

Ungated Spillway Design Example

Module D

Problem

Design an uncontrolled overflow ogee crest for a spillway to discharge

$$Q = 5.66 \times 10^2 \text{ m}^3/\text{s}.$$

The upstream face of the crest is sloped 3:3 (i.e. 1H:1V). A bridge is to span the crest:

- Bridge piers are 0.46 m wide with rounded noses.
- The pier-contraction coefficient is $K_p = 0.05$.
- The abutment coefficient is $K_a = 0.10$.

The bridge span (maximum center to center of the piers) is not to exceed 9.14 m. The maximum expected head over the crest is

$$H_e = 3.05 \text{ m}$$

Neglect the velocity of approach head. The distance of the spillway crest to the reservoir bottom at the dam is

$$P = 5.49 \text{ m}$$

Your design should be based upon the design head being equal to the maximum expected head. What will be the discharges for different expected heads of up to 4.9 m.

Solution

1. Design head and basic discharge coefficient We take the design head as the maximum expected head:

$$H_d = H_e = 3.05 \text{ m}.$$

The ratio of crest height to design head is

$$\frac{P}{H_d} = \frac{5.49}{3.05} \approx 1.8.$$

From the standard ogee-spillway design curves for this value of P/H_d , obtain a basic discharge coefficient for a vertical upstream face.

From the curves:

$$C_0 \approx 2.18$$

Then apply a correction for the actual upstream slope 3:3.

$$\frac{C_{\text{inclined}}}{C_{\text{vert}}} \approx 0.99,$$

Thus the design coefficient is

$$C = C_0 \left(\frac{C_{\text{inclined}}}{C_{\text{vert}}} \right) \approx (2.18)(0.99) \approx 2.15.$$

This value C corresponds to the design head H_d .

2. Effective crest length for the design discharge The discharge relation is

$$Q = C L_e H_e^{3/2},$$

where H_e is the actual total head over the crest.

At the design condition, where $H_e = H_d$, the effective crest length is

$$L_e = \frac{Q}{C H_d^{3/2}}.$$

Compute

$$H_d^{3/2} = 3.05^{1.5} \approx 5.33 \text{ m}^{3/2},$$

so

$$L_e = \frac{5.66 \times 10^2}{(2.15)(5.33)} \approx 49.5 \text{ m}.$$

This is the effective crest length (after end contractions).

3. Number of piers and total net crest length The net crest length L_0 (between abutments, i.e., $L_0 = L - wN$) is related to the effective length by the USBR contraction expression:

$$L_e = L_0 - (2NK_p + K_a) H_e.$$

At the design condition $H_e = H_d$:

$$L_e = L_0 - (2NK_p + K_a) H_d.$$

We don't know how many pillars to build for the bridge a priori. We need to find a number that satisfy both the requirement of a maximum bridge span (center to center of the piers) not exceeding 9.14 m, while at the same time allowing for the calculated L_e

Trial $N = 4$.

$$2NK_p + K_a = 2(4)(0.05) + 0.10 = 0.50, \quad L_0 = 49.5 + 0.50(3.05) \approx 51.0 \text{ m}.$$

There are $N + 1 = 5$ spans, so

$$S = \frac{51.0}{5} \approx 10.2 \text{ m} > 9.14 \text{ m},$$

which is not acceptable.

Trial $N = 5$.

$$2NK_p + K_a = 2(5)(0.05) + 0.10 = 0.60, \quad L_0 = 49.5 + 0.60(3.05) \approx 51.3 \text{ m}.$$

Now there are $N + 1 = 6$ spans, so

$$S = \frac{51.3}{6} \approx 8.6 \text{ m} < 9.14 \text{ m},$$

which is acceptable.

4. Crest length as a function of actual head For any operating head H_e ,

$$L_e(H_e) = L_0 - (2NK_p + K_a) H_e.$$

With $L_0 = 51.3 \text{ m}$ and $2NK_p + K_a = 0.60$,

$$L(H_e) = 51.3 - 0.60 H_e.$$

5. Discharge coefficient for heads $H_e \neq H_d$ Use the curve giving

$$\phi(H_e/H_d) = \frac{C(H_e)}{C_0}.$$

Thus

$$C(H_e) = \phi(H_e/H_d) C_0.$$

Representative values of ϕ are

H_e/H_d	0.10	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60
ϕ	0.820	0.852	0.900	0.940	0.972	1.000	1.025	1.050	1.070

See figure (c) in slide 65 and 67 of the class notes.

6. Ogee crest profile The downstream profile of the ogee crest is expressed in non-dimensional form as

$$\frac{y}{H_d} = \frac{1}{K} \left(\frac{x}{H_d} \right)^n,$$

where:

- x is measured along the crest from the vertical through the highest point of the crest (origin),
- y is measured downward from the crest,
- H_d is the design head.

For this design where $P/H_d \approx 1.8$. we adopt

$$K = 2, \quad n = 1.85.$$

Hence the non-dimensional equation of the crest profile becomes

$$\frac{y}{H_d} = \frac{1}{2} \left(\frac{x}{H_d} \right)^{1.85},$$

or, in dimensional form,

$$y = \frac{H_d}{2} \left(\frac{x}{H_d} \right)^{1.85}.$$

With $H_d = 3.05$ m, this profile defines the downstream shape of the ogee crest used in the design.

7. Discharge for various operating heads For any operating head H_e , the discharge over the crest is given by

$$Q(H_e) = C(H_e) L(H_e) H_e^{3/2},$$

where

$$L(H_e) = 51.3 - 0.60 H_e, \quad H_d = 3.05 \text{ m.}$$

and because the heads now considered are different from H_e , the correction to C is made by

$$C = \phi(H_e/H_d) C_0 \left(\frac{C_{\text{inclined}}}{C_{\text{vert}}} \right)$$

with $C_0 = 2.18$ and $C_0 \left(\frac{C_{\text{inclined}}}{C_{\text{vert}}} \right) = (2.18)(0.99) \approx 2.15$. The factor $\phi(H_e/H_d)$ is obtained from standard ogee-spillway curves and accounts for the variation of the discharge coefficient with the ratio H_e/H_d as shown before.

Table 1: Head–discharge relationship for the designed ogee spillway.

H_e (m)	H_e/H_d	ϕ	$C(H_e)$	$L(H_e)$ (m)	$H_e^{3/2}$ ($\text{m}^{3/2}$)	Q (m^3/s)
0.30	0.10	0.820	1.76	51.12	0.164	14.8
0.61	0.20	0.852	1.83	50.93	0.476	44.5
1.22	0.40	0.900	1.94	50.57	1.35	131.9
1.83	0.60	0.940	2.02	50.20	2.48	251.2
2.44	0.80	0.972	2.09	49.84	3.81	396.9
3.05	1.00	1.000	2.15	49.47	5.33	566.5
3.66	1.20	1.025	2.20	49.10	7.00	757.7
4.27	1.40	1.050	2.26	48.74	8.82	970.8
4.88	1.60	1.070	2.31	48.37	10.78	1199.6

The resulting head–discharge relationship is summarized in Table 1.

At the design head $H_d = 3.05$ m, the discharge is

$$Q(H_d) \approx 5.66 \times 10^2 \text{ m}^3/\text{s},$$

which satisfies the required design discharge.

If we plot Q and H_e we obtain the discharge rating curve for the designed ungated overflow spillway. See plot below:

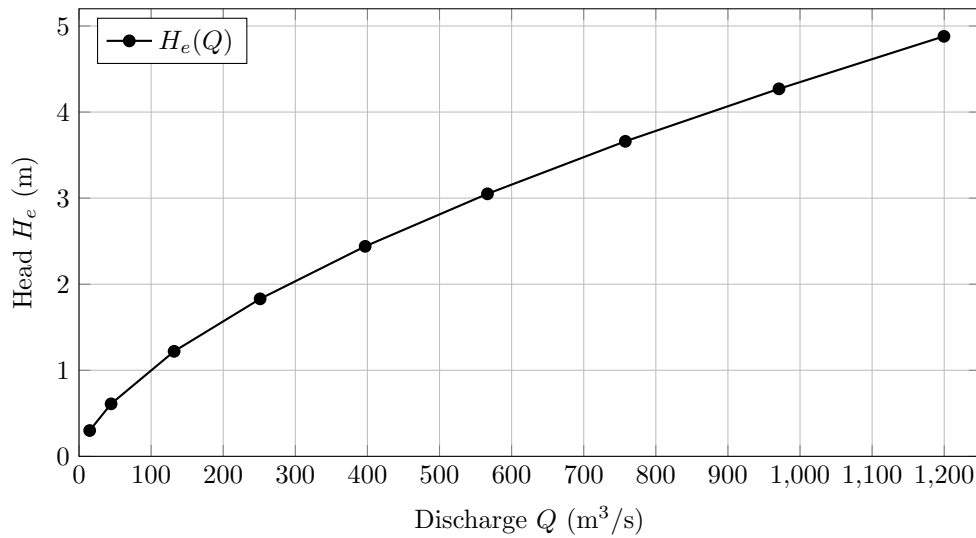


Figure 1: Relationship between discharge Q and head H_e for the designed ogee spillway.