Traffic Engineering (CIVIL-349)
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Exercise 1 - Solution
Queueing systems

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Problem 1: Note that $\lambda = 10$ cars per hour, and $\mu = \frac{60}{4} = 15$ cars per hour. Therefore, $\rho = \frac{\lambda}{\mu} = \frac{10}{15} \approx 0.67$.

a.

This system can be described as an M/M/1 queueing system.

b.

The probability of the system being empty is $P_0 = 1 - \rho \approx 0.33$.

c.

The average number of cars in the system is $L = \frac{\rho}{1-\rho} = 2$ cars.

The average number of cars in the queue is $L_Q = \frac{\rho^2}{1-\rho} \approx 1.33$ cars.

d.

The average time spent in the system is $W = \frac{L}{\lambda} = 0.2$ hours (or 12 minutes).

The average time spent in the queue is $W_Q = W - \frac{1}{\mu} \approx 0.13$ hours (or 8 minutes).

e.

The probability of having 2 or more cars in the system is:

$$P(L \geq 2) = \rho^2 \approx 0.44$$

Problem 2:

a.

Note that, here, $\lambda = 10$ cars per hour, and $\mu = \frac{60}{2} = 30$ cars per hour. Therefore, $\rho = \frac{10}{30} \approx 0.33$.

The probability that the system is empty becomes $P_0 = 1 - \rho \approx 0.67$.

The average number of cars in the system is $L = \frac{\rho}{1-\rho} = 0.5$ cars.

The average number of cars in the queue is $L_Q = \frac{\rho^2}{1-\rho} = 0.17$ cars.

The average time spent in the system is $W = \frac{L}{\lambda} = 0.05$ hours (or 3 minutes).

The average time spent in the queue is $W_Q = W - \frac{1}{\mu} \approx 0.017$ hours (or 1 minute).

b.

Note that, here, we have two identical M/M/1 systems with $\lambda = 5$

cars per hour, and $\mu = \frac{60}{4} = 15$ cars per hour. Therefore, $\rho = \frac{5}{15} = 0.33$.

The probability that the system is empty becomes $P_0 = 1 - \rho \approx 0.67$.

The average number of cars in the system is $L = \frac{\rho}{1-\rho} = 0.5$ cars.

The average number of cars in the queue is $L_Q = \frac{\rho^2}{1-\rho} = 0.17$ cars.

The average time spent in the system is $W = \frac{L}{\lambda} = 0.1$ hours (or 6 minutes).

The average time spent in the queue is $W_Q = W - \frac{1}{\mu} \approx 0.033$ hours (or 2 minutes).

Problem 3:

a.

The probability of losing customers is $P_{loss} = P_5$, where $P_5 = \frac{\rho^5(1-\rho)}{1-\rho^6} \approx 0.048$. Therefore, the rate in which the restaurant loses customers is $P_5 \cdot \lambda \approx 0.481$ cars per hour.

The average utilization rate is $\rho = \frac{\lambda_e}{\mu} = \frac{\lambda \cdot (1-P_5)}{\mu} \approx 0.635$.

b.

The number of cars in the queue is $L_Q = \sum_{i=2}^{5}(i-1)P_i \approx 0.788$ cars.

The waiting time is $W_Q = \frac{L_Q}{\lambda_e} \approx 0.083$ hours (or 4.967 minutes).

Problem 4:

Note that, in this case, the queueing system can be described as M/M/2.

The utilization rate becomes $\rho = \frac{\lambda}{2\mu} \approx 0.33$.

Therefore, the probability of the system being empty becomes:

$$\begin{aligned} P_0 &= \frac{1}{\sum_{n=0}^{2-1} \frac{(2\rho)^n}{n!} + \frac{(2\rho)^2}{2!}\left(\frac{1}{1-\rho}\right)} \\ &= \frac{1}{1 + 2\rho + 2\rho^2\left(\frac{1}{1-\rho}\right)} \\ &= \frac{1}{2} = 0.5 \end{aligned}$$

The average number of cars in the system is $L = 2\rho + \frac{(2\rho)^2\rho}{2!(1-\rho)^2} P_0 = \frac{2}{3} + \frac{(2/3)^2 \cdot (1/3)}{2(1-1/3)^2} \cdot 0.5 = 0.75$ cars.

The average number of cars in the queue is $\frac{(2\rho)^2\rho}{2!(1-\rho)^2} P_0 \approx 0.083$ cars.

The average time spent in the system is $W = \frac{L}{\lambda} = 0.075$ hours (or 4.5 minutes).

The average time spent in the queue is $W_Q = \frac{L_Q}{\lambda} \approx 0.008$ (or 0.5 minutes).

Problem 5:

a.

This system can be described as an M/D/1 queueing system.

b.

We still have the same values of $\lambda = 10$ and $\mu = 15$. However, $\sigma^2 = 0$.

Therefore, we have that the probability that the system is empty as $P_0 = 1 - \rho = 1 - \frac{10}{15} \approx 0.33$.

The number of cars in the system is $L = \rho + \frac{\rho^2(1+\sigma^2\mu^2)}{2(1-\rho)} \approx 1.33$ cars.

The number of cars in the queue is $L_Q = \frac{\rho^2(1+\sigma^2\mu^2)}{2(1-\rho)} \approx 0.67$ cars.

The average time spent in the system is $W = \frac{L}{\lambda} \approx 0.133$ hours (or 8 minutes).

The average time spent in the queue is $W_Q = \frac{L_Q}{\lambda} \approx 0.067$ hours (or 4 minutes).