 \Leftrightarrow u_f\frac{d\left(k\exp\left(-\alpha\left(\frac{k}{k_{jam}}\right)^2\right)\right)}{dk} = 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{d\left(k\exp\left(-\alpha\left(\frac{k}{k_{jam}}\right)^2\right)\right)}{dk} = 0 \Leftrightarrow$$

$$\Leftrightarrow \exp\left(-\alpha\left(\frac{k}{k_{jam}}\right)^2\right) + k\frac{d\left(\exp\left(-\alpha\left(\frac{k}{k_{jam}}\right)^2\right)\right)}{dk} = 0 \Leftrightarrow$$

$$\Leftrightarrow \exp\left(-\alpha\left(\frac{k}{k_{jam}}\right)^2\right) + k\exp\left(-\alpha\left(\frac{k}{k_{jam}}\right)^2\right)\frac{d\left(-\alpha\left(\frac{k}{k_{jam}}\right)^2\right)}{dk} = 0 \Leftrightarrow$$

$$1 + k\left(-\frac{\alpha}{k_{jam}^2}\right)2k = 0 \Leftrightarrow 1 = \frac{2\alpha k^2}{k_{jam}^2} \Leftrightarrow k^2 = \frac{k_{jam}^2}{2\alpha} \Leftrightarrow k = k_{crit} = \frac{k_{jam}}{\sqrt{2\alpha}}$$

Therefore, for $k_{jam} = 0.15$ veh/m, we have $k_{crit} = \frac{0.15}{\sqrt{2\alpha}}$ (veh/m).

c. The maximum flow q_{crit} is observed for $k = k_{crit}$, so by substituting the value of k_{crit} (found in **b**) in Equation 9, we get:

$$q_{crit} = q\left(k_{crit}\right) = 30\frac{0.15}{\sqrt{2\alpha}}\exp\left(-\alpha\left(\frac{0.15}{0.15\sqrt{2\alpha}}\right)^2\right) = \frac{4.5}{\sqrt{2\alpha}}\exp\left(-\frac{1}{2}\right) \quad (10)$$

d. For $q_{crit} = 2100$ veh/h $= \frac{2100}{3600}$ veh/sec $= 0.583$ veh/sec, by solving Equation 10 for α we get:

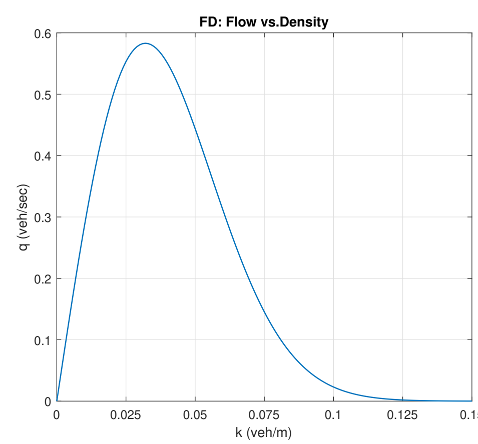

Figure 1: The Drake's Fundamental Diagram between flow and density (see Equation 9) for the case of question **d**, where $\alpha = 10.96$ and $q_{crit} = 0.583$ veh/sec = 2100 veh/h.

$$q_{\text{crit}} = 0.583 = \frac{4.5}{\sqrt{2\alpha}} \exp\left(-\frac{1}{2}\right) \Leftrightarrow \alpha = 10.96$$

In Figure 1 we see the FD for the case where $\alpha = 10.96$, according to Equation 9.

The relationship between the critical flow q_{crit} and the parameter α can be observed in Figure 2.

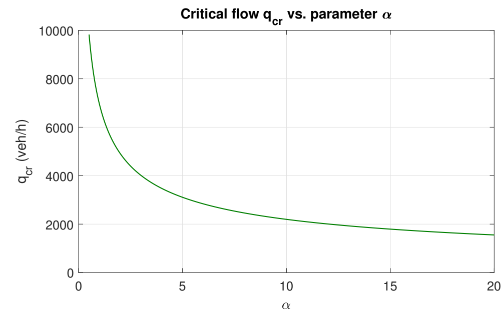

Figure 2: The critical flow q_{crit} of Drake's FD as a function of parameter α, see Eq. 10