CIVIL-312: Hydraulic Engineering and Infrastructures

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Pipe Networks

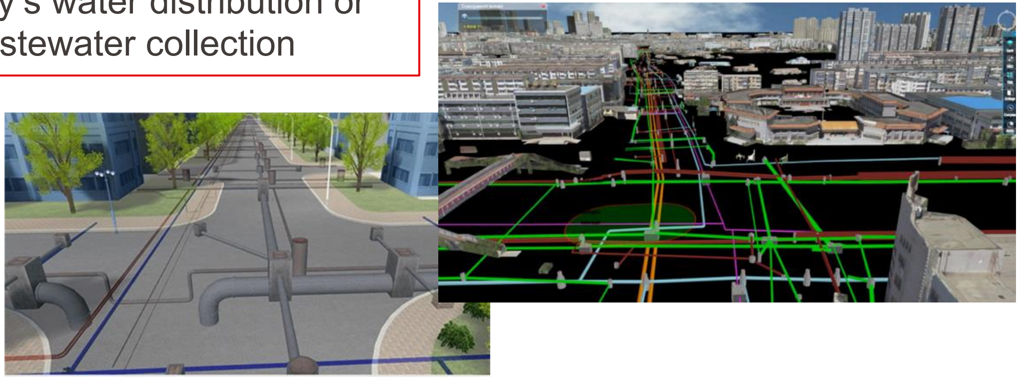
Last week

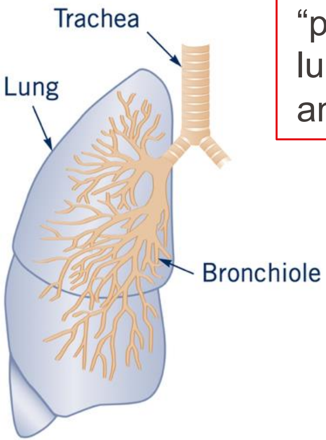
- Energy balance in pipes
- Velocity & stress distribution
- Losses h_L: major and minor

Pipe Networks

Typically pipe systems are composed by several pipes with different sizes, junction, etc.

City's water distribution or wastewater collection





"pipe" system in our lungs, branching and changing sizes

We'll see two types of networks:

- **Simple networks**: pipes in series, in parallel, and branching
- **Complex networks**: systems and loops (we'll solve using Hardy Cross method and software EPANET)

Pipe Networks

To resolve pipe networks, both simple and complex, we need system of equations that are essentially derived from

- ❑ Mass continuity
- ❑ Energy balance

Let's simplify the headloss (neglect minor losses for now) and rewrite the expression as a generic:

$$h_L = KQ^n$$

friction losses are proportional to the discharge raised to some power n.

So, for for the formulation we've seen so far (Darcy-Weisbach)

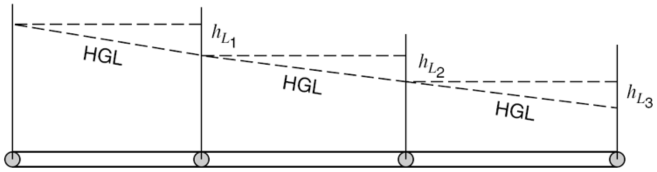
$$h_L = \frac{fL}{D}\frac{V^2}{2g} = \frac{fL}{D}\frac{Q^2}{2gA^2}$$

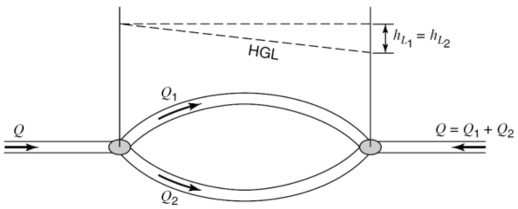
$$(Q = V * A)$$

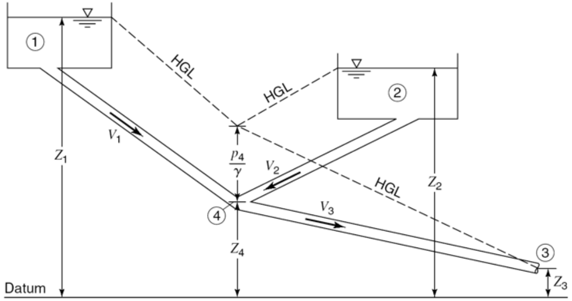
$$\Rightarrow \quad K = \frac{fL}{D}\left(\frac{1}{2gA^2}\right) \;\&\; n = 2$$

Simple Networks

Pipes in series | **Pipes in parallel** | **Branching pipes**



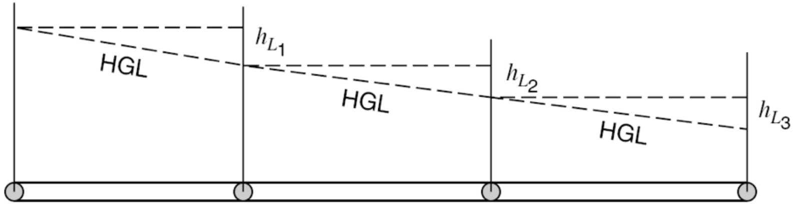




These simple networks can be converted to an *equivalent pipe* to simplify and analyze the network. Two pipes are equivalent when for the same headloss, both deliver the same rate of flow.

Simple Networks

- Pipe in Series



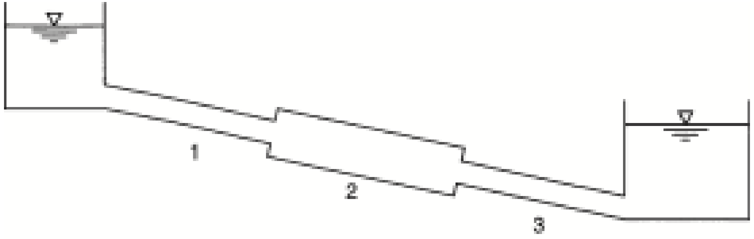
Every fluid particle that passes through the system passes through each of the pipes

- Flow continuity: the discharge Q is equal in each pipe

$$Q_{in} = Q_1 = Q_2 = Q_3 = \cdots = Q_n$$

- Through energy, the headloss is the sum of headlosses in each pipe

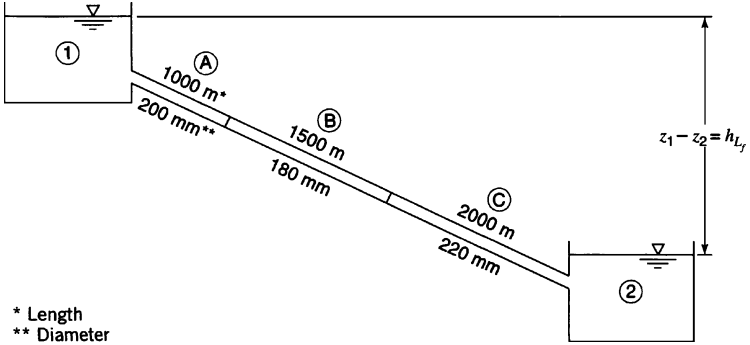
$$h_L = h_{L_1} + h_{L_2} + h_{L_3} + \cdots + h_{L_n}$$



Note: if the size of the pipe changes, the velocity will change (and so the headloss)!

Simple Networks

Example 1:



$Q = 0.03\ m^3/s$
f = 0.025 for the three pipes

Neglect minor losses and determine difference of height in reservoirs.

Energy balance between 1 and 2:

$$z_1 + \cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} = z_2 + \cancel{\frac{P_2}{\gamma}} + \cancel{\frac{V_2^2}{2g}} + h_{L\,tot}$$

$$z_1 - z_2 = h_{L\,tot}$$

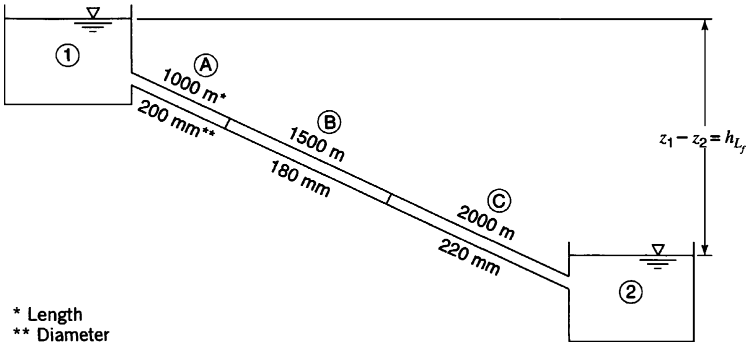
The total headloss is the sum of the three:

$$h_{L\,tot} = h_A + h_B + h_C = z_1 - z_2$$

$$\frac{fL_A}{D_A}\frac{V_A^2}{2g} + \frac{fL_B}{D_B}\frac{V_B^2}{2g} + \frac{fL_A}{D_C}\frac{V_C^2}{2g} = z_1 - z_2$$

Simple Networks

Example:



Q = 0.03 m³/s
f = 0.025 for the three pipes

Neglect minor losses and determine difference of height in reservoirs.

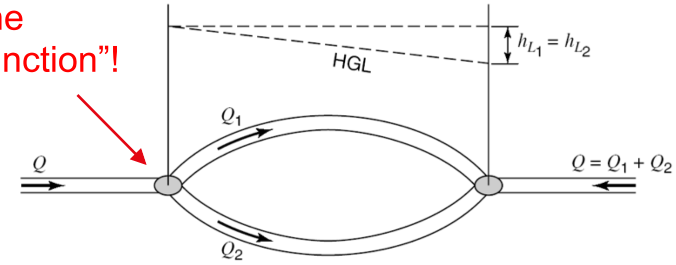
Applying continuity: $Q = V_A A_A = V_B A_B = V_C A_C$

$$\frac{f L_A}{D_A} \frac{Q^2}{2g\left(\frac{\pi D_A^2}{4}\right)^2} + \frac{f L_B}{D_B} \frac{Q^2}{2g\left(\frac{\pi D_B^2}{4}\right)^2} + \frac{f L_C}{D_C} \frac{Q^2}{2g\left(\frac{\pi D_C^2}{4}\right)^2} = z_1 - z_2$$

$$z_1 - z_2 = 27.78\, m$$

Simple Networks

Introducing the concept of “junction”!



- Pipe in Parallel

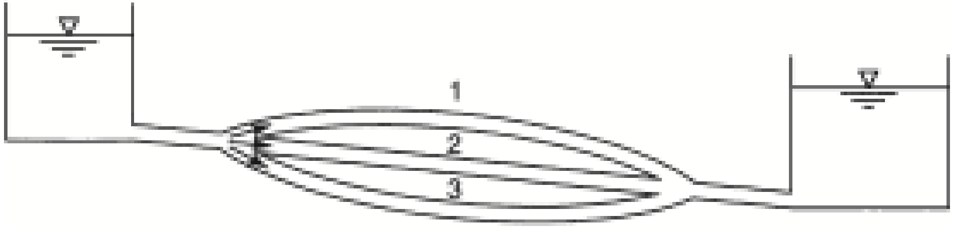
A fluid particle may take any of the paths available, with the total flowrate equal to the sum of the flowrates in each pipe

- Flow continuity: the total flow is the sum of the flow in each pipe (i.e. the flow splits in the pipes)

$$Q_{in} = Q_1 + Q_2 + Q_3 + \cdots + Q_n = \sum Q_i = Q_{out}$$

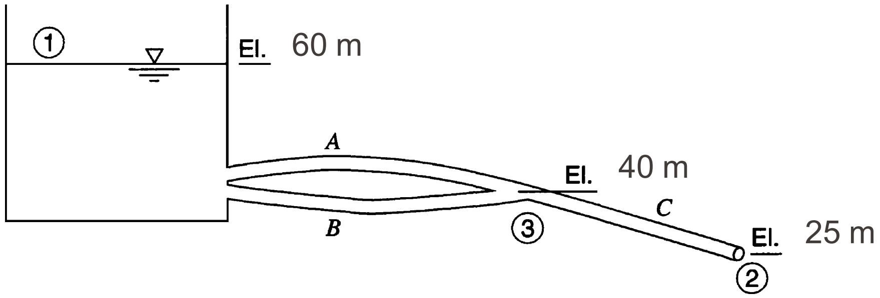
- Through energy balance, the flow distribution in the parallel pipes is such that the headloss in each pipe is equal!

$$h_L = h_{L_1} = h_{L_2} = h_{L_3} = \cdots + h_{L_n}$$



Simple Networks

Example 2:



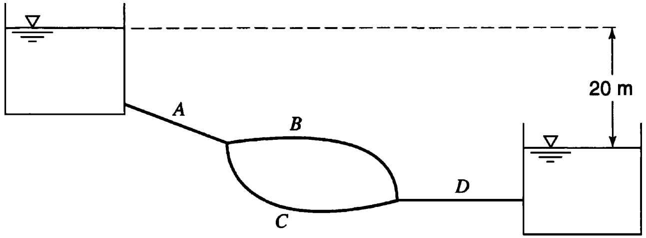
Pipe	D (cm)	L (m)	f
A	20	450	0.020
B	15	600	0.025
C	25	900	0.030

Find the flow rate Q in each pipe and the pressure in point 3.

Neglect minor losses.

Simple Networks

Example 3:

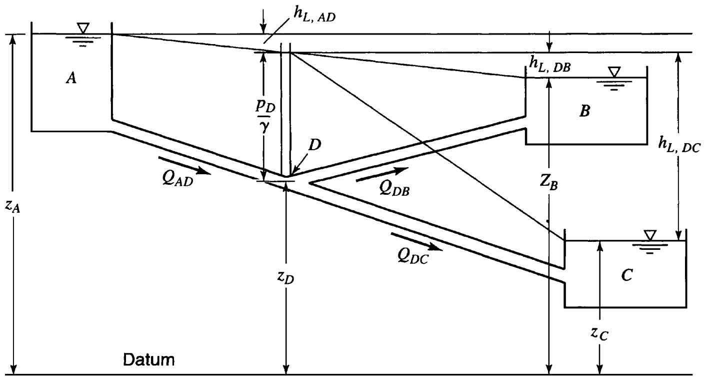


The pipe system connects two reservoir that have an elevation difference of 200 m. This pipe system consists of 200 m of 50 cm concrete pipe (pipe A), that branches into 400 m of 20 cm pipe (pipe B) and 400 m of 40 cm pipe (pipe) in parallel. Pipe B and C join into a single 50 cm pipe that is 500 m long (pipe D).

For f=0.03 for all the pipes, what is the flow rate in the system?

Simple Networks

- Branching pipes



- Flow continuity at the **junction**: the flux of water in the conjunction must be balanced, i.e., what goes in is = to what goes out

$$\sum Q_{in} = \sum Q_{out}$$

- Heads: head at junction is the common value for al the branches. Also, neglect velocity heads.

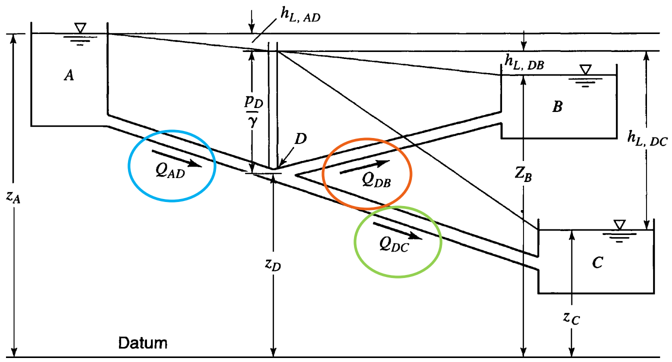
- Write system of equations:

  - ❑ *n*-energy equation (one for each branch)

  - ❑ Continuity equation at the junction

Simple Networks

- Branching pipes



For example, for the system in this figure

- Energy:

Pipe between A and junction D → $$z_A = \frac{P_D}{\gamma} + z_D + h_{L_{AD}}$$

Pipe between B and junction D → $$z_B = \frac{P_D}{\gamma} + z_D - h_{L_{DB}}$$

Pipe between C and junction D → $$z_C = \frac{P_D}{\gamma} + z_D - h_{L_{DC}}$$

- Continuity: $$Q_{AD} - Q_{DB} - Q_{DC} = 0$$

4 equations for 4 unknowns

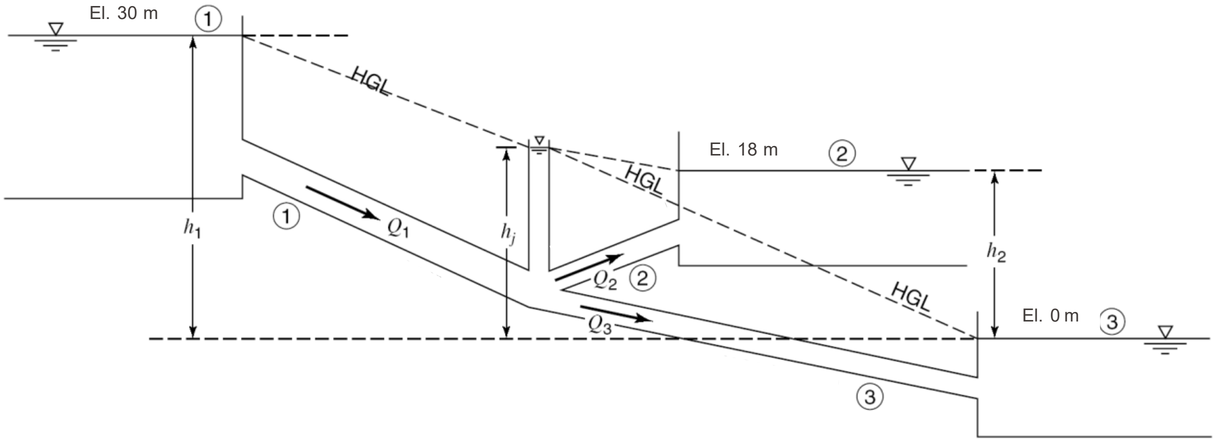
with:

$$h_{L_i} = \frac{f_i L_i}{D_i}\frac{V_i^2}{2g} = \frac{f_i L_i}{D_i}\frac{Q_i^2}{2gA_i^2}$$

Simple Networks

- Branching pipes – Example 4

The reservoirs are connected by riveted steel pipes ($\epsilon = 0.9$ mm). Determine the flow rate in each pipe assuming fully turbulent flow.



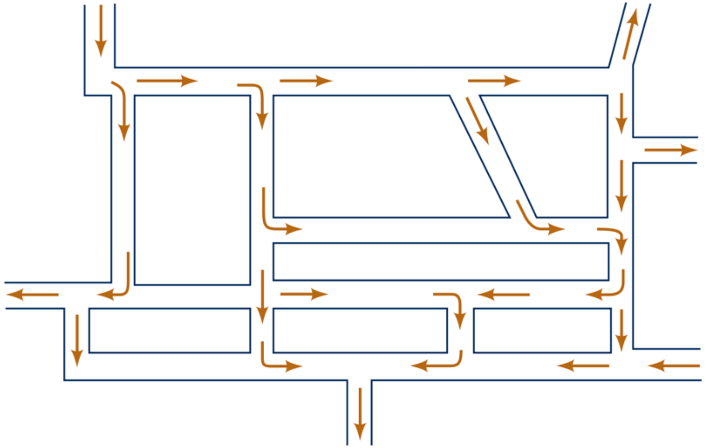
Pipe 1: D_1 = 60 cm, L_1 = 1 km

Pipe 2: D_2 = 50 cm, L_2 = 300 m

Pipe 3: D_3 = 40 cm, L_3 = 1.2 km

Remember $h_L = KQ^2$ with $K = \frac{fL}{D}\left(\frac{1}{2gA^2}\right)$

Complex networks



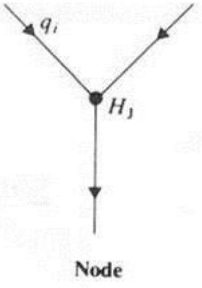
- Systems of pipes with many inlets and outlets
- The direction of the flow is by no means obvious—in fact flow may revert in some cases the depending on the use
- Networks like these are typical of city water distribution system

Solutions of pipe networks uses *nodes* and *loops* equations:

Nodes:

The continuity equation requires that for each *node* (the junction of two or more pipes) the net flowrate is zero

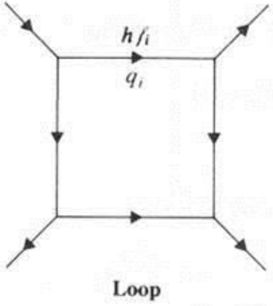
$$\sum_{i=1}^{n} q_i = 0$$



Loops:

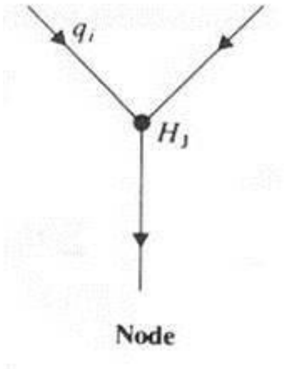
The net pressure difference completely around a *loop* (starting at one location in a pipe and returning to that location) must be zero.

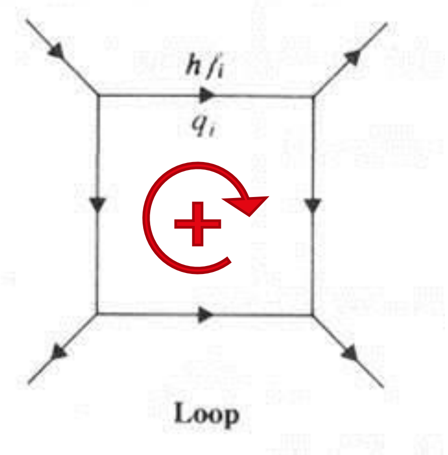
$$\sum_{i=1}^{n} h_{f_i} = 0$$



Note $h_f = h_L$!

Complex networks





Conservation of mass for steady flow: At any node (or junction) in the network

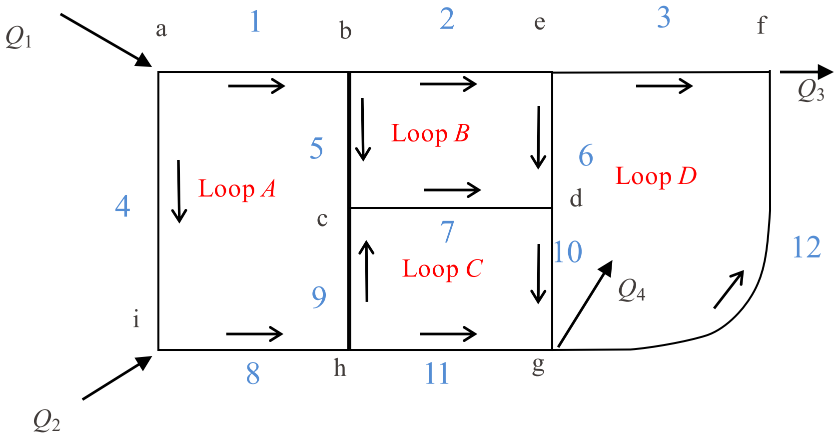
$$\sum_{i=1}^{n} q_i = 0$$

In the context of pipe networks, this expression is called the "junction equation."

Conservation of energy for steady flow: The headloss between any two junctions must be the same regardless of the path taken to get from one junction to another. Following the conversion that in a given loop, clockwise flow is positive and counterclockwise is negative. **The implication of this condition is that the algebraic sum of headlosses around a given loop must be zero.** An equation giving the headloss around a loop is called a "loop equation."

$$\sum_{i=1}^{n} h_{f\,i} = 0$$

Complex networks



For Example:

Consider Loop *B* in the Figure here. The flow in the loop must satisfy each of the governing principles:

- At each junction *b*, *c*, *d*, or *e*, the total inflow must equal the total outflow.
- The head loss due to flow in the counter-clockwise direction (pipes *5* and *7*) must be equal to the headloss due to the flow in the clockwise direction (pipes *2* and *6*).

This can be expressed as

$$h_{f,5} + h_{f,7} = h_{f,2} + h_{f,6} \Rightarrow h_{f,2} + h_{f,6} - h_{f,5} - h_{f,7} = 0$$

which shows that $\sum h_f = 0$

As a general rule, a network with *m* loops and *n* junctions provides a total of *m+(n-1)* independent equations where *m* is the number of "loops" in the network and *n* is the number of "nodes." The network shown in the figure above has m=4 and n=9 thus *m+(n-1)=12*

The Hardy Cross method

- Hardy-Cross (1936) is an algorithm for predicting water flow through pipe networks (multiple pipelines).
- Also known as the "loop" or "head balance" method.
  - **This is used when the total volume rate of flow through the network is known but the heads or pressures at junctions within the network are unknown.**
- Overall procedure:
  - **Step 1**: assume values of q_i in pipes to satisfy $\sum q_i = 0$ at the nodes.
  - **Step 2**: calculate h_{fi} from q_i (using any suitable method for calculating head loss).
  - **Step 3**: If $\sum h_{fi} = 0$, then the solution is correct.
  - **Step4**: If $\sum h_{fi} \neq 0$, then apply a correction δq to q_i factor and return to step 2
- Let's see how to calculate the correction factor δq

The Hardy-Cross method: Derivation

- Let's start from the simplified version of the headloss equation introduced before:

$$h_f = KQ^2$$

with $K = \frac{fL}{D}\left(\frac{1}{2gA^2}\right)$

- Let's assume that the correct value of Q is composed by an initially assumed value q_i and a minor correction value δq for each pipe i
- So we can write:

$$h_f = K(q_i + \delta q)^2$$

which can be expanded in

$$h_f = Kq_i^2 + 2Kq_i\delta q + K\delta q^2$$

- Since δq is small, we can neglect the second order term → $\delta q^2 \cong 0$

so:

$$h_f = K(q_i + \delta q)^2 \approx Kq_i^2 + 2Kq_i\delta q = Kq_i|q_i| + 2Kq_i\delta q$$

- Note that here I rewrote Kq_i^2 as $Kq_i|q_i|$ to keep track of the sign of the flow!! (so $\mathrm{h_f} = \mathrm{Kq_i^2} = Kq_i|q_i|$)

The Hardy-Cross method: Derivation

- Summing h_f for all links in the loop being evaluated resulting in

$$\sum h_f = \sum(Kq_i|q_i| + 2Kq_i\delta q) = \sum Kq_i|q_i| + \delta q \sum 2Kq_i$$

- Now remember that for any loop in a pipe network, conservation of energy requires that $\sum h_f = 0$, so:

$$\sum Kq_i|q_i| + \delta q \sum 2Kq_i = 0$$

- Rearranging to express δq:

$$\delta \mathrm{q} = \frac{-\sum Kq_i|q_i|}{\sum 2Kq_i} = \frac{-\sum Kq_i|q_i|}{\sum 2Kq_i \frac{|q_i|}{|q_i|}}$$

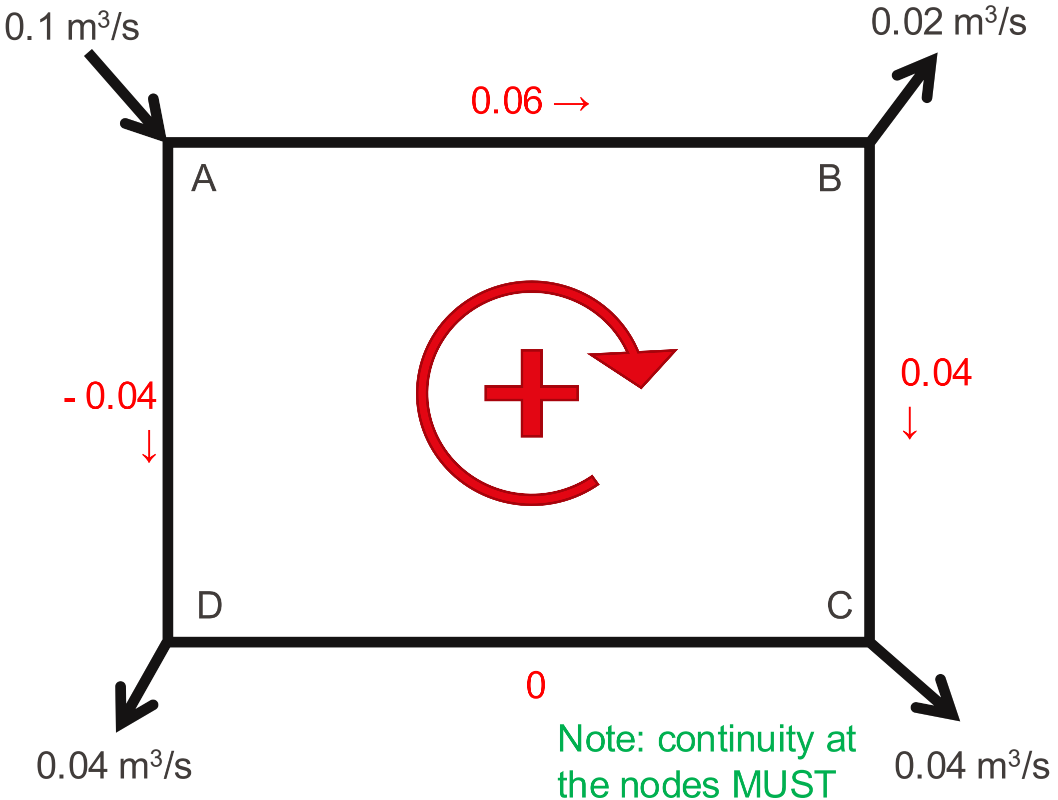
Mutliply and divide by $|q_i|$ the denominator

and remembering that $\mathrm{h_f} = \mathrm{Kq_i^2} = Kq_i|q_i|$, then:

$$\delta \mathrm{q} = -\frac{\sum h_f}{2\sum \left|\frac{h_f}{q_i}\right|}$$

The absolute value in the denominator is required because δq was given the same sign in all pipes in the loop.
δq is then applied to the next iteration with the sign

Example: single loop



Note: continuity at the nodes MUST match!!

- In this example all the pipes are 1000 m long. AB and DA have a diameter of 150 mm while BC and CD have a diameter of 125 mm. All have a roughness ϵ of 0.03 mm.
- The first stage of the calculation is to assume flows in the pipes AB, BC, CD and DE.
- It does not matter what flows are assumed provided they are consistent with the known inflows and outflows.
- The sign convention (clockwise flows are positive) must be observed.

- We set up a tabulation of the results and calculate the flow velocities and corresponding Re from the estimated flows and pipe diameters.

- The next stage is to calculate the head loss for each pipe based on u, L and D, using any suitable method to find f (Moody, C&W, etc…) and then calculate $K = \frac{fL}{D}\left(\frac{1}{2gA^2}\right)$

Pipe	q_i (m^3/s)	L (m)	D (m)	A	u (m/s)	ϵ	Re	f	K	h_{fi} (m)	$\lvert h_{fi}/q_i \rvert$
AB	0.06	1000	0.15	0.018	3.40	0.00003	509296	0.015474	16837.1093		
BC	0.04	1000	0.125	0.012	3.26	0.00003	407437	0.016147	43718.2747		
CD	0	1000	0.125	0.012	0.00	0.00003	0	0	0		
DA	-0.04	1000	0.15	0.018	-2.26	0.00003	339531	0.015994	17402.9163		

- We can now calculate $h_{f_i} = K q_i^2 (m)$ and the ratio $| h_{f_i} / q_i |$

- Note that if the flow is negative in the sign convention, the head loss is negative (i.e. it is a head gain!) too.

- Note also that the numbers in this example are not especially realistic – the head losses are massive.

Pipe	q_i (m^3/s)	L (m)	D (m)	A	u (m/s)	ϵ	Re	f	K	h_{fi} (m)	$\|h_{fi}/q_i\|$
AB	0.06	1000	0.15	0.018	3.40	0.00003	509296	0.015474	16837.1093	60.61	1010.2
BC	0.04	1000	0.125	0.012	3.26	0.00003	407437	0.016147	43718.2747	69.95	1748.7
CD	0	1000	0.125	0.012	0.00	0.00003	0	0	0	0.00	0.0
DA	-0.04	1000	0.15	0.018	-2.26	0.00003	339531	0.015994	17402.9163	-27.84	696.1

- we can now calculate the correction factors which involves summing each of the last two columns

Pipe	q_i (m^3/s)	L (m)	D (m)	A	u (m/s)	ϵ	Re	f	K	h_{fi} (m)	$\lvert h_{fi}/q_i \rvert$
AB	0.06	1000	0.15	0.018	3.40	0.00003	509296	0.015474	16837.1093	60.61	1010.2
BC	0.04	1000	0.125	0.012	3.26	0.00003	407437	0.016147	43718.2747	69.95	1748.7
CD	0	1000	0.125	0.012	0.00	0.00003	0	0	0	0.00	0.0
DA	-0.04	1000	0.15	0.018	-2.26	0.00003	339531	0.015994	17402.9163	-27.84	696.1
										$\sum h_{f,i}$	$\sum h_{f,i}/q_i$
										102.72	**3455.07**

$$\delta q = -\frac{\sum h_f}{2\sum\left|\frac{h_f}{q_i}\right|} = \frac{102.72}{2 \times 3455.07} = -0.015$$

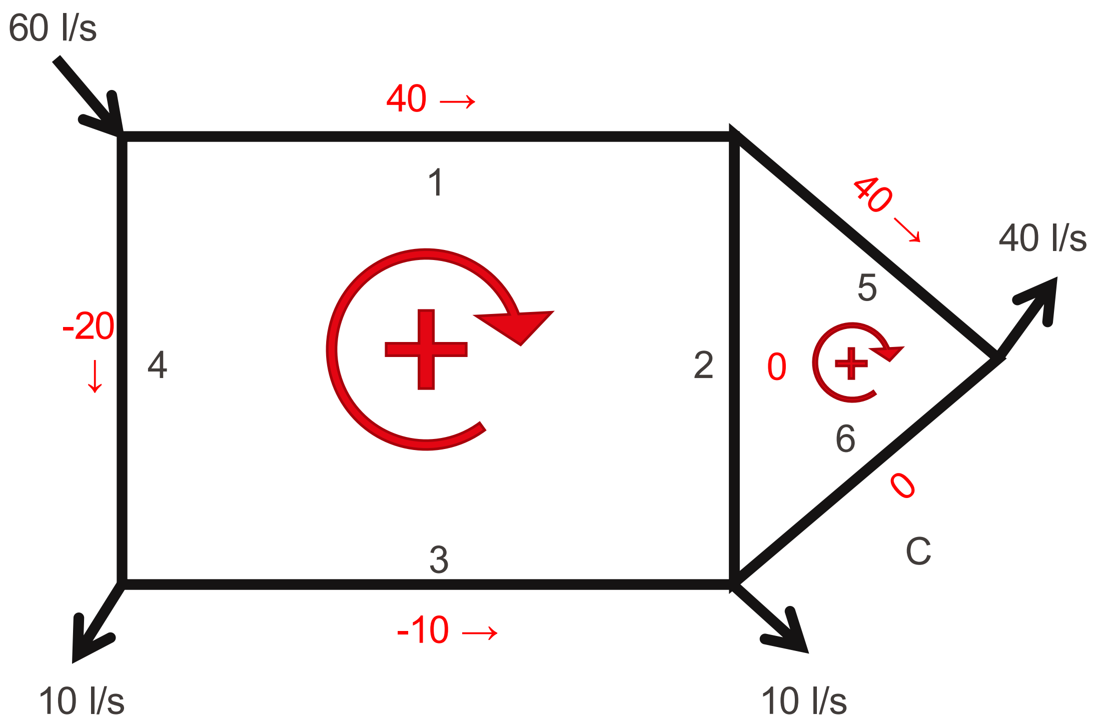
- **The correction factor is now *added* to the original estimate of the flow for *every* pipe in the system**. For example, the new flow in AB becomes 0.06 + -0.015 = 0.045, and you start over, recalculating all the parameters and the new correction factor

Pipe	q_i (m³/s)	L (m)	D (m)	A	u (m/s)	ϵ	Re	f	K	h_{fi} (m)	$\lvert h_{fi}/q_i \rvert$
AB	0.045	1000	0.15	0.018	2.55	0.00003	383119	0.015822	17215.7647	35.07	777.0
BC	0.025	1000	0.125	0.012	2.05	0.00003	256025	0.016814	45524.1884	28.76	1144.3
CD	-0.015	1000	0.125	0.012	-1.21	0.00003	151412	0.01788	48410.4014	-10.70	0.0
DA	-0.055	1000	0.15	0.018	-3.10	0.00003	465707	0.015575	16947.0064	-51.01	929.8
										$\sum h_{f,i}$	$\sum h_{f,i}/q_i$
										2.12	**2851.09**

- In this new iteration, $\delta q = -0.0004$

- The iterations continue until the sum of the penultimate column is close to zero and the correction factor is negligibly small. **Typically, 3 or 4 iterations are sufficient**. Here we're already pretty close.

Example: complex loops



(the black numbers next to the pipes are the # of the pipe, red is the guessed flow in it)

- Same procedure, but setting up *two tabulations* and thus calculating *two different correction factors* δq_i, one of each loop
- The important new tricky thing to note here are **the pipes that are shared between neighboring loops!** In this example is pipe 2.

Note that because of the sign convention, the flow direction is technically opposite (going down in the left-hand side (LHS) and up in the right-hand side (RHS)). Therefore, in that pipe the flow must be THE SAME BUT OPPOSITE SIGN, i.e., if we take it positive in the LHS, in the RHS is negative.

Left Hand Loop				Right Hand Loop			
Pipe	**q_i (litres/s)**	**h_{fi} (m)**	**h_{fi}/q_i**	**Pipe**	**q_i (litres/s)**	**h_{fi} (m)**	**h_{fi}/q_i**
1	40	28.5	0.71	**5**	40	28.5	0.71
2	0	0	0	**6**	0	0	0
3	-10	-2.2	0.22	**2**	0	0	0
4	-20	-8	0.40				
	Σ=	**18.3**	**1.33**		**Σ=**	**28.5**	**0.71**

Note: I'm not traducing l/s in m^3/s in the ratio h_{fi}/q_i because I will calculate δq in l/s

These two must always be equal and opposite – same pipe in the two loops, but opposite sign conventions

- We now calculate *two* correction factors, one for each loop: $\delta q\ (LHS) = -6.87$ l/s and $\delta q\ (RHS) = -20$ l/s
- We then apply each correction factor to the flow estimates for its own loop, so the new flow in pipe 1 is as follows:

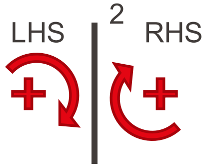
  40 + - 6.87 = 33.1 litres/s

- While that for pipe 5 is:

  40 + -20 = 20.0 litres/s

- **BUT (and this is the tricky part) pipes that appear in both loops have both correction factors applied, taking into account the sign convention**.
- So, the new flow for pipe 2 in the LHS loop becomes:

  0 + -6.87 - -20 = 13.1 litres/s



  while that for pipe 2 in the RHS loop is:

  0 + -20 - -6.87 = -13.1 litres/s

  which is of course still equal and opposite to its equivalent in the LHS loop where the sign convention dictates its flow has the opposite sign.

- The calculation continues as before until the sum of the head losses around the loops becomes small and the correction factors negligible.

Procedure for implementing Hardy Cross (recap)

1) Estimate (guess) the flow q_i in each pipe in the system applying the constraint that conservation of mass is satisfied at each junction.

2) The flow distribution chosen in Step 1 is not likely to satisfy the conservation of energy requirement, therefore, the chosen discharges, q_i, should be adjusted by adding corrections, δq, to the assumed discharges. Starting with Loop 1, the procedure for making corrections is as follows:

   a) Compute values of h_{f_i}, $|h_{f_i}/q_i|$, and δq

   b) Compute new discharge estimates as $q_{i_{new}} = q_{i_{old}} + \delta q$

   c) Update the discharge estimates in loops that are adjacent to the current loop

3) Repeat Step 2 for each loop in the pipe network before going to step 4

4) You have just completed on iteration of calculations. Review the δq values for each loop. If each δq value is less than what you consider to be an acceptable error then a solution has been found. If one or more of the loops has a δq value that is unacceptable, then return to Step 2 to begin a new iteration of calculations.

Network-solving Software

- In practice, these methods are typically implemented in commercial software that are used in practical designs and companies.
- Later we will see one of these software: EPANET

📢 Announcement for next week 📢

Next Friday will be dedicated to the learning and use of the software EPANET which will be used for HOMEWORK 1

- Bing a laptop if you can
  - You can go ahead and install EPANET or you can use the EPFL Virtual Machine VDI (you'll find the software already installed)
  - You can access VDI on a table BUT it's going to be tricky because you need a keyboard to enter parameters
- More instructions on this will be posted on Moodle in the next couple of days