Hydraulic Engineering and Infrastructures

Civil Engineering Department

Pressurized pipe flow

1 Pressure drop ††

Water $\nu = 1.12 \times 10^{-6}\,\mathrm{m^2/s}$ flows from the basement to the second floor through the 1.9 cm diameter copper pipe (a drawn tubing with threaded elbows $K_L = 1.5$) at a rate of $Q = 0.76\,\mathrm{L/s}$, flows through a globe valve (with $K_L = 10$) and exits through a faucet of diameter 1.27 cm as shown in Figure 1.

Determine the pressure at point (1) if

1. all losses are neglected,
2. the only losses included are major losses, or
3. all losses are included.

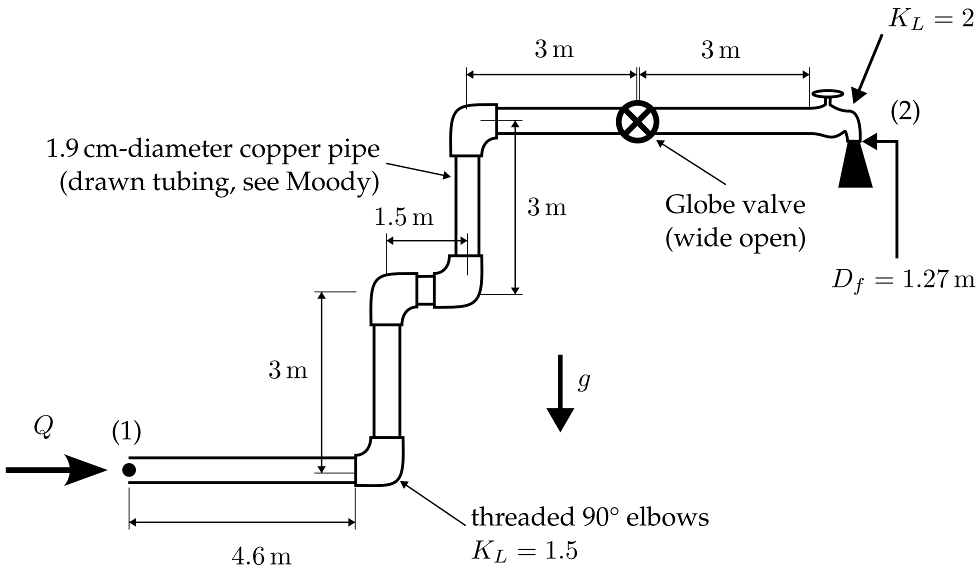

Figure 1: Pressure drop through a pipe.

Solution 1 – Pressure drop †

1. The energy equation in the absence of losses is as follows :

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g}.$$

   Substituting the discharge and the atmospheric pressure (≈ 0),

$$p_1 = \gamma \cdot \left(z_2 - z_1 + \frac{8Q^2}{\pi^2 g} \left[\frac{1}{D_f^4} - \frac{1}{D^4} \right] \right) = 74.2\,\mathrm{kPa}.$$

2. The energy loss is added to the equation :

$$p_1 = \gamma \cdot \left(z_2 - z_1 + \frac{8Q^2}{\pi^2 g} \left[\frac{1}{D_f^4} - \frac{1}{D^4} \right] \right) + h_L \quad \text{with} \quad h_L = f \frac{L}{D} \frac{V_1^2}{2g}.$$

   To compute the friction factor, we can either use the Moody diagram or solve the Colebrook-White equation numerically with $\varepsilon / D = 8 \times 10^{-5}$ (since the rugosity of a drawn tubing is $\varepsilon = 0.0015\,\mathrm{mm}$, see the provided Moody diagram) the and $Re = 45 \times 10^3$). In both cases, $f = 0.0216$. The head loss then has a value of $7.5\,\mathrm{m}$ and the supplied pressure is $p_1 = 148\,\mathrm{kPa}$.

3. Adding minor losses increases the head loss :

$$p_1 = \gamma \cdot \left(z_2 - z_1 + \frac{8Q^2}{\pi^2 g} \left[\frac{1}{D_f^4} - \frac{1}{D^4} \right] \right) + h_L,$$

$$h_L = \left(K_{L,\text{Valve}} + 4K_{L,\text{elbow}} + K_{L,\text{Faucet}} + f\frac{L}{D} \right) \frac{V_1^2}{2g} = 14.1\,\mathrm{m}.$$

Which needs a pressure $p_1 = 213\,\mathrm{kPa}$.

2 No minor losses †

Air at standard temperature and pressure flows through a horizontal, galvanised iron pipe ($\varepsilon = 0.000152$ m) at a rate of 0.0566 m^3/s. The pressure drop is to be no more than 3.45 kPa per 30.5 m of pipe. Assume the flow is incompressible with $\rho = 1.2266$ kg/m^3 and $\mu = 1.7907 \times 10^{-5}$ Pa·s.

Determine the minimum pipe diameter.

Solution 2 – No minor losses

Note that if the pipe was too long, the pressure drop from one end to the other, $p_1 - p_2$, would not be small relative to the pressure at the beginning, and compressible flow considerations would be required.

For example, if we consider p_1 to be the atmospheric pressure (so, $p_1 = 101$ kPa) a pipe of length 61 m gives

$$\frac{p_1 - p_2}{p_1} = \frac{(3.45 \text{ kPa}/30.5 \text{ m})(61 \text{ m})}{101 \text{ kPa}} = 0.068 = 6.8\%,$$

which is probably small enough to justify the incompressible assumption.

With $z_1 = z_2$ and $V_1 = V_2$, the energy equation becomes:

$$p_1 = p_2 + f\frac{l}{D}\frac{\rho V^2}{2} \tag{1}$$

where $V = Q/A = 4Q/(\pi D^2) = 4(0.0566 \text{ m}^3/\text{s})/\pi D^2 = 0.0721/D^2$, with D in meters. Thus, with $p_1 - p_2 = 3.45$ kPa and $l = 30.5$ m, Eq. (1) becomes:

$$\begin{aligned} p_1 - p_2 &= 3.45 \text{ kPa} \\ &= f\frac{30.5 \text{ m}}{D}(1.23 \text{ kg/m}^3)\frac{1}{2}\left(\frac{0.0721}{D^2}\frac{\text{m}}{\text{s}}\right)^2 \end{aligned}$$

This yields:

$$D = 0.123\, f^{1/5} \tag{2}$$

Also, using the Reynolds number ($\text{Re} = \rho V D/\mu$) and the roughness, we have:

$$\text{Re} = \frac{8.8 \times 10^4}{D} \tag{3}$$

$$\frac{\varepsilon}{D} = \frac{0.000152}{D} \tag{4}$$

Thus, we have four equations (Eqs. (2), (3), (4), and either the Moody chart, the Colebrook equation) and four unknowns (f, D, ε/D and Re) from which the solution can be obtained by trial-and-error methods. If we use the Moody chart, it is probably easiest to assume a value of f, use Eqs. (2), (3), and (4) to calculate D, Re, and ε/D and then compare the assumed f with that from the Moody chart. If they do not agree, try again.

Thus we assume $f = 0.02$, a typical value, and obtain $D = 0.056$ m, which gives $\varepsilon/D = 0.0027$ and $\text{Re} = 8.8 \times 10^4$. From the Moody chart, for these values of ε/D and Re, we get $f = 0.027$, which is different from the value we initially chose. This means we need to redo this process for another value of f.

If we try again for $f = 0.027$, we get $D = 0.060$ m, $\varepsilon/D = 0.0025$, and $\text{Re} = 8.3 \times 10^4$, which in turn give $f = 0.027$, in agreement with the chosen value of f. Thus, the diameter of the pipe should be:

$$D = 0.060 \text{ m}$$

3 Minor losses †

Water at 15°C ($\nu = 1.21 \times 10^{-6}$ m^2/s) flows from reservoir A to reservoir B through a pipe of length $L = 518$ m and roughness $\varepsilon = 0.000152$ m at a rate of $Q = 0.737$ m^3/s. The system contains a square-edged entrance, four regular 45° elbows and a submerged exit.

Determine the pipe diameter needed.

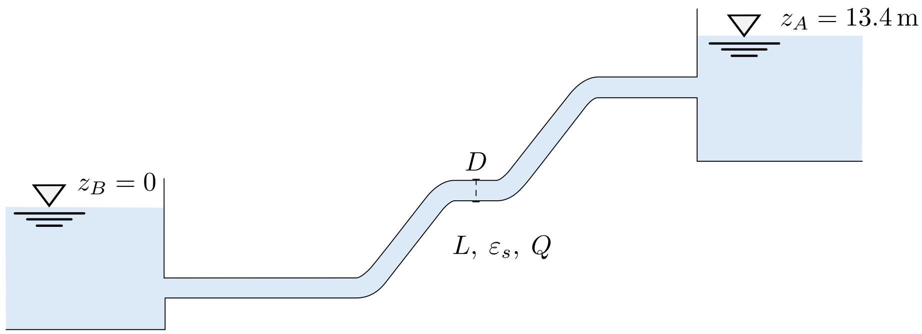

Figure 2: Illustration of the problem.

Solution 3 – Minor losses ††

The energy equation can be applied between two points on the surfaces of the reservoirs ($p_1 = p_2 = V_1 = V_2 = z_2 = 0$) as follows:

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \\ z_1 &= \frac{V^2}{2g}\left(f\frac{\ell}{D} + \sum K_i\right) \end{aligned} \tag{1}$$

where we have:

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2} \tag{2}$$

The loss coefficients are obtained from the table (*Minor loss coefficients for pipe flow (Mays, 2019)*) as $K_{\text{entrance}} = 0.5$, $K_{\text{elbow}} = 0.2$, and $K_{\text{exit}} = 1$. Thus, Eq. (1) can be written as:

$$13.4 = \frac{V^2}{2g}\left(\frac{L}{D}f + \sum_i K_i\right)$$

or, when combined with Eq. (2) to eliminate V,

$$f = f(D) = \frac{\pi^2 g z_1}{8Q^2\ell}D^5 - \frac{D}{\ell}\sum_i K_i \tag{3}$$

Where D and f are the only unknowns.

To determine D we must know f, which is a function of Re and ε/D, such as:

$$\begin{aligned} \mathrm{Re} = \mathrm{Re}(D) &= \frac{VD}{\nu} = \frac{4Q}{\pi D^2}\frac{D}{\nu} \\ r = r(D) &= \frac{\varepsilon}{D} \end{aligned}$$

Where D is the only unknown.

Again, we have four equations (Eqs. (3), (4), (5) and the Moody chart or the Colebrook equation) for the four unknowns D, f, Re, and $r = \varepsilon/D$.

Consider the solution by using the Moody chart. Although it is often easiest to assume a value of f and make calculations to determine if the assumed value is the correct one, with the inclusion of minor losses this may not be the simplest method. For example, if we assume $f = 0.02$ and calculate D from Eq. (3), we would have to solve a fifth-order equation. With only major losses, the term proportional to D in Eq. (3) is absent, and it is easy to solve for D if f is given. With both major and minor losses included, this solution for D (given f) would require a trial-and-error or iterative technique.

Thus, for this type of problem it is perhaps easier to assume a value of D, calculate the corresponding f from Eq. (3), and with the values of Re and ε/D determined from Eqs. (4) and (5), look up the value of f in the Moody chart (or the Colebrook equation).

The solution is obtained when the two values of f are in agreement. A few rounds of calculation will reveal that the solution is given by:

$$D \approx 0.497 \text{ m}$$

4 Cavitation in siphon †

Siphons are pipes that allow to connect two or more reservoirs winding around obstacles. When they go higher than the highest surface, the siphon will experience low pressures $p = H - z_{\text{max}}$ and might cavitate[1].

Consider a pipe with a diameter $D = 20\,\text{cm}$, a rugosity ε =0.5 mm and the kinematic viscosity of the water ν =1.16 × 10^{-6} m^2/s. What's the maximum head difference ΔH between the reservoirs for which $p/\gamma \geq -10.25\,\text{m}$ throughout the pipe ? What's the corresponding discharge ?

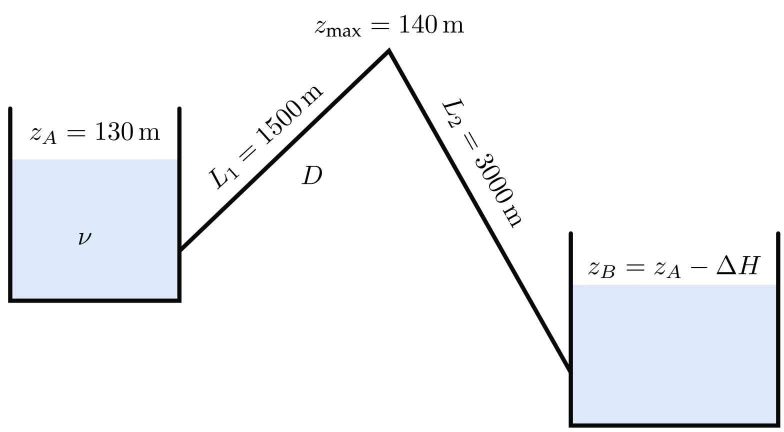

Figure 3: Two reservoirs connected by a siphon.

Solution 4 – Cavitation in siphon

The energy conservation between A and C gives a relation between V and f:

$$z_A = z_C + \frac{p_C}{\gamma} + \frac{V^2}{2g} + f\frac{L}{D}\frac{V^2}{2g}$$

$$\Rightarrow V = \sqrt{\frac{2g}{1 + fL/D}\left(z_A - z_C - \frac{p_C}{\gamma}\right)}$$

Inserting this expression for the speed in the Reynolds number, we have an implicit equation for f.

[1] Cavitation is when the the pressure is low enough for the water to vaporize. Here's a cool video introducing the topic: Cavitation - Physics girl.

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon_s}{3.71D} + \frac{2.51\nu}{D\sqrt{\frac{2gf}{1+fL/D}\left(z_A - z_C - \frac{p_C}{\gamma}\right)}}\right).$$

The term p_C/γ being set to −10.25 m, the friction factor is $f = 0.0296$. The head loss has a value of 0.249 m and gives a discharge of 4.66 L/s. Finally, $z_B = 129.25\,\mathrm{m}$.