Hydraulic Engineering and Infrastructures

Civil Engineering Department

Hydraulic Structures

1 Free and submerged flow through a vertical sluice gate

We will use the notation and theoretical background presented in the Gates and Hydraulic Jumps notes available on Moodle.

This exercise concerns flow under a vertical sluice gate in a horizontal, rectangular channel of width b. Upstream of the gate the water depth is uniform and given by y_1. The gate opening is a, and due to contraction the jet thickness at the vena contracta is

$$y_g = C_c\, a,$$

where C_c is the contraction coefficient. The downstream uniform depth is denoted y_d (tailwater depth). We consider two operating regimes: free flow and submerged flow.

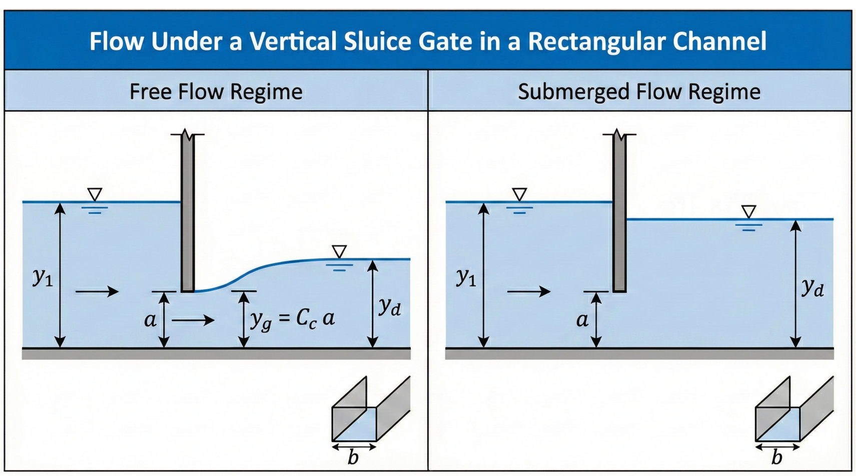

Figure 1: Free and submerged flow under a vertical sluice gate, showing upstream depth y_1, gate opening a, contracted jet thickness $y_g = C_c a$, and tailwater depth y_d.

Consider a sluice gate with geometric opening $a = 0.10$ m in a rectangular channel of width $b = 1.5$ m. Upstream depth is $y_1 = 1.0$ m, and the contraction coefficient is $C_c = 0.61$. Take $g = 9.81\ \mathrm{m/s^2}$.

(i) Using conservation of specific energy between the upstream section and the vena contracta, show that the free-flow discharge under the gate may be expressed as

$$Q_{\text{free}} = C_d\, b\, a\, \sqrt{2gy_1}, \qquad \text{with} \qquad C_d = \frac{C_c}{\sqrt{1 + \dfrac{C_c a}{y_1}}}.$$

Start by writing the specific energy at the upstream depth y_1 and at the contracted jet thickness $y_g = C_c a$, then set $E_1 = E_g$ to eliminate q. Show that this leads to the expression above for the discharge coefficient C_d. Finally, evaluate y_g and C_d numerically for the given data.

(ii) Compute the free discharge Q_{free} through the gate.

(iii) Determine the tailwater depth $y_{d,\text{crit}}$ for which the hydraulic jump would attach to the gate and compare it with y_1.

Solution

Given:

$$a = 0.10 \text{ m}, \qquad y_1 = 1.0 \text{ m}, \qquad b = 1.5 \text{ m}, \qquad C_c = 0.61, \qquad g = 9.81 \text{ m/s}^2.$$

The contracted jet thickness at the vena contracta is

$$y_g = C_c\, a = 0.61 \times 0.10 = 0.061 \text{ m}.$$

(i) *Derivation of* Q_{free} *and* C_d

Let $q = Q/b$ be the discharge per unit width. The specific energy at the upstream section and at the contracted jet section are

$$E_1 = y_1 + \frac{q^2}{2g\, y_1^2}, \qquad E_g = y_g + \frac{q^2}{2g\, y_g^2}.$$

Neglecting losses between section 1 and the vena contracta, conservation of specific energy gives

$$E_1 = E_g.$$

Hence

$$y_1 - y_g = \frac{q^2}{2g}\left(\frac{1}{y_g^2} - \frac{1}{y_1^2}\right) = \frac{q^2}{2g}\,\frac{y_1^2 - y_g^2}{y_g^2 y_1^2}.$$

Solving for q^2,

$$\frac{q^2}{2g} = \frac{y_g^2 y_1^2}{y_1^2 - y_g^2}\,(y_1 - y_g) = \frac{y_g^2 y_1^2}{(y_1 - y_g)(y_1 + y_g)}\,(y_1 - y_g) = \frac{y_g^2 y_1^2}{y_1 + y_g}. \tag{1}$$

For free flow under the gate we define the discharge coefficient C_d by

$$Q_{\text{free}} = C_d\, b\, a\,\sqrt{2gy_1}, \qquad q_{\text{free}} = \frac{Q_{\text{free}}}{b} = C_d\, a\,\sqrt{2gy_1}.$$

Thus

$$\frac{q_{\text{free}}^2}{2g} = C_d^2 a^2 y_1. \tag{2}$$

Equating (1) and (2) gives

$$C_d^2 a^2 y_1 = \frac{y_g^2 y_1^2}{y_1 + y_g} \quad\Rightarrow\quad C_d^2 = \frac{y_g^2 y_1}{a^2(y_1 + y_g)}.$$

Using $y_g = C_c a$ so that $y_g^2 = C_c^2 a^2$, we obtain

$$C_d^2 = \frac{C_c^2 a^2 y_1}{a^2(y_1 + y_g)} = \frac{C_c^2 y_1}{y_1 + y_g} = \frac{C_c^2}{1 + \dfrac{y_g}{y_1}} = \frac{C_c^2}{1 + \dfrac{C_c a}{y_1}}.$$

Taking the square root,

$$C_d = \frac{C_c}{\sqrt{1 + \dfrac{y_g}{y_1}}} = \frac{C_c}{\sqrt{1 + \dfrac{C_c a}{y_1}}},$$

which is the required expression.

Numerically,

$$\frac{y_g}{y_1} = \frac{0.061}{1.0} = 0.061, \qquad C_d = \frac{0.61}{\sqrt{1 + 0.061}} \approx \frac{0.61}{\sqrt{1.061}} \approx \frac{0.61}{1.03} \approx 0.59.$$

So

$$\boxed{y_g = 0.061 \text{ m}}, \qquad \boxed{C_d \approx 0.59}.$$

(ii) *Free discharge through the gate*

From the result above,

$$Q_{\text{free}} = C_d\, b\, a\, \sqrt{2 g y_1}.$$

Substituting the numerical values

$$Q_{\text{free}} = 0.592 \times 1.5 \times 0.10 \times \sqrt{2 \times 9.81 \times 1.0}.$$

Compute step by step:

$$\sqrt{2 \times 9.81 \times 1.0} = \sqrt{19.62} \approx 4.43,$$

$$0.592 \times 1.5 \times 0.10 = 0.0888,$$

so

$$Q_{\text{free}} \approx 0.0888 \times 4.43 \approx 0.39 \text{ m}^3/\text{s}.$$

Thus

$$\boxed{Q_{\text{free}} \approx 0.39 \text{ m}^3/\text{s}}.$$

(iii) *Tailwater depth for which the jump attaches to the gate*

The limit of free flow corresponds to a hydraulic jump that is just attached to the gate. In that case, the upstream depth of the jump is the jet depth y_g and the downstream depth is the critical tailwater depth $y_{d,\text{crit}}$.

First compute the unit discharge and velocity in the jet:

$$q = \frac{Q_{\text{free}}}{b} \approx \frac{0.39}{1.5} \approx 0.26 \text{ m}^2/\text{s},$$

$$V_g = \frac{q}{y_g} \approx \frac{0.26}{0.061} \approx 4.3 \text{ m/s}.$$

The Froude number at the contracted section is

$$Fr_g = \frac{V_g}{\sqrt{g y_g}} = \frac{4.3}{\sqrt{9.81 \times 0.061}} \approx 5.6.$$

For a hydraulic jump in a rectangular channel, the conjugate depth relation is

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right],$$

where y_1 and Fr_1 are the depth and Froude number on the supercritical side. Here $y_1 = y_g$ and $y_2 = y_{d,\text{crit}}$, so

$$y_{d,\text{crit}} = y_g \, \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_g^2}\right].$$

With $Fr_g \approx 5.6$ one finds numerically

$$y_{d,\text{crit}} \approx 0.45 \text{ m}.$$

Hence

$$\boxed{y_{d,\text{crit}} \approx 0.45 \text{ m}}.$$

Comparison with the upstream depth,

$$y_1 = 1.0 \text{ m},$$

shows that

$$y_{d,\text{crit}} < y_1.$$

Therefore only relatively shallow tailwater depths allow free flow under the gate. If the tailwater exceeds about 0.45 m, the hydraulic jump is drowned and the gate operates under submerged conditions.

2 Ogee Spillway Design

A high overflow spillway with $P/H_d > 1.5$ has a maximum discharge of $283.2\,\mathrm{m}^3/\mathrm{s}$ with a maximum expected head of $6.10\,\mathrm{m}$. Using the USBR recommendation for the under-design procedure mentioned in class, determine the design head, spillway crest length (neglect contractions), and the minimum pressure (expressed in kPa) on the spillway. Plot the complete spillway crest shape for a compound circular curve in the upstream quadrant of the crest.

Solution

To under design the spillway while ensuring no cavitation, the USBR recommend to adopt an expected-head to design-head ratio no more than 1.33. Thererfore:

$$H_d = H_e/1.33 = 6.10/1.33 = 4.58\,\text{m} \tag{1}$$

From the discharge coefficient plots, for high spillways $P/H_d > 1.5$, the basic discharge coefficient $C_0 \approx 2.18$. Using the condition of $H_e/H_d = 1.33$, the basic discharge coefficient must be corrected using the ratio $C/C_o = 1.04$.

Therefore, the final overall design coefficient is:

$$C = 2.18 \times 1.04 = 2.27$$

The spillway crest length is given by the standard discharge equation:

$$L = \frac{Q}{CH_{\text{max}}^{3/2}} = \frac{283.2}{2.28 \times (6.10)^{3/2}} = 8.26\,\text{m} \tag{2}$$

The minimum pressure head is obtained from standard design charts (Figure 6.3a) to be -0.43 times the design head with the result:

$$\left(\frac{p}{\gamma}\right)_{\text{min}} = -0.43 \times 4.57 = -1.97\,\text{m} \tag{3}$$

The minimum pressure is calculated using $\gamma_{water} \approx 9.81\,\text{kN/m}^3$:

$$p_{\text{min}} = -1.97 \times 9.81 = -19.3\,\text{kPa}$$

For high spillways with $P/H_d \gg 1$, the downstream crest shape recommended by USBR can be expressed in nondimensional form as

$$\left(\frac{y}{H_d}\right) = \frac{1}{K}\left(\frac{x}{H_d}\right)^n,$$

with standard values $n = 1.85$ and $K = 2$. Using the design head $H_d = 4.58\,\text{m}$, the dimensional downstream profile becomes

$$y = \frac{H_d}{K}\left(\frac{x}{H_d}\right)^{1.85} = \frac{H_d^{1-1.85}}{2}\,x^{1.85} = \frac{H_d^{-0.85}}{2}\,x^{1.85}.$$

Numerically,

$$H_d^{-0.85} = 4.58^{-0.85} \approx 0.274, \qquad \frac{0.274}{2} \approx 0.137,$$

so that

$$\boxed{y_{\mathrm{d/s}} = 0.137\,x^{1.85}.}$$

Representative downstream ordinates:

x (m)	0	1	2	3	4	5	6
y (m)	0.000	0.137	0.494	1.047	1.783	2.694	3.774

The upstream profile is described by the USBR recommended elliptical form

$$\frac{x^2}{A^2} = \frac{(B-y)^2}{B^2} = 1,$$

with chart-based coefficients appropriate for high spillways,

$$\frac{A}{H_d} = 0.28, \qquad \frac{B}{H_d} = 0.165.$$

Using $H_d = 4.58$ m:

$$A = 0.28(4.58) \approx 1.28 \text{ m}, \qquad B = 0.165(4.58) \approx 0.76 \text{ m}.$$

Thus,

$$\boxed{\frac{x^2}{1.28^2} + \frac{(0.76-y)^2}{0.76^2} = 1.}$$

Representative upstream ordinates (for $H_d = 4.58$ m):

x/H_d	0	−0.05	−0.10	−0.15	−0.20	−0.25	−0.28
x (m)	0.000	−0.229	−0.458	−0.687	−0.916	−1.145	−1.282
y (m)	0.000	0.012	0.050	0.118	0.227	0.415	0.756

The upstream and downstream ogee crest profiles for the design head $H_d = 4.58$ m are plotted below. The coordinate system follows hydraulic convention, with positive y downward.

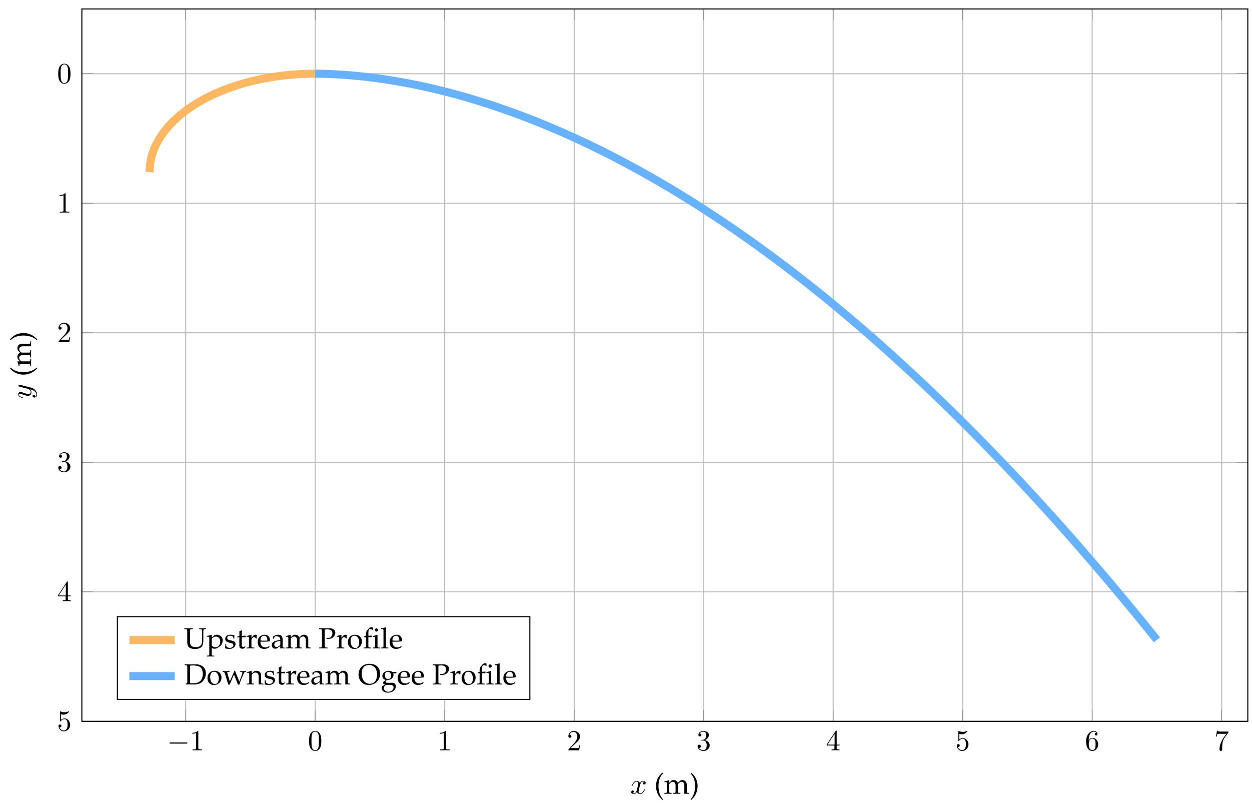

Figure 2: Computed upstream and downstream ogee crest profiles for $H_d = 4.58$ m (positive y downward).