Hydraulic Engineering and Infrastructures

Civil Engineering Department

Pressurized pipe flow

1 Pipes fueled by a tank †

From reservoir D, water must be supplied for irrigation to farms F1 and F2. The minimum flow rates to be supplied are 500 L/s and 200 L/s, respectively, with a minimum pressure of 50 m.w.c. (meters of water column) at both points.

By concession, the maximum flow rate that can leave reservoir D is 850 L/s.

Determine:

1. The commercial diameter that the BF2 pipe should have (available commercial diameters: 100, 125, 150, 200, 300, 400, 500, 700, 800, 900, 1000 mm) and the actual flow rate that will circulate through the pipe with the selected commercial diameter. To start, you can assume that Q_1 and Q_2 are at their minimum value.
2. For the selected commercial diameter of BF2, find the maximum and minimum levels that the water in reservoir D must reach so that the maximum and minimum operating flow conditions are satisfied, and calculate their values.
3. Draw the energy gradient lines for each pipe under both operating conditions.

	ϕ (mm)	L (m)	f
AB	800	10000	0.025
BF1	600	5000	0.025
BF2	?	3000	0.025

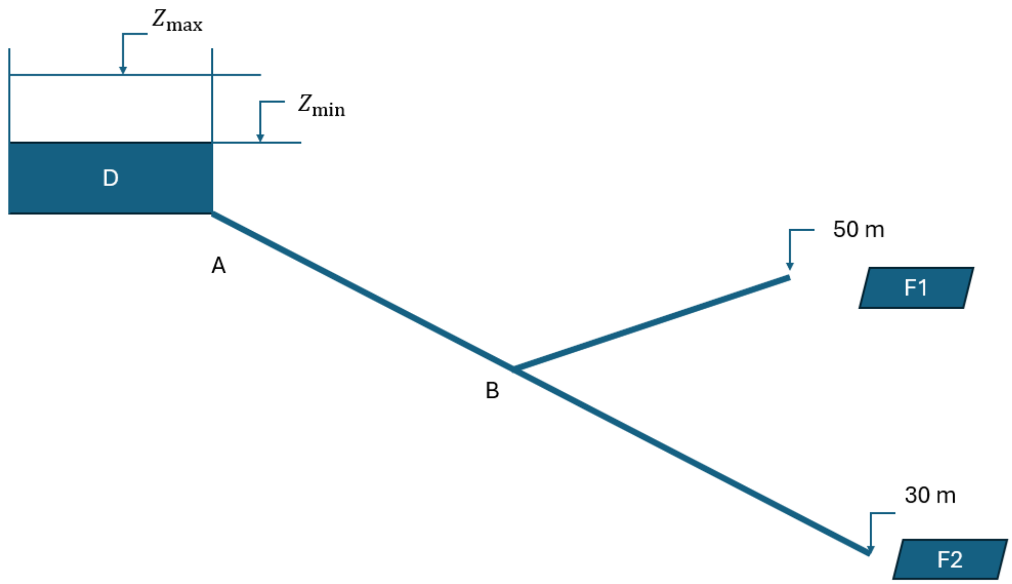

Figure 1: Tank fueling the two pipes.

Solution – Pipes fueled by a tank

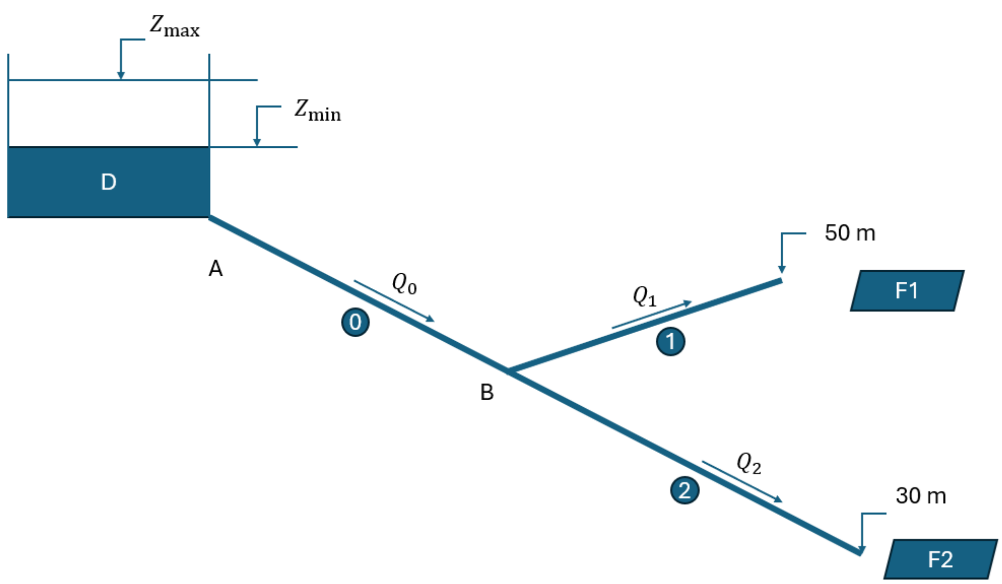

Figure 2: Tank fueling the two pipes.

Using the notation as in Fig. 2, we know that:

$$Q_0 < 850\ \mathrm{L/s} \qquad \frac{P_1}{\gamma} \geq 50\,\mathrm{m.a.s.l.}$$
$$Q_1 > 500\ \mathrm{L/s} \qquad \frac{P_2}{\gamma} \geq 50\,\mathrm{m.a.s.l.}$$
$$Q_2 > 200\ \mathrm{L/s}$$

1. Using $\Delta H = \beta \frac{Q_i^2}{D_i^5} L_i$, with $\beta = \frac{8 \cdot 0.025}{\pi^2 \cdot g} = 2.06 \cdot 10^{-3}$, we get:

$$\Delta H_0 = 2.06 \cdot 10^{-3} \cdot \frac{Q_0^2}{0.8^5} \cdot 10000 = 62.86 \cdot Q_0^2$$
$$\Delta H_1 = 2.06 \cdot 10^{-3} \cdot \frac{Q_1^2}{0.6^5} \cdot 5000 = 132.46 \cdot Q_1^2$$
$$\Delta H_2 = 2.06 \cdot 10^{-3} \cdot \frac{Q_2^2}{D_2^5} \cdot 3000 = \frac{6.18}{D_2^5} \cdot Q_2^2$$

- Applying Bernoulli between tank D and F1:

$$z + 0 + 0 = 50 + \frac{P_{F1}}{\gamma} + \frac{V_{F1}^2}{2g} + \Delta H_0 + \Delta H_1$$
$$z = 100 + \frac{V_{F1}^2}{2g} + \Delta H_0 + \Delta H_1 \tag{1}$$

Then, between tank D and F2:

$$z + 0 + 0 = 30 + \frac{P_{F2}}{\gamma} + \frac{V_{F2}^2}{2g} + \Delta H_0 + \Delta H_2$$
$$z = 80 + \frac{V_{F2}^2}{2g} + \Delta H_0 + \Delta H_2 \tag{2}$$

By equating (1) and (2), we obtain the following:

$$100 + \frac{V_{F1}^2}{2g} + \Delta H_0 + \Delta H_1 = 80 + \frac{V_{F2}^2}{2g} + \Delta H_0 + \Delta H_1$$
$$20 + \frac{V_{F1}^2}{2g} + \Delta H_1 = \frac{V_{F2}^2}{2g} + \Delta H_2 \tag{3}$$

For the smallest z, we get the smallest discharges: $Q_1 = 0.500$ m^3/s and $Q_2 = 0.200$ m^3/s. Then:

$$V_{F1} = \frac{Q_1}{S_1} = \frac{0.500}{\frac{0.6^2}{4}\pi} = 1.77 \text{ m/s} \Rightarrow \frac{V_{F1}^2}{2g} = 0.16 \text{ m/s}$$
$$V_{F2} = \frac{Q_2}{S_2} = \frac{0.200}{\frac{D_2^2}{4}\pi} = \frac{0.25}{D_2^2} \Rightarrow \frac{V_{F2}^2}{2g} = \frac{0.0033}{D_2^4}$$

Substituting everything into (3), we get $D_2 = 0.341$ m. The commercial diameter to choose is then $D_2 = 400$ mm. We choose a diameter larger than what is necessary so that $Q_0 = Q_1 + Q_2 < 850$ L/s.

Using this new value of $D_2 = 400$ mm in equation (3) and $Q_1 = 0.500$ m^3/s, we find $Q_2 = 0.297$ m^3/s and $Q_0 = 0.797$ m^3/s. These are the new minima values for Q_1, Q_2 and Q_0 as the discharge of Q_2 cannot be lower when $D_2 = 0.400$ mm.

2. To find the minimum value of z, we can use equations (1) and (2) with minimum values of Q_1 and Q_2:

$$\text{From (1): } z = 173.1 \,\text{m}$$
$$\text{From (2): } z = 173.2 \,\text{m}$$

- The minimal head required in the tank D is thus 173.09 m.a.s.l..

  Then, to find the maximum value, we use the maximum value of Q_0, $Q_0 = Q_1 + Q_2$ and equation (3). By finding the values of Q_1 and Q_2 in that case, we can obtain z.

$$Q_1 = 0.539 \text{ m}^3/\text{s}$$
$$Q_2 = 0.311 \text{ m}^3/\text{s}$$
$$z = 183.9 \,\text{m}$$

- The energy gradient lines are represented in red in Fig. 3. In order to draw them, the ΔH need to be computed for $z = z_{max}$ and $z = z_{min}$.

$$z_{min} = 173.1 \,\text{m}; \Delta H_0 = 39.9 \,\text{m}$$
$$\Delta H_1 = 33.1 \,\text{m}$$
$$\Delta H_2 = 53.2 \,\text{m}$$

$$z_{max} = 183.9\,\mathrm{m}; \Delta H_0 = 45.4$$
$$\Delta H_1 = 38.5\,\mathrm{m}$$
$$\Delta H_2 = 58.4\,\mathrm{m}$$

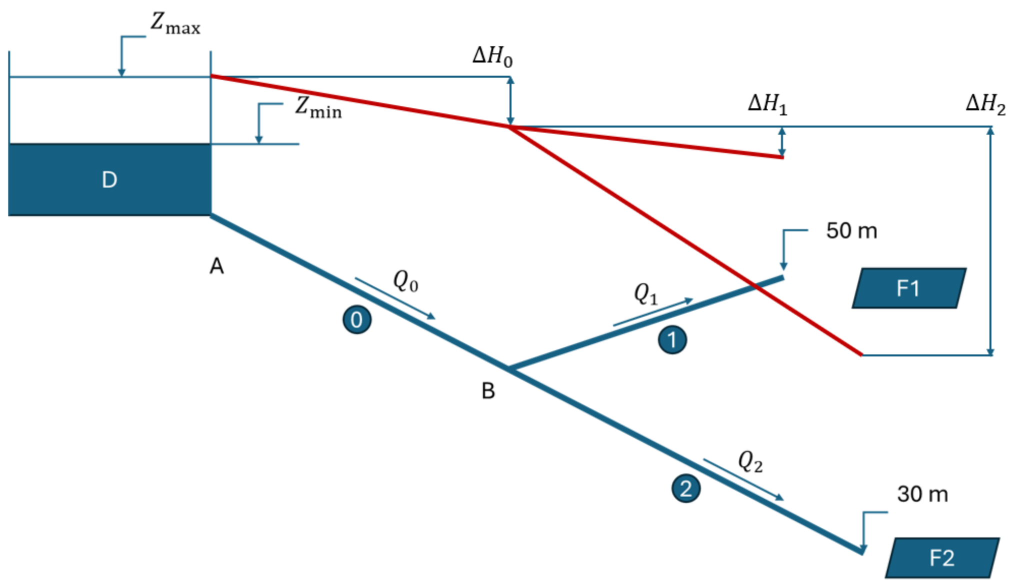

Figure 3: Energy gradient lines (red). This figure is when $z = z_{max}$

2 Hardy cross †

Solve the following network using the Hardy Cross method, but instead of the Darcy–Weisbach equation (which gives the head loss as proportional to Q^2), apply the Hazen–Williams equation. This empirical relation expresses the head loss in pressurized pipes as

$$h_f = \frac{10.7L}{C_{HW}^{1.852} D^{4.87}} Q^{1.852}$$

where h_f is the head loss (m), L is the pipe length (m), D is the internal diameter (m), Q is the discharge (L/s), and C_{HW} is the Hazen–Williams roughness coefficient. Accordingly, the discharge correction in the Hardy Cross method changes to

$$\Delta Q = \frac{-\sum h_f}{1.852 \sum \frac{h_f}{Q}}$$

instead of the Darcy–Weisbach formulation with exponent 2. Use a Hazen–Williams factor of $C_{HW} = 100$.

Pipe	L	D
1	305 m	150 mm
2	305 m	150 mm
3	610 m	200 mm
4	457 m	150 mm
5	153 m	200 mm

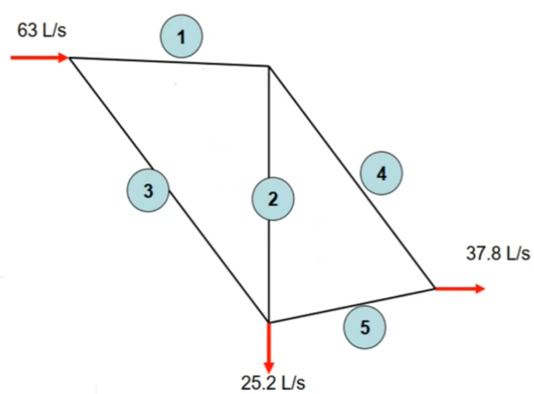

Figure 4: Pipeline system.

Solution – Hardy coss

1. Water is supplied and withdrawn from junction nodes only.
2. Discharge entering the system is equal to discharge leaving the system:

$$\sum Q_{\text{in}} = \sum Q_{\text{out}}$$

3. For long pipe systems, neglect minor losses.
4. Assume flow for each individual pipe network.
5. For any closed loop, the sum of head loss must equal zero:

$$\sum h_f = 0$$

Clockwise flows in a loop are considered positive. Counterclockwise flows are considered negative.

6. To balance head around each loop, flow rate correction, ΔQ, for each loop in the network is computed:

$$\Delta Q = \frac{-\sum h_f}{1.85 \sum \dfrac{h_f}{Q}}$$

7. Discharge is adjusted and another iteration is carried out until summation of head loss approximates to zero:

$$\sum h_f \approx 0$$

The flow in common pipes in two loops is *positive in one loop* and *negative in the other loop*. Pipes in common loops receive both corrections, such that:

For the first loop:	$\Delta Q = \Delta Q_1 - \Delta Q_2$
For the second loop:	$\Delta Q = \Delta Q_2 - \Delta Q_1$

To compute the head losses, we use the Hazen Williams equation:

$$h_f = \frac{10.7L}{C_{HW}^{1.852} D^{4.87}} Q^{1.852}$$
$$h_f = KQ^{1.852}$$

By continuity, we have:

$$63\ \text{L/s} = Q_1 + Q_3$$
$$Q_1 = Q_2 + Q4$$
$$Q_3 + Q_2 = 25.2\ \text{L/s} + Q_5$$
$$Q_4 + Q_5 = 37.8\ \text{L/s}$$

First iteration

For the first iteration, ensuring that continuity is respected, we assume, in m^3/s:

$$Q_1 = 0.024$$
$$Q_2 = 0.0114$$
$$Q_3 = 0.039$$
$$Q_4 = 0.0126$$
$$Q_5 = 0.0252$$

Table 1: First iteration results.

Loop	**Pipe**	**Diameter (m)**	**Length (m)**	**K**	**Q (m^3/s)**	h_f	h_f/Q	$\Delta Q \times 10^3$	$Q_{corrected}$ **(m^3/s)**
1	1	0.15	305	6639	0.024	6.64	276	-0.23	0.02377
	2	0.15	305	6639	0.0114	1.67	146	$+0.35$	0.01175
	3	0.20	610	-3271	-0.039	-8.04	206	-0.23	-0.03923
Iteration 1									
2	2	0.15	305	6639	-0.0114	-1.67	146	-0.35	-0.01175
	4	0.15	457	9947	0.0126	3.02	239	-0.58	0.01202
	5	0.20	153	820.5	-0.0252	-0.89	36.0	-0.58	-0.02578

After computing h_f for each pipe, we have:

For the first loop: $h_f = 6.64 + 1.67 - 8.04 = 0.27$

$$\Rightarrow \frac{h_f}{Q} = 276 + 146 + 206 = 628$$

For the second loop: $h_f = -1.67 + 3.02 - 0.89 = 0.46$

$$\Rightarrow \frac{h_f}{Q} = 146 + 239 + 36.0 = 421$$

Then, with these values, we can compute the corrections ΔQ:

$$\Delta Q_1 = \frac{-0.27}{1.85 \times 628} = -0.23 \times 10^{-3}$$
$$\Delta Q_2 = \frac{-0.46}{1.85 \times 421} = -0.58 \times 10^{-3}$$

For the first loop: $\Delta Q = -0.23 \times 10^{-3} + 0.58 \times 10^{-3} = 0.35 \times 10^{-3}$

For the second loop: $\Delta Q = -0.58 \times 10^{-3} + 0.23 \times 10^{-3} = -0.35 \times 10^{-3}$

Second iteration

For the second iteration, we use the corrected values of Q we obtained with the first iteration.

Table 2: Second iteration results.

Loop	**Pipe**	**Diameter (m)**	**Length (m)**	**K**	**Q (m^3/s)**	h_f	h_f/Q	$\Delta Q \times 10^3$	$Q_{corrected}$ **(m^3/s)**
1	1	0.15	305	6639	0.02377	6.52	274	−0.13	0.02363
	2	0.15	305	6639	0.01175	1.77	150	−0.05	0.01169
	3	0.20	610	3271	−0.03923	−8.13	207	−0.13	−0.03937
Iteration 2									
2	2	0.15	305	6639	−0.01175	−1.77	150	+0.05	−0.01169
	4	0.15	457	9947	0.01202	2.77	230	−0.08	0.01194
	5	0.20	153	820.5	−0.02578	−0.94	36.3	−0.08	−0.02586

Using the same methodology as for the first iteration, we get the following corrections:

$$\Delta Q_1 = \frac{-0.16}{1.85 \times 631} = -0.13 \times 10^{-3}$$
$$\Delta Q_2 = \frac{-0.06}{1.85 \times 416.3} = -0.08 \times 10^{-3}$$

For the first loop: $\Delta Q = -0.13 \times 10^{-3} + 0.08 \times 10^{-3} = -0.05 \times 10^{-3}$

For the second loop: $\Delta Q = -0.08 \times 10^{-3} + 0.13 \times 10^{-3} = 0.05 \times 10^{-3}$