Hydraulic Engineering and Infrastructures

Civil Engineering Department

Quantitative flow profiles

1 Irrigation channel

A hydraulic engineering company has hired you to design an irrigation channel from the field of Mr. Favre (point A, in Figure 1) to the field of Mr. Bianchi (point C, in Figure 1). Here, the channel discharges into a river under free fall conditions. The channel should convey a discharge of $Q = 10\ \mathrm{m}^3/\mathrm{s}$, have a uniform rectangular cross-section with base width $b_0 = 2$ m, and Manning roughness coefficient of $n_0 = 0.01\ \mathrm{m}^{-1/3}\mathrm{s}$.

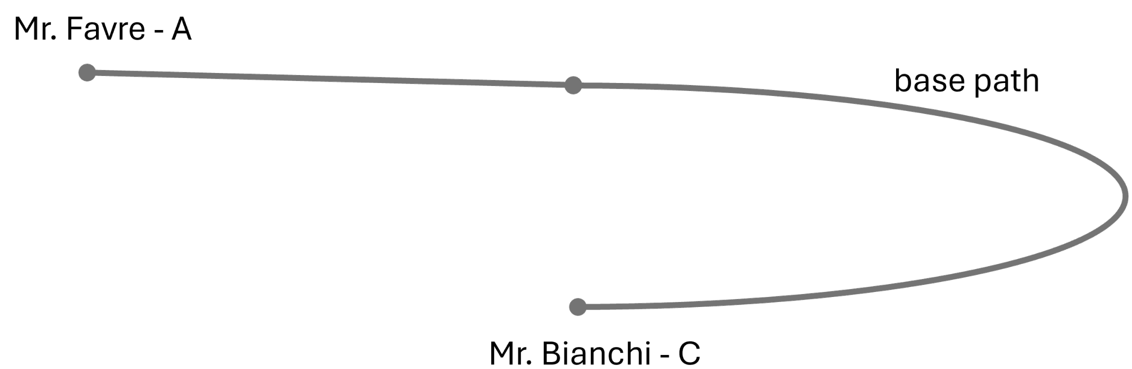

Figure 1: Planview of the base configuration.

The engineering company suggests you a base configuration and some possible alternatives, varying one or more design parameters. **Be careful with the possible changes in critical and normal depth between one configuration and another. Channels can be assumed to have large width in order to compute the normal depth.**

a. Base configuration

The slope between the fields of Mr. Favre and Mr. Bianchi is $S_0 = 0.0002$. You can assume that the channel is long enough for the flow to develop normal conditions. Follow the four steps you learned in class to compute the hydraulic profile, i.e. (i) compute the critical and normal depth, (ii) identify the control points and the flow type conditions (i.e., M_i, S_i, uniform), (iii) draw the qualitative profile, and (iv) compute the quantitative profile with the Direct Step Method (DSM).

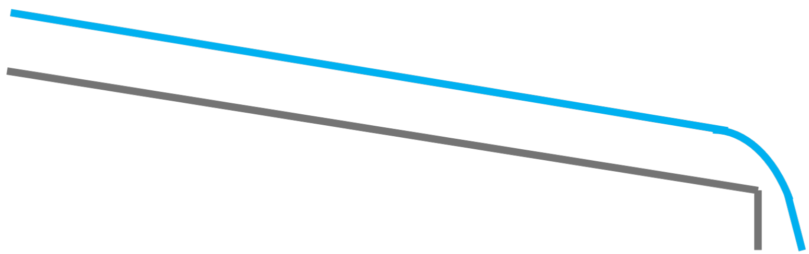

Figure 2: Longitudinal profile of the base configuration.

b. Section increase

The first suggested alternative is to increase the channel cross-section. While keeping a rectangular shape, at a certain point upstream of Mr. Bianchi field, the channel width increases to $b_1 = 4$ m for a length of $L = 800$ m. You can assume that the width increase is smooth and there is no energy loss. Follow the four steps you learned in class and compute the hydraulic profile with the DSM.

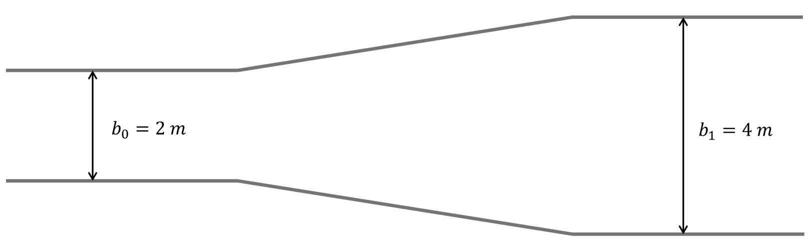

Figure 3: Width increase.

c. Slope increase

The engineering company also suggests a shorter path passing from the terrain of Mr. Schneider (point B). The first reach between Mr. Favre and Mr. Schneider maintains the same slope as the base configuration $S_0 = 0.0002$, whereas for the new reach between the house of Mr. Schneider and Mr. Bianchi the slope suddenly increases by a factor 200. Consider that the alternative path is long $L = 90$ m. Follow the four steps you learned in class and compute the hydraulic profile with the DSM.

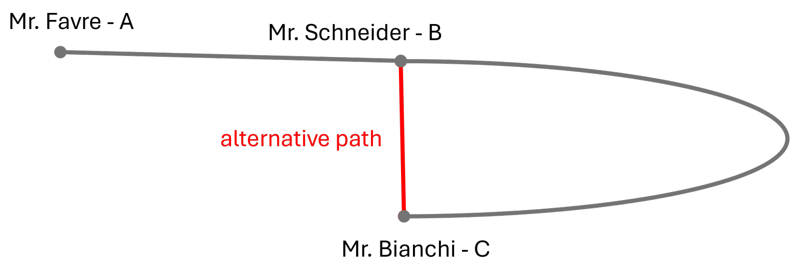

Figure 4: Planview of the second alternative configuration with a different path and a slope increase.

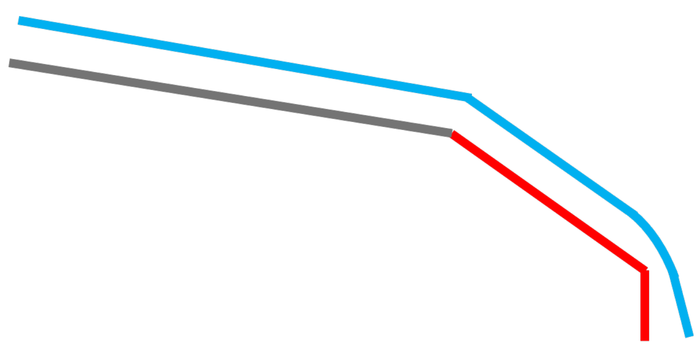

Figure 5: Longitudinal profile of the configuration with the slope change.

d. Roughness increase

Since Mr. Bianchi is particularly careful with ecology, he requires to include a more ecofriendly solution. For this, the engineering company suggests to add some vegetation to the reach between Mr. Schneider and Mr. Bianchi, which increases the Manning roughness to $n_1 = 0.12$. Consider the same channel length as for case 3. Follow the four steps you learned in class and compute the hydraulic profile with the DSM.

Solution

The water profiles have to be resolved using the Direct Step method (DSM). Since the channel has a rectangular cross-section, you can immediately compute the critical as

$$y_c = \left(\frac{Q^2}{b^2 g}\right)^{1/3}$$

and the normal depths as

$$y_n = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5}$$

For channels with different cross sections (i.e., trapezoidal), the computation would have been iterative because both critical and normal depth would depend on the channel section, which again depends on the the flow depth.

1. Base configuration

For the base configuration we have $y_{c,0} = 1.366$ m and $y_{n,0} = 2.133$ m, hence we have a mild slope. For this, the only control is the downstream point C. At this point the flow is subject to free fall conditions, hence is critical. The water profile is computed going upstream, starting from $y = y_{c,0} = 1.366$ m up to $y = y_{n,0} = 2.133$ m, which results in an M2 profile, that becomes uniform upstream. Once normal depth is reached, the flow remains uniform.

Table 1: Profile for the base configuration.

Section	y_i **(m)**	dy **(m)**	y_{med} **(m)**	A **(m^2)**	v **(m/s)**	Fr **(-)**	$1 - Fr^2$ **(-)**	P **(m)**	R **(m)**	S_f **(-)**	$S_0 - S_f$ **(-)**	dx **(m)**	x **(m)**	**sign**
1	1.36591	0.05	1.39	2.781829955	3.595	0.973	0.053	4.782	0.582	0.00266	-0.00246	1.08	0	-1
2	1.41591	0.05	1.44	2.881829955	3.470	0.923	0.148	4.882	0.590	0.00243	-0.00223	3.32	-1.08	-1
3	1.46591	0.05	1.49	2.981829955	3.354	0.877	0.231	4.982	0.599	0.00223	-0.00203	5.69	-4.40	-1
4	1.51591	0.05	1.54	3.081829955	3.245	0.835	0.303	5.082	0.606	0.00205	-0.00185	8.20	-10.09	-1
5	1.56591	0.05	1.59	3.181829955	3.143	0.796	0.367	5.182	0.614	0.00189	-0.00169	10.84	-18.28	-1
6	1.61591	0.05	1.64	3.281829955	3.047	0.759	0.423	5.282	0.621	0.00175	-0.00155	13.64	-29.13	-1
7	1.66591	0.05	1.69	3.381829955	2.957	0.726	0.473	5.382	0.628	0.00162	-0.00142	16.60	-42.77	-1
8	1.71591	0.05	1.74	3.481829955	2.872	0.695	0.517	5.482	0.635	0.00151	-0.00131	19.72	-59.37	-1
9	1.76591	0.05	1.79	3.581829955	2.792	0.666	0.556	5.582	0.642	0.00141	-0.00121	23.02	-79.09	-1
10	1.81591	0.1	1.87	3.731829955	2.680	0.626	0.608	5.732	0.651	0.00127	-0.00107	56.66	-102.11	-1
11	1.91591	0.1	1.97	3.931829955	2.543	0.579	0.665	5.932	0.663	0.00112	-0.00092	72.30	-158.78	-1
12	2.01591	0.12	2.07	4.14931848	2.410	0.534	0.715	6.149	0.675	0.00098	-0.00078	107.45	-231.07	-1
13	2.13340												-339	-1

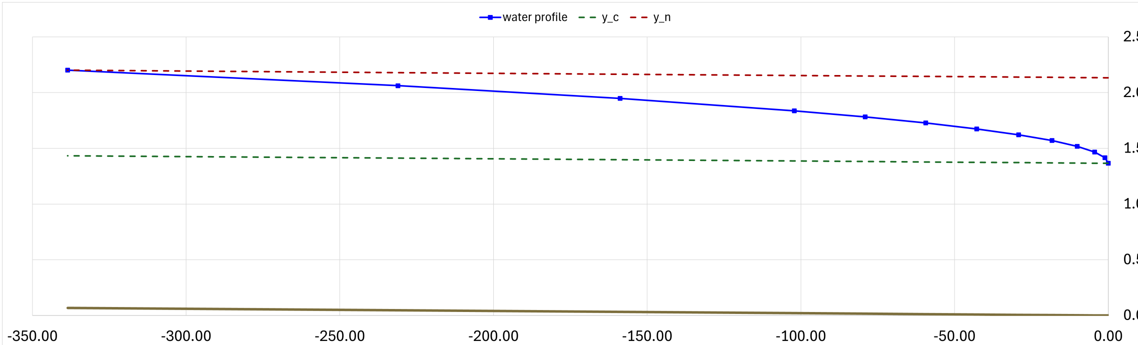

Figure 6: Water profile for the base configuration.

2. Section increase

In this case the profile should be split in two parts. For the upstream part, for which the section has not changed, the critical and the normal depth are the same as for the base configuration. For the downstream part, both values will change. Respectively, we obtain $y_{c,2} = 0.861$ m and $y_{n,2} = 1.408$ m. Again, we notice that the downstream profile maintains a mild slope. For this reason we identify two control points: (i) point C (the most downstream one), where the flow undergoes a free fall and therefore reaches $y = y_{c,2}$; (ii) point of section change 800 m upstream of point C. Both profiles are computed backwards and they identify M2 curves again. Additionally, we can assume that the section change is smooth and does not influence the water profile nor the specific energy. We need to verify whether the most downstream part of the channel will reach normal conditions.

Table 2: Downstream profile for the configuration with section increase.

Section	y_i (m)	dy (m)	y_{med} (m)	A (m^2)	v (m/s)	Fr (-)	$1-Fr^2$ (-)	P (m)	R (m)	S_f (-)	$S_0 - S_f$ (-)	dx (m)	x (m)	sign
1	0.86047	0.01	0.87	3.461890064	2.889	0.991	0.017	5.731	0.604	0.00163	-0.00143	0.12	0	-1
2	0.87047	0.01	0.88	3.501890064	2.856	0.974	0.051	5.751	0.609	0.00158	-0.00138	0.37	-0.12	-1
3	0.88047	0.05	0.91	3.621890064	2.761	0.926	0.142	5.811	0.623	0.00143	-0.00123	5.76	-0.49	-1
4	0.93047	0.05	0.96	3.821890064	2.617	0.855	0.270	5.911	0.647	0.00122	-0.00102	13.16	-6.24	-1
5	0.98047	0.05	1.01	4.021890064	2.486	0.792	0.373	6.011	0.669	0.00106	-0.00086	21.79	-19.40	-1
6	1.03047	0.05	1.06	4.221890064	2.369	0.736	0.458	6.111	0.691	0.00092	-0.00072	31.88	-41.19	-1
7	1.08047	0.05	1.11	4.421890064	2.261	0.687	0.528	6.211	0.712	0.00080	-0.00060	43.71	-73.07	-1
8	1.13047	0.05	1.16	4.621890064	2.164	0.643	0.587	6.311	0.732	0.00071	-0.00051	57.65	-116.78	-1
9	1.18047	0.05	1.21	4.821890064	2.074	0.603	0.636	6.411	0.752	0.00063	-0.00043	74.20	-174.43	-1
10	1.23047	0.05	1.26	5.021890064	1.991	0.567	0.678	6.511	0.771	0.00056	-0.00036	94.02	-248.63	-1
11	1.28047	0.05	1.31	5.221890064	1.915	0.535	0.714	6.611	0.790	0.00050	-0.00030	118.05	-342.65	-1
12	1.33047	0.08	1.37	5.475987832	1.826	0.498	0.752	6.738	0.813	0.00044	-0.00024	241.61	-460.70	-1
13	1.40752												-702	-1

We notice that the profile reaches normal conditions approximately 700 m upstream of the control point. For the remaining 100 m, the flow will be uniform. The first point of the upstream profile will be equal to the normal depth of the downstream profile. Hence, the first point for the computation of the upstream profile will be $y = y_{n,2} = 1.408$ m.

Table 3: Upstream profile for the configuration with section increase.

Section	y_i (m)	dy (m)	y_{med} (m)	A (m^2)	v (m/s)	Fr (-)	$1-Fr^2$ (-)	P (m)	R (m)	S_f (-)	S_0-S_f (-)	dx (m)	x (m)	sign
1	1.40752	0.05	1.43	2.865042799	3.490	0.931	0.133	4.865	0.589	0.00247	-0.00227	2.93	0	-1
2	1.45752	0.05	1.48	2.965042799	3.373	0.884	0.218	4.965	0.597	0.00226	-0.00206	5.28	-2.93	-1
3	1.50752	0.05	1.53	3.065042799	3.263	0.841	0.292	5.065	0.605	0.00208	-0.00188	7.77	-8.22	-1
4	1.55752	0.05	1.58	3.165042799	3.160	0.802	0.357	5.165	0.613	0.00192	-0.00172	10.39	-15.98	-1
5	1.60752	0.05	1.63	3.265042799	3.063	0.765	0.414	5.265	0.620	0.00177	-0.00157	13.16	-26.37	-1
6	1.65752	0.05	1.68	3.365042799	2.972	0.731	0.465	5.365	0.627	0.00164	-0.00144	16.09	-39.54	-1
7	1.70752	0.05	1.73	3.465042799	2.886	0.700	0.510	5.465	0.634	0.00153	-0.00133	19.18	-55.63	-1
8	1.75752	0.05	1.78	3.565042799	2.805	0.671	0.550	5.565	0.641	0.00142	-0.00122	22.46	-74.81	-1
9	1.80752	0.05	1.83	3.665042799	2.728	0.644	0.586	5.665	0.647	0.00133	-0.00113	25.91	-97.27	-1
10	1.85752	0.1	1.91	3.815042799	2.621	0.606	0.633	5.815	0.656	0.00121	-0.00101	62.95	-123.18	-1
11	1.95752	0.1	2.01	4.015042799	2.491	0.561	0.685	6.015	0.668	0.00106	-0.00086	79.34	-186.13	-1
12	2.05752	0.08	2.10	4.190924903	2.386	0.526	0.723	6.191	0.677	0.00096	-0.00076	72.39	-265.47	-1
13	2.13340												-338	-1

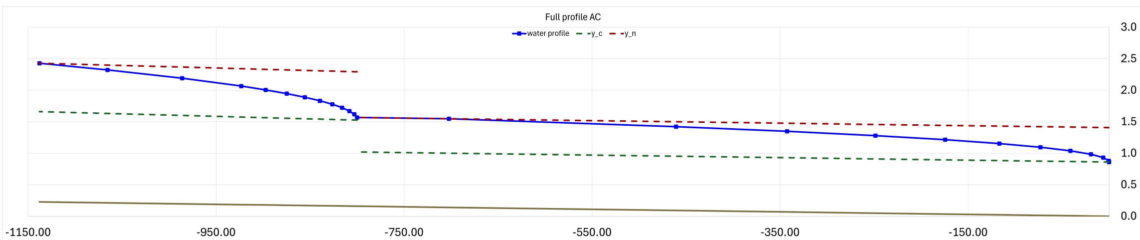

Figure 7: Water profile for the configuration with section increase.

3. Slope increase

As the slope increases, we know that the normal depth will change compared to the base configuration. However, the critical depth will stay the same. Hence, for this case we obtain $y_{c,3} = y_{c,0} = 1.366$ m and $y_{n,3} = 0.435$ m. Therefore, the alternative path has steep conditions and it identifies an S2 curve. The only control point is point B, where the flow goes from mild to steep conditions. Hence, in point B the flow has to be critical. The alternative path BC will be computed from point B to downstream, for a total length of $L = 90$ m as specified in the problem description. We don't know beforehand whether the flow will reach normal conditions in point C, but this is not important as we compute the profile in the downstream direction anyways. For sure, we know that it will not go back to critical.

We notice that the flow in the alternative path BC does not reach normal conditions before $x = 90$ m, hence the depth before the free fall will not be normal.

Table 4: Profile AB for the configuration with larger slope.

Section	y_i (m)	dy (m)	y_{med} (m)	A (m^2)	v (m/s)	Fr (-)	$1-Fr^2$ (-)	P (m)	R (m)	S_f (-)	S_0-S_f (-)	dx (m)	x (m)	sign
1	1.36591	0.05	1.39	2.781829955	3.595	0.973	0.053	4.782	0.582	0.00266	-0.00246	1.08	0	-1
2	1.41591	0.05	1.44	2.881829955	3.470	0.923	0.148	4.882	0.590	0.00243	-0.00223	3.32	-1.08	-1
3	1.46591	0.05	1.49	2.981829955	3.354	0.877	0.231	4.982	0.599	0.00223	-0.00203	5.69	-4.40	-1
4	1.51591	0.05	1.54	3.081829955	3.245	0.835	0.303	5.082	0.606	0.00205	-0.00185	8.20	-10.09	-1
5	1.56591	0.05	1.59	3.181829955	3.143	0.796	0.367	5.182	0.614	0.00189	-0.00169	10.84	-18.28	-1
6	1.61591	0.05	1.64	3.281829955	3.047	0.759	0.423	5.282	0.621	0.00175	-0.00155	13.64	-29.13	-1
7	1.66591	0.05	1.69	3.381829955	2.957	0.726	0.473	5.382	0.628	0.00162	-0.00142	16.60	-42.77	-1
8	1.71591	0.05	1.74	3.481829955	2.872	0.695	0.517	5.482	0.635	0.00151	-0.00131	19.72	-59.37	-1
9	1.76591	0.05	1.79	3.581829955	2.792	0.666	0.556	5.582	0.642	0.00141	-0.00121	23.02	-79.09	-1
10	1.81591	0.1	1.87	3.731829955	2.680	0.626	0.608	5.732	0.651	0.00127	-0.00107	56.66	-102.11	-1
11	1.91591	0.1	1.97	3.931829955	2.543	0.579	0.665	5.932	0.663	0.00112	-0.00092	72.30	-158.78	-1
12	2.01591	0.12	2.07	4.14931848	2.410	0.534	0.715	6.149	0.675	0.00098	-0.00078	107.45	-231.07	-1
13	2.13340												-338.5	-1

Table 5: Profile BC for the configuration with larger slope.

Section	y_i (m)	dy (m)	y_{med} (m)	A (m^2)	v (m/s)	Fr (-)	$1-Fr^2$ (-)	P (m)	R (m)	S_f (-)	S_0-S_f (-)	dx (m)	x (m)	sign
1	1.36591	0.1	1.32	2.631829955	3.800	1.058	-0.118	4.632	0.568	0.00307	0.03693	0.32	0	1
2	1.26591	0.1	1.22	2.431829955	4.112	1.191	-0.418	4.432	0.549	0.00376	0.03624	1.15	0.32	1
3	1.16591	0.1	1.12	2.231829955	4.481	1.354	-0.834	4.232	0.527	0.00471	0.03529	2.36	1.47	1
4	1.06591	0.1	1.02	2.031829955	4.922	1.559	-1.431	4.032	0.504	0.00604	0.03396	4.21	3.84	1
5	0.96591	0.1	0.92	1.831829955	5.459	1.821	-2.317	3.832	0.478	0.00797	0.03203	7.23	8.05	1
6	0.86591	0.07	0.83	1.661829955	6.017	2.108	-3.442	3.662	0.454	0.01038	0.02962	8.14	15.28	1
7	0.79591	0.05	0.77	1.541829955	6.486	2.358	-4.562	3.542	0.435	0.01275	0.02725	8.37	23.42	1
8	0.74591	0.05	0.72	1.441829955	6.936	2.608	-5.802	3.442	0.419	0.01535	0.02465	11.77	31.79	1
9	0.69591	0.05	0.67	1.341829955	7.453	2.905	-7.439	3.342	0.402	0.01875	0.02125	17.50	43.56	1
10	0.64591	0.0505	0.62	1.241329955	8.056	3.265	-9.659	3.241	0.383	0.02333	0.01667	29.27	61.06	1
11	0.59541												90	1

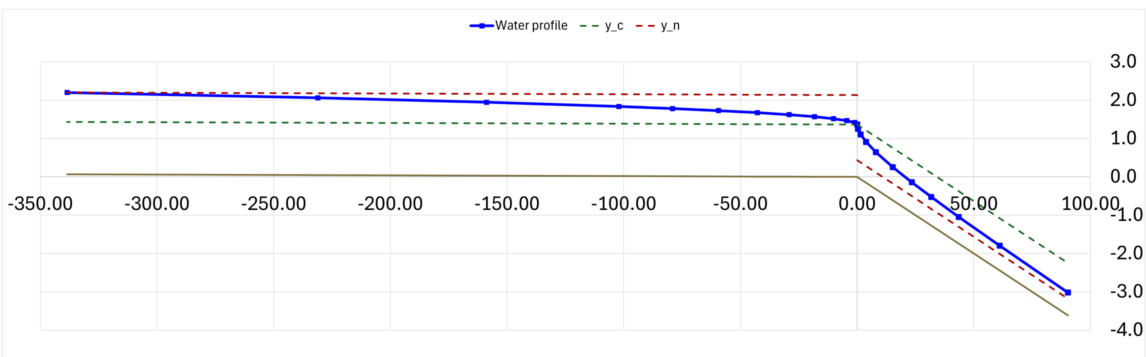

Figure 8: Water profile for the configuration with larger slope.

4. Slope and roughness increase

The last configuration is similar to the previous one but includes an increase of the channel roughness in the alternative path (profile BC). The increase of roughness will not cause any change in the critical depth, whereas it will change the normal depth. Hence, for this configuration we get $y_{c,4} = y_{c,0} = 1.366$ m and $y_{n,4} = 1.933$ m. In this case, we notice how an increase of roughness causes the downstream profile to become mild. Therefore, we identify two control points: (i) point C, where the flow is critical because of the free fall condition; (ii) point B, which represents the downstream boundary for the reach AB. Both profiles are computed backwards and identify M2 curves. The downstream profile (BC) will tend to normal depth, but given the limited length of $L = 90$ m it is not possible to state beforehand whether it will reach it. The last point of this profile will represent the first point of the profile AB, which again will tend to the normal depth.

Table 6: Profile BC for the configuration with larger slope and larger roughness.

Section	y_i **(m)**	dy **(m)**	y_{med} **(m)**	A **(m^2)**	v **(m/s)**	**Fr (-)**	$1 - Fr^2$ **(-)**	P **(m)**	R **(m)**	S_f **(-)**	$S_0 - S_f$ **(-)**	dx **(m)**	x **(m)**	**sign**
1	1.36591	0.02	1.38	2.751829955	3.634	0.989	0.022	4.752	0.579	0.39395	-0.35395	0.00	0	-1
2	1.38591	0.02	1.40	2.791829955	3.582	0.968	0.063	4.792	0.583	0.37967	-0.33967	0.00	0.00	-1
3	1.40591	0.02	1.42	2.831829955	3.531	0.948	0.102	4.832	0.586	0.36612	-0.32612	0.01	0.00	-1
4	1.42591	0.02	1.44	2.871829955	3.482	0.928	0.139	4.872	0.589	0.35326	-0.31326	0.01	-0.01	-1
5	1.44591	0.02	1.46	2.911829955	3.434	0.909	0.174	4.912	0.593	0.34104	-0.30104	0.01	-0.02	-1
6	1.46591	0.02	1.48	2.951829955	3.388	0.890	0.207	4.952	0.596	0.32942	-0.28942	0.01	-0.03	-1
7	1.48591	0.05	1.51	3.021829955	3.309	0.860	0.261	5.022	0.602	0.31042	-0.27042	0.05	-0.05	-1
8	1.53591	0.05	1.56	3.121829955	3.203	0.819	0.330	5.122	0.610	0.28591	-0.24591	0.07	-0.09	-1
9	1.58591	0.05	1.61	3.221829955	3.104	0.781	0.390	5.222	0.617	0.26411	-0.22411	0.09	-0.16	-1
10	1.63591	0.05	1.66	3.321829955	3.010	0.746	0.444	5.322	0.624	0.24464	-0.20464	0.11	-0.25	-1
11	1.68591	0.05	1.71	3.421829955	2.922	0.713	0.491	5.422	0.631	0.22718	-0.18718	0.13	-0.36	-1
12	1.73591	0.05	1.76	3.521829955	2.839	0.683	0.533	5.522	0.638	0.21147	-0.17147	0.16	-0.49	-1
13	1.78591	0.05	1.81	3.621829955	2.761	0.655	0.571	5.622	0.644	0.19729	-0.15729	0.18	-0.64	-1
14	1.83591	0.05	1.86	3.721829955	2.687	0.629	0.605	5.722	0.650	0.18445	-0.14445	0.21	-0.83	-1
15	1.88591	0.047	1.91	3.819097022	2.618	0.605	0.634	5.819	0.656	0.17310	-0.13310	0.23	-1.03	-1
16	1.93318												-1.26	-1

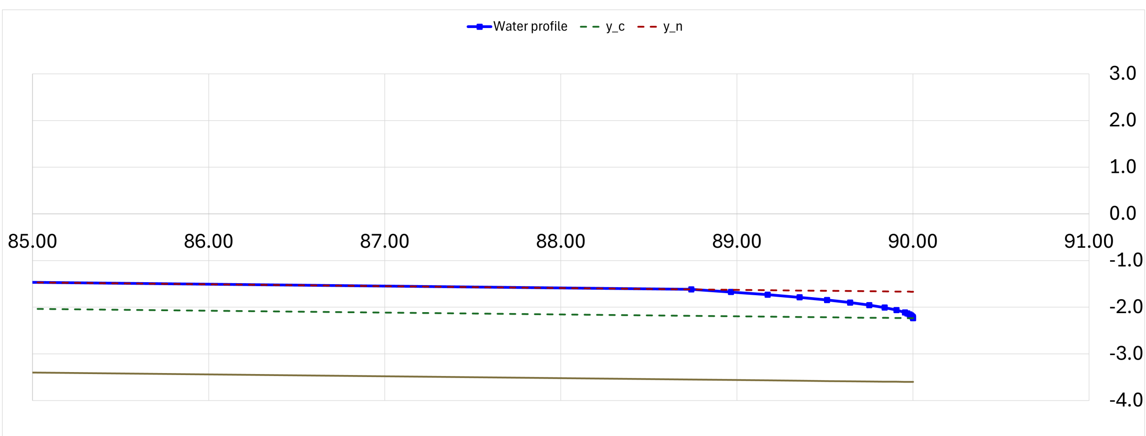

Figure 9: Zoom of the water profile on the downstream end of the reach with larger slope and larger roughness.

We notice that the profile BC quickly (after just 1.26 m) reaches normal conditions. Therefore, the depth along the rest of the reach will remain normal. For this reason, the first point for computing the water profile of reach AB will be the normal depth of reach BC.

Note: keep in mind that this may not always be the case. It is possible that the downstream profile does not reach normal depth. In this case, the last value of the depth would be anyways the first point used to compute the water profile of the upstream reach, regardless of it having reached normal depth or not.

Table 7: Profile AB for the configuration with larger slope and larger roughness.

Section	y_i (m)	dy (m)	y_{med} (m)	A (m^2)	v (m/s)	Fr (-)	$1 - Fr^2$ (-)	P (m)	R (m)	S_f (-)	$S_0 - S_f$ (-)	dx (m)	x (m)	sign
1	1.93318	0.01	1.94	3.87636409	2.580	0.592	0.650	5.876	0.660	0.00116	-0.00096	6.78	0	-1
2	1.94318	0.01	1.95	3.89636409	2.566	0.587	0.655	5.896	0.661	0.00114	-0.00094	6.94	-6.78	-1
3	1.95318	0.01	1.96	3.91636409	2.553	0.583	0.661	5.916	0.662	0.00113	-0.00093	7.10	-13.72	-1
4	1.96318	0.01	1.97	3.93636409	2.540	0.578	0.666	5.936	0.663	0.00112	-0.00092	7.27	-20.82	-1
5	1.97318	0.01	1.98	3.95636409	2.528	0.574	0.671	5.956	0.664	0.00110	-0.00090	7.43	-28.09	-1
6	1.98318	0.01	1.99	3.97636409	2.515	0.569	0.676	5.976	0.665	0.00109	-0.00089	7.60	-35.52	-1
7	1.99318	0.01	2.00	3.99636409	2.502	0.565	0.681	5.996	0.666	0.00108	-0.00088	7.77	-43.12	-1
8	2.00318	0.01	2.01	4.01636409	2.490	0.561	0.685	6.016	0.668	0.00106	-0.00086	7.95	-50.90	-1
9	2.01318	0.01	2.02	4.03636409	2.477	0.557	0.690	6.036	0.669	0.00105	-0.00085	8.12	-58.84	-1
10	2.02318	0.05	2.05	4.09636409	2.441	0.545	0.703	6.096	0.672	0.00101	-0.00081	43.28	-66.96	-1
11	2.07318	0.05	2.10	4.19636409	2.383	0.525	0.724	6.196	0.677	0.00095	-0.00075	47.96	-110.24	-1
12	2.12318	0.01	2.13	4.256585548	2.349	0.514	0.736	6.257	0.680	0.00092	-0.00072	10.41	-158.21	-1
13	2.13340												-169	-1

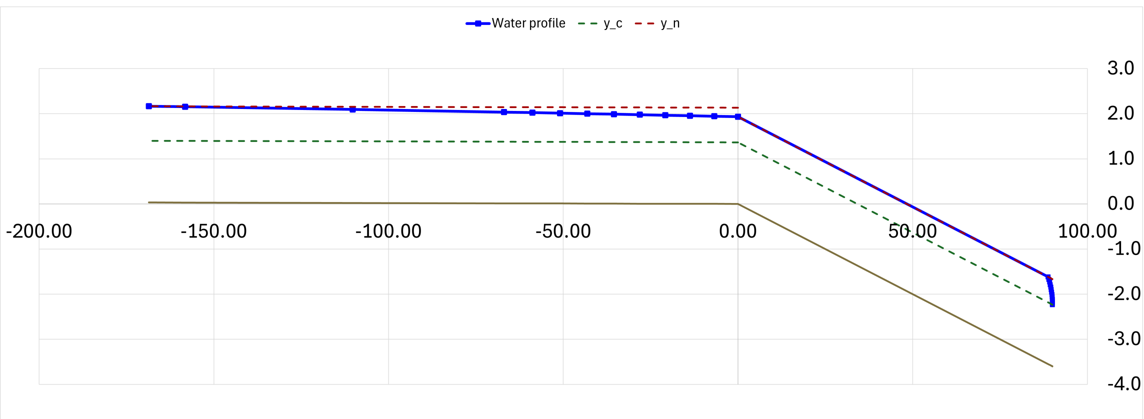

Figure 10: Water profile for the configuration with larger slope and larger roughness.