Hydraulic Engineering and Infrastructures

Civil Engineering Department

Flow profiles

1 Sea Level Variation

A very wide river that discharges its waters into the sea has a discharge per unit width of $q = 5\ \mathrm{m}^2/\mathrm{s}$, a bed slope of $i = 0.0005$, and a roughness coefficient of $n = 0.03$.

As shown in Figure 1, after the river mouth it can be assumed that the seabed has an 8% slope. The tidal level may vary from elevation $+0.5$ m up to $+4.5$ m. Determine the normal and critical depths in the river before the mouth, and perform a qualitative analysis of the hydraulic profile in the river under the extreme tidal conditions.

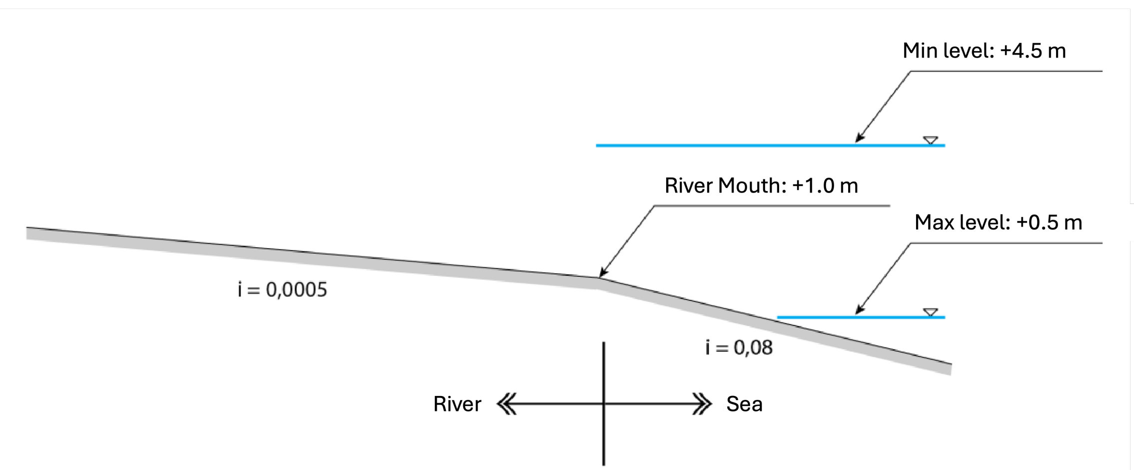


Figure 1: Schematic representation of the river–sea system.

Recommended procedure

1. Compute y_c via the Froude number and y_n via the Chézy or Manning equation.

2. Determine the control point(s) (CP) which correspond to where the regime changes (eg. change of slope). When drawing the qualitative profiles, each CP will be the starting point.

3. Determine the direction in which the drawing will be done, starting from the CP. If $y > y_c$, the flow is subcritical meaning that you move upstream from the CP. If $y < y_c$, the flow is supercritical meaning that you move downstream from the CP.

Solution

In the case of a very wide river (of width b and depth y such as $b >> y$), we have:

$$A = by$$
$$P = 2y + b$$
$$R_h = \frac{A}{P} = \frac{by}{2y+b} \approx y$$

We are given q, the flow rate per unit width. Thus, $q = \frac{Q}{b}$.

The bed slopes are such that $S_{01} = 0.0005$ and $S_{02} = 0.08$.

Critical depth

To determine the critical depth, we need to use the criticity criterion given by the Froude number $Fr = 1$.

$$Fr = \frac{V}{\sqrt{gy}} \Leftrightarrow Fr^2 = \frac{Q^2}{gyA^2}$$
$$\Leftrightarrow Fr^2 = \frac{Q^2}{gy(by)^2}$$
$$\Leftrightarrow Fr^2 = \frac{q^2}{gy^3}$$

with y being the water depth and b the width of the river.

For $Fr = 1$, we have $y = y_c$. This gives us:

$$Fr^2 = 1 \Leftrightarrow \frac{q^2}{gy_c^3} = 1$$
$$\Leftrightarrow y_c = (\frac{q^2}{g})^{1/3}$$
$$\Leftrightarrow y_c = (\frac{5^2}{9.81})^{1/3}$$
$$\Leftrightarrow y_c = 1.366 \text{ m}$$

As q is constant throughout the length of the river, the critical depth y_c is the same everywhere.

Normal depths

We can compute the normal depth using the Chézy equation:

$$V = C\sqrt{R_h S} = \frac{y^{2/3}}{n}\sqrt{S} \tag{1}$$

with $C = \frac{R_h^{1/6}}{n} = \frac{y^{1/6}}{n}$, the Chézy constant and S, the slope of the energy line.

By continuity,

$$V = \frac{Q}{A} = \frac{qb}{by} = \frac{q}{y} \tag{2}$$

Then, equating (1) and (2):

$$\frac{q}{y} = \frac{y^{2/3}}{n}\sqrt{S} \Leftrightarrow y^{5/3} = \frac{nq}{\sqrt{S}}$$

To compute the normal depth, we have to assume $S = S_0$ as the normal depth is the depth for which the flow is uniform, meaning that the energy slope, S, is the same as the bed slope, S_0.

As the bed slope changes along the length of the river, we have:

$$y_{n1}^{5/3} = \frac{nq}{\sqrt{S_{01}}} \Rightarrow y_{n1} = 3.133\text{ m } > y_c$$
$$y_{n2}^{5/3} = \frac{nq}{\sqrt{S_{02}}} \Rightarrow y_{n1} = 0.683\text{ m } < y_c$$

Qualitative drawing of the water profile

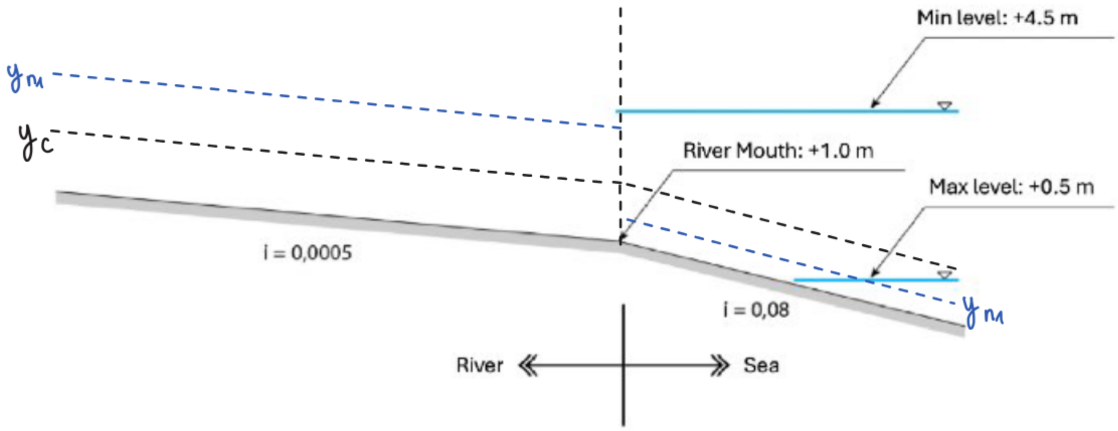


Figure 2: Schematic representation of the river–sea system with the control point (green) and y_{n1}, y_{n2} and y_c.

The control point (CP) is located where there is a change of slope, as shown in Fig. 2. On the left of the CP, $y_{n1} > y_c$, meaning that we are in a subcritical regime. This indicates that we need to draw the water profile from the CP upstream. On the right of the CP, $y_{n2} < y_c$, meaning that we are in a supercritical regime. This indicates that we need to draw the water profile from the CP downstream.

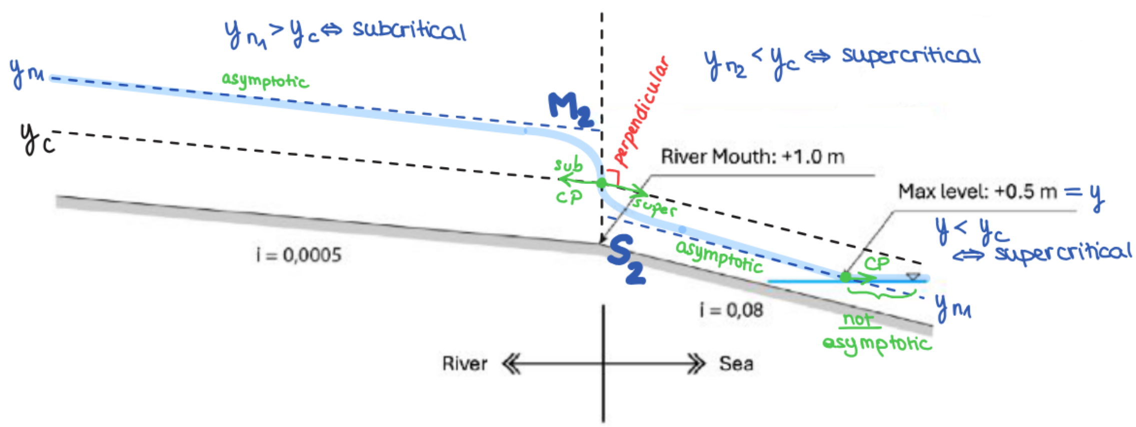


Figure 3: Schematic representation of the river–sea system, when the water reaches the maximum level of 0.5 m.

As is shown in Fig. 3, the water profile on the left side of the CP reaches y_{n1} asymptotically. At the CP, the water profile crosses y_c perpendicularly. Then, on the right of the CP, the water profile once again reaches y_{n2} asymptotically. Finally, it reaches the maximum level of 0.5 m without changing profile as it is in the supercritical regime.

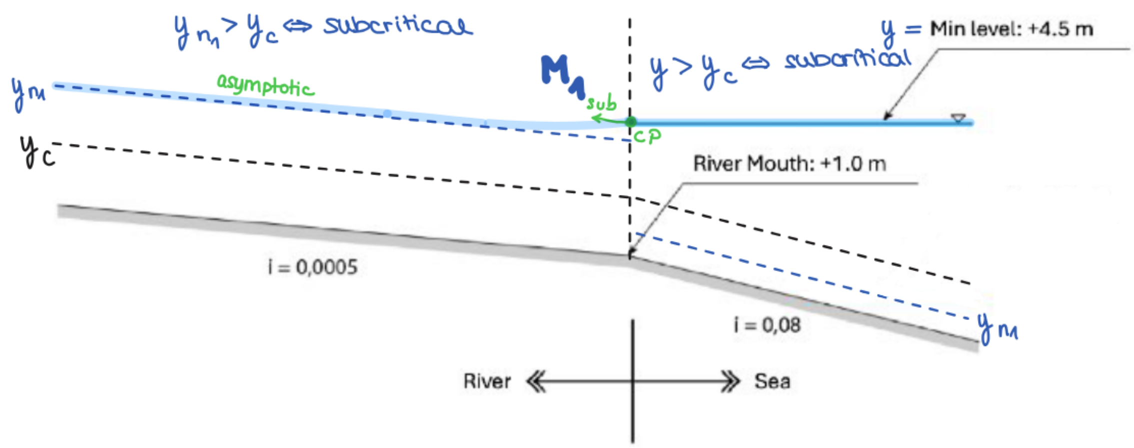


Figure 4: Schematic representation of the river–sea system, when the water reaches the minimum level of 4.5 m.

In Fig. 4, the water level reaches the minimum level of 4.5 m asymptotically from y_{n1}.

The reasons why the y_c line must always be crossed perpendicularly and the y_n line must always be reached asymptotically are explained by the following equation:

$$\frac{dy}{dx} = \frac{S_0 - S}{1 - Fr^2}$$

with S_0 being the slope of the bed and S, the slope of the energy line.

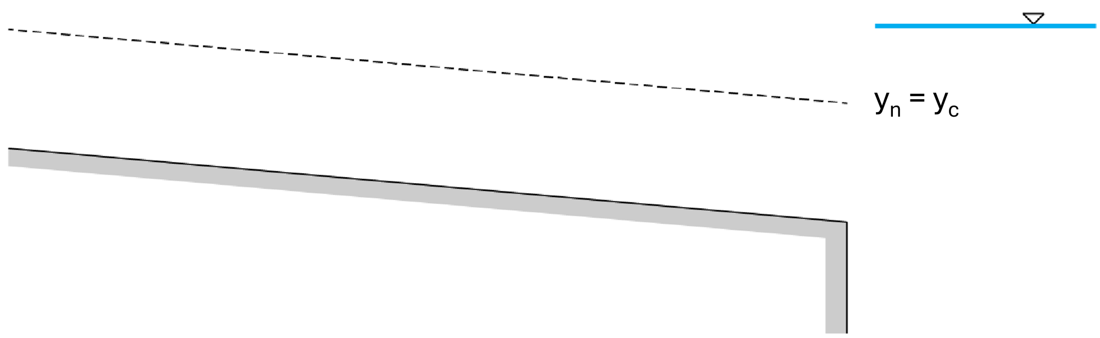
When in the supercritical regime ($y = y_c \Leftrightarrow Fr = 1$), the denominator $1 - Fr^2$ of the above equation tends to 0, which causes $\frac{dy}{dx}$ to diverge to $+\infty$.

When reaching y_n, we are reaching uniform flow, meaning that $S_0 = S$ which, in turn, means $\frac{dy}{dx} = 0$.

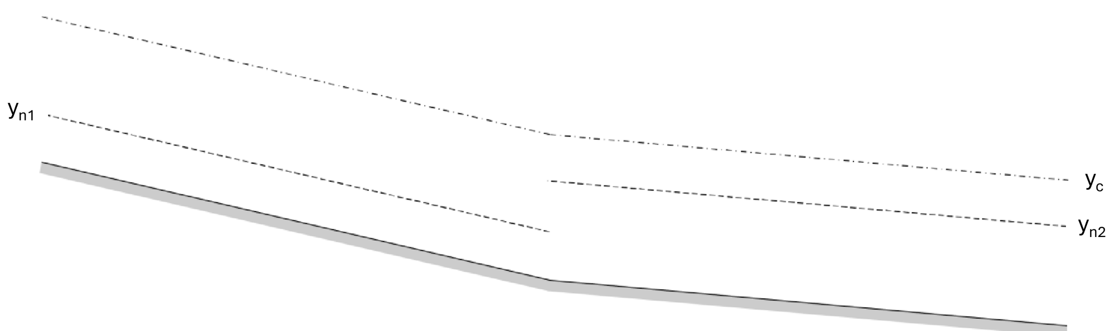
2 Qualitative Profiles

In the following cases, perform a qualitative analysis of the water-surface profile (hydraulic grade line), identifying the classification (type), control points, and starting points together with their computation direction. Draw your sketches on the diagrams provided below.

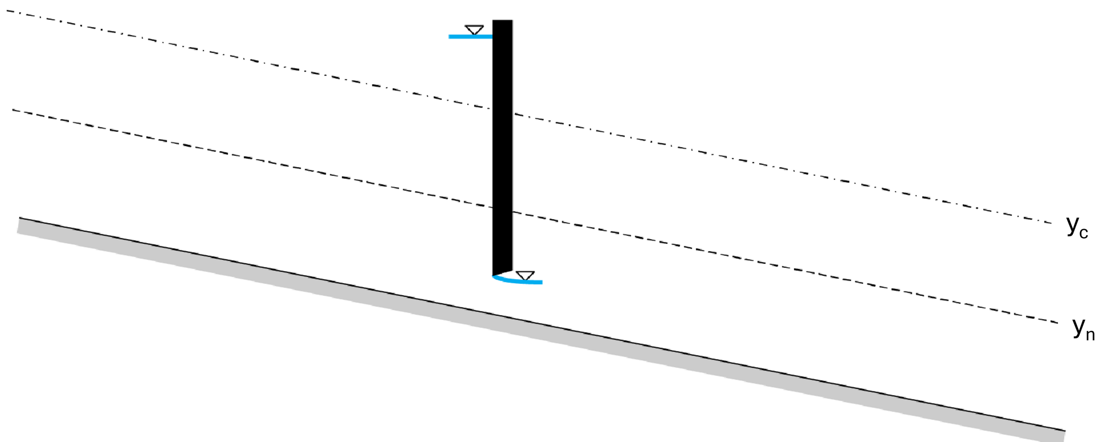
(a)



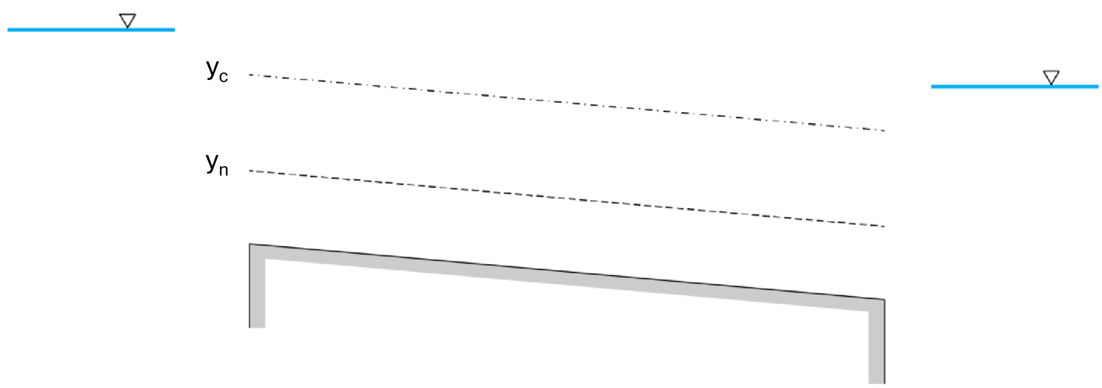
(b)



(c)



(d)



Solution

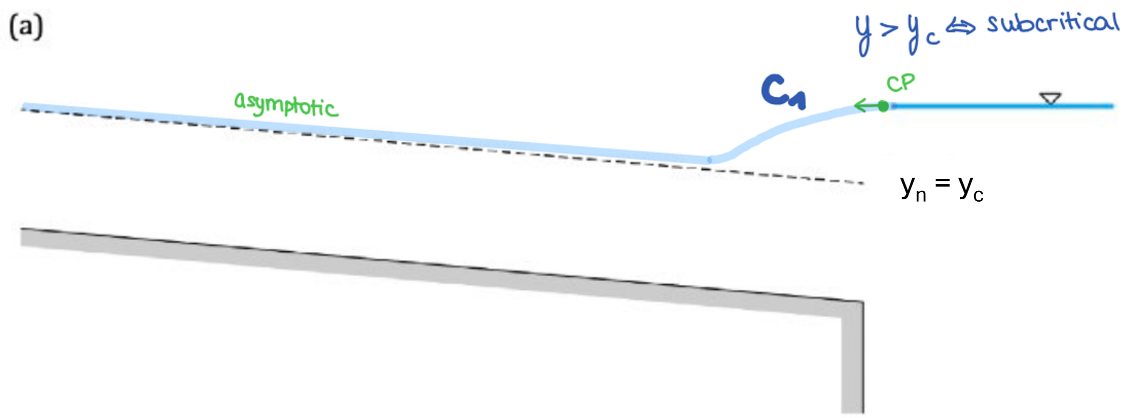


Figure 5

Fig. 5

The control point is placed at the reservoir of height $y > y_c$. There, we are in the subcritical regime. This means that the reservoir has an influence on the upstream profile. As $y_c = y_n$, we follow a C1 profile to reach y_n from y.

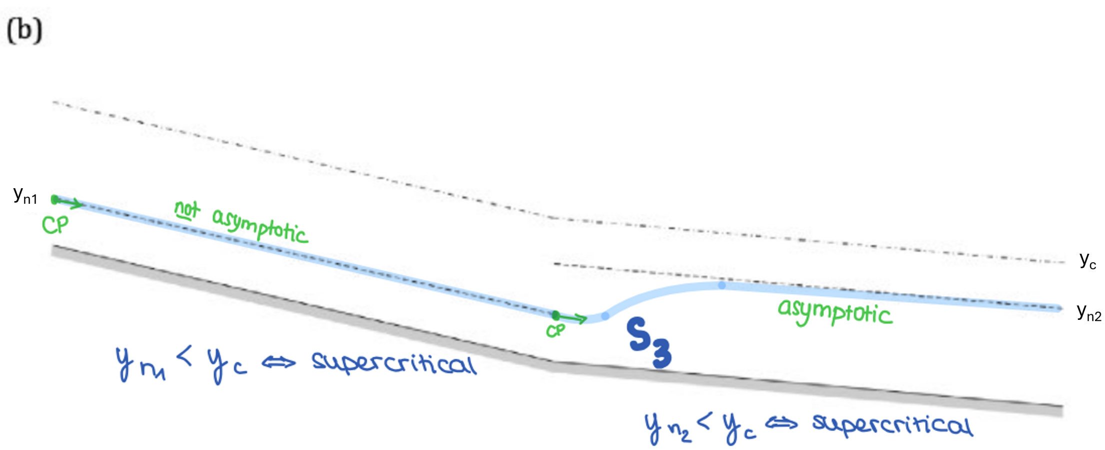


Figure 6

Fig. 6

For both slopes, we have $y_{n1} < y_c$ and $y_{n2} < y_c$, which means that we are in the supercritical regime throughout the length of the channel. We then need to draw downstream from the control points and the change of slope will have no influence on the profile upstream. This is why we first follow y_{n1} before suddenly following a S3 profile to reach y_{n2}.

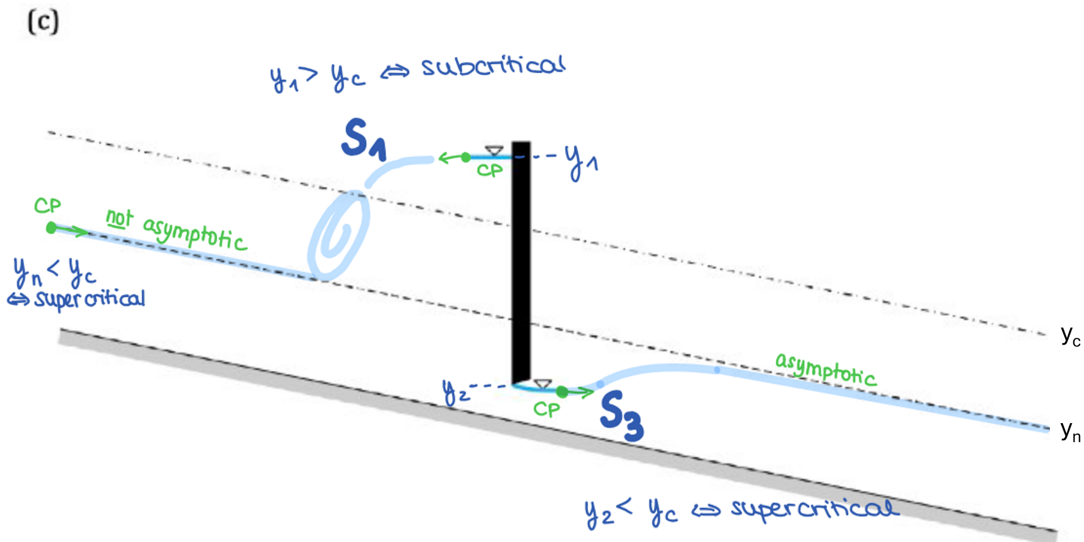


Figure 7

Fig. 7

We assume that we are at $y = y_n$ at the start. As $y_n < y_c$, we are in the supercritical regime and nothing downstream impacts the profile, which is why the profile is not asymptotic. To reach y_1, we need to perform a hydraulic jump as $y_1 > y_c > y_n$, following a S1 profile.

On the other side of the wall, we start from $y_2 < y_n < y_c$ therefore, we are in the supercritical regime. The goal of the water profile is always to reach y_n, which is always done asymptotically. The water profile is thus S3.

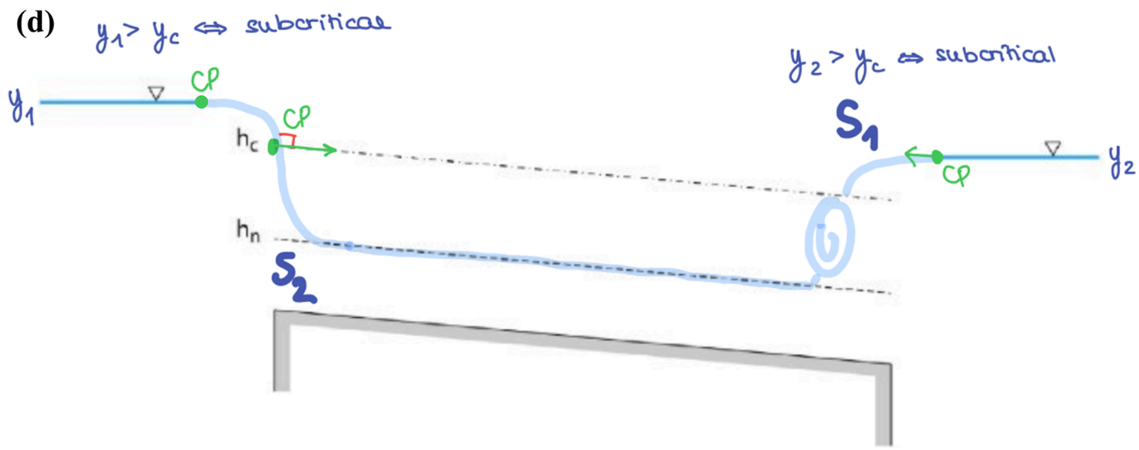


Figure 8

Fig. 8

We know that we need to reach y_n from the reservoir of height y_1. As $y_1 > y_c$, the regime is subcritical. However, $y_n < y_c$, meaning that to reach y_n, we need to cross y_c, which is always done perpendicularly. We then follow a S1 profile to cross y_c and then a S2 profile to reach y_n asymptotically.

Then, from y_n, we need to reach the second reservoir of height y_2. As we are leaving y_n, we need to perform a hydraulic jump. To rejoin the jump from the reservoir, we follow a S1 profile as we are in the subcritical regime ($y_2 > y_c$).

3 Spillway into stilling basin

The spillway of a reservoir has an approach channel with a rectangular cross-section with a (base of $10\,\mathrm{m}$, side slopes of 1/1 (H/V)) that ends in a weir of height $a = 2\,\mathrm{m}$.

There is a spillway with a rectangular cross-section after the weir with same width of 10 m, dropping 20 m and ending in a stilling basin where a hydraulic jump is supposed to form.

Downstream of the basin, a step of 4 m discharges the water into a river with a steep slope.

a) Draw the profile qualitatively, noting the supercritical or subcritical states. What happens when the step's height c is small? What happens when it is big? Where are the main head losses?

b) Estimate the water depth y_2 above the weir and the corresponding Froude number.

c) Compute the discharge when the water depth y_1 in the approaching channel is of 5 m.

d) Assuming regular losses along the spillway are small, compute the water depth y_3 at the end of the spillway and the corresponding Froude number.

e) From now on, we assume the step is big. Compute the water depth y_4 after the hydraulic jump.

f) What should be the water depth y_5 over the step? Estimate the value of the necessary step height c to force a hydraulic jump in the stilling basin. Assume the energy losses on the step are negligible.

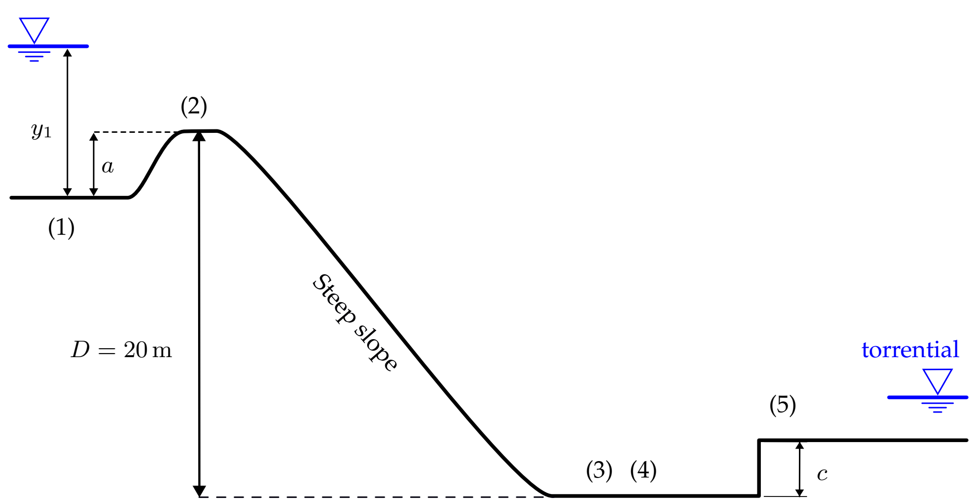


Figure 9: Schema of the spillway.

Objectives and guidance

This exercise focuses on using the concepts of specific energy, critical flow, the solving for water depth from a known head (iterative process) and considering head losses around a hydraulic jump. Combining these points, the hydraulic profile can be drawn for this exercise.

Solution

a) The starting point is the spillway which is long enough for the flow to become supercritical. When the step is small, the flow remains supercritical. This possibility is shown in Figure 10.

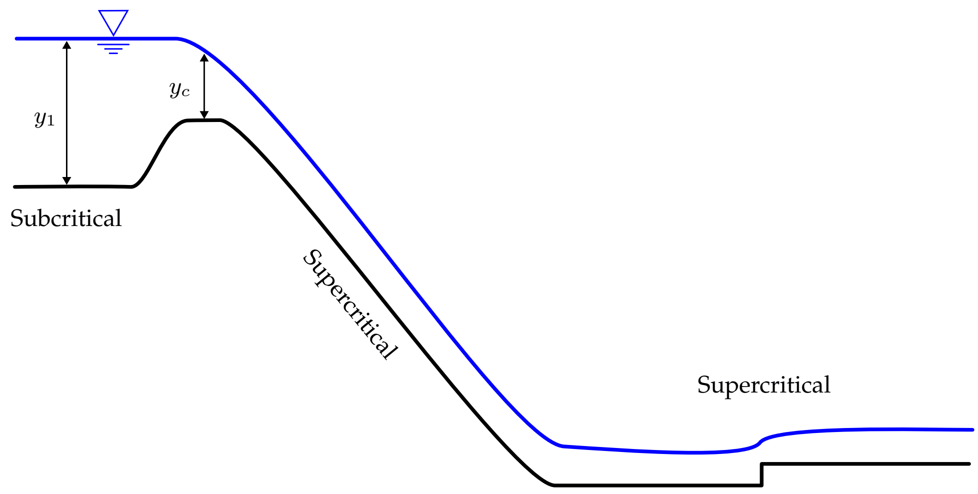


Figure 10: Hydraulic profile of the spillway for a small step downstream.

When the step is big enough, the flow before it is subcritical, meaning that there's a transition from supercritical (in the spillway) to subcritical (before the step) which implies a hydraulic jump somewhere in between. There is a chute (subcritical $\rightarrow$ supercritical) after the step. This possibility is shown in Figure 11.

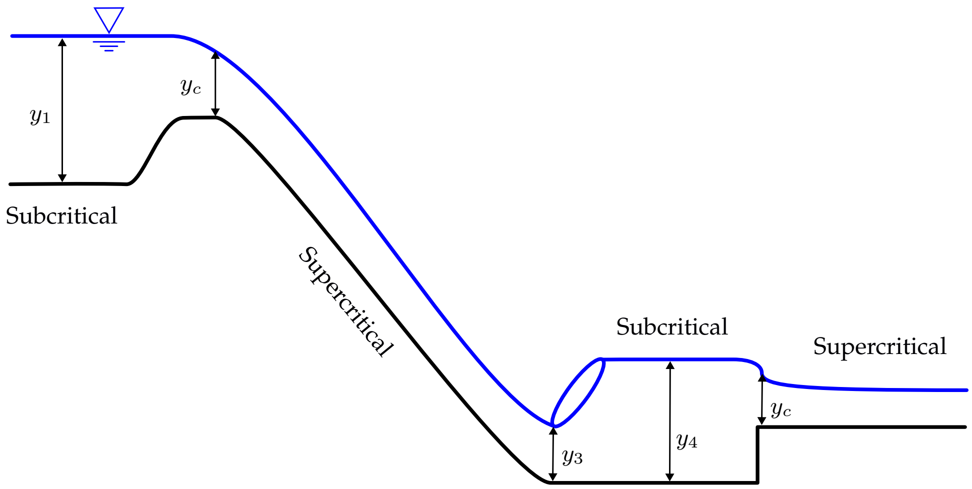


Figure 11: Hydraulic profile of the spillway for a big step downstream.

The main head losses happen at the hydraulic jump. Without more information on the roughness of the bed and the shapes of the weir and the step, it is also the only quantifiable head loss.

b) Let's assume there are no head losses between the end of the approach channel and the weir. Then, the difference in specific energy between those two points should be the height of the weir a. And since the flow passes through the critical state over the weir, the height y_2 is simply $y_2 = \left(\frac{Q}{\sqrt{g}b}\right)^{2/3}$.

$$E_1 = y_1 + \frac{Q^2}{2gA_1^2}$$

$$E_2 + a = \frac{3}{2}\left(\frac{Q}{\sqrt{g}b}\right)^{2/3} + a$$

$$\Rightarrow y_1 + \frac{Q^2}{2gA_1^2} = \frac{3}{2}\left(\frac{Q}{\sqrt{g}b}\right)^{2/3} + a$$

Solving for Q yields the solution

$$Q = 92.0\,\mathrm{m}^3/\mathrm{s}.$$

c) Now that the discharge is known, the critical depth can be computed.

$$y_2 = y_c = \sqrt[3]{\frac{Q^2}{gb^2}} = 2.05\,\mathrm{m}, \quad \mathrm{Fr}_2 = 1$$

d) If the losses are small, the heads should be the same at the top of the weir (H_2) and at the bottom right before the jump (H_3).

$$H_3 = H_2 \Rightarrow E_3 = E_2 + D$$

$$\Rightarrow y_3 + \frac{Q^2}{2gb^2y_3^2} = \frac{3}{2}y_c + 20$$

$$\Rightarrow y_3 = 0.437\,\mathrm{m}, \quad \mathrm{Fr}_3 = \frac{Q/(by_3)}{\sqrt{gy_3}} = 10.2$$

Where the supercritical solution is kept for a hydraulic jump to form in the basin.

e) The depth after the hydraulic jump y_4 is the conjugated depth. Using this concept, the head losses (which are often important) can be accounted for.

$$y_4 = \frac{y_3}{2}\left(\sqrt{1 + 8\mathrm{Fr}_3^2} - 1\right)$$

$$y_4 = 6.07\,\mathrm{m}, \quad \mathrm{Fr}_4 = 0.20$$

f) The water depth over the step should be the critical depth (since the flow goes from subcritical in the basin to supercritical in the downstream river.). Therefore, $y_5 = y_c = 2.05\,\mathrm{m}$. Considering that the head losses around the step are small, we can relate the specific energies with the step's height.

$$E_5 + c = E_4$$

$$\Rightarrow c = y_4 + \frac{Q^2}{2gb^2y_4^2} - \frac{3}{2}y_5$$

$$y_5 = 2.05\,\mathrm{m}, \quad c = 3.11\,\mathrm{m}$$