Homework 2 - Solutions

Exersize 1

- B dissolved in solvent C
- catalytic reaction $A+B \xrightarrow{D} E$ (catalyst D)
- 1st order reaction, rate constant k, forward reaction
- bubbling gas A into liquid C

i) $T = 25°C \rightarrow 35°C$

Arrehnius Equation: $k = k_0 \cdot \exp\left(-\frac{E_a}{RT}\right)$

→ exponential increase of k when T increasing, so when T from $25°C \rightarrow 35°C$ we expect much higher than 10% increase to k

BUT, because the overall consumption rate of gas A is 10% with the increase of T then the mass transfer of the component limits the kinetics of the reaction

ii) When starting stirring the mass transfer of gas A is expected to be enhanced, so more molecules of A would be able to react. In this way, the limiting step of mass transfer is not so strict and closer to the time scale of the reaction, that is why we expect an increase of the consumption rate of gas A under these conditions.

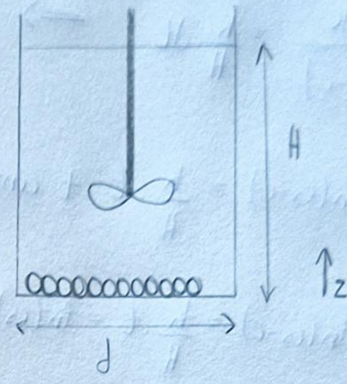
Exersize 2

$T = 25^\circ C = 298K$

$d = 10cm$

$H = 20cm$

- no stirring: $t = 60min$, $z = 1mm$, $c(t,z) = 1.5 g/L$
  $c(t,\infty) = 0 g/L$



- with stirring: $t = 5min$,

- $c_s = 3 g/L$ (saturation concentration of benzoic acid in water, $25^\circ C$)

→ In the absence of stirring, we can assume trasient diffusion across a semi-infinite slab

$$\frac{c(t,z) - c_s}{c_\infty - c_s} = \text{erf}\, J \overset{t=60min}{\underset{z=1mm}{\Longrightarrow}}$$

$$\frac{1.5-3}{0-3} = \text{erf}\, J \Rightarrow \text{erf}\, J = 0.5 \Rightarrow \left.\begin{matrix} J = 0.48 \\ J = \dfrac{z}{\sqrt{4Dt}} \end{matrix}\right\} \Rightarrow \frac{z}{\sqrt{4Dt}} = 0.48 \Rightarrow$$

$$\frac{z^2}{4Dt} = (0.48)^2 = 0.23$$

System: tank of height H

Element for mass balance: $dV = A dz = \frac{\pi d^2}{4} dz$

Initial conditions: $t = 0$, $c = 0$

$$D = \frac{z^2}{4t \cdot 0.23} = \frac{(10^{-3})^2}{4 \cdot (60 \cdot 60) \cdot 0.23} \Rightarrow$$

$$\boxed{D = 3 \cdot 10^{-4} \cdot 10^{-6} = 3 \cdot 10^{-10}\ m^2/s}$$

i) Mass balance in the case of stirred tank:

$$\text{Accumulation} \cdot dV = \cancelto{0}{\text{Flux}_{in}} \cdot dA - \cancelto{0}{\text{Flux}_{out}} \cdot dA + \overset{\text{mass gained due to dilution of benzoic acid}}{\text{Generation}} \cdot dV - \cancelto{0}{\text{Consumption}} \cdot dV \Rightarrow$$

$$\frac{d(cV)}{dt} = kA(c_s - c) \Rightarrow \frac{V\, dc}{dt} = kA(c_s - c) \Rightarrow$$

$$\frac{dc}{dt} = \frac{h}{H}(c_s - c) \Rightarrow$$

$$\frac{dc}{(c_s - c)} = \frac{h}{H}\,dt \Rightarrow$$

$$-\ln(c_s - c) = \frac{h}{H}\cdot t + \text{const.} \xrightarrow[\text{conditions}]{\text{initial}} \text{const.} = -\ln(c_s)$$

$$-\ln(c_s - c) = \frac{h}{H}\cdot t - \ln(c_s) \Rightarrow$$

$$\ln(c_s - c) - \ln(c_s) = -\frac{h}{H}\cdot t \Rightarrow$$

$$\boxed{\ln\left(\frac{c_s - c}{c_s}\right) = -\frac{h}{H}\cdot t} \quad (1)$$

$\rightarrow t = 5\,\text{min} = 5\cdot 60\,s$, $c = 1.5\,g/L$

$$(1):\ \ln\left(\frac{3-1.5}{3}\right) = -\frac{h}{20\cdot 10^{-2}}\cdot(5\cdot 60) \Rightarrow$$

$$-\ln 2 = -h\cdot 1500 \Rightarrow h = \frac{\ln 2}{1500} = 4.6\cdot 10^{-4}\,m/s$$

ii) Film theory : $\delta = \frac{D}{h} = \frac{3\cdot 10^{-10}\,(m^2/s)}{4.6\cdot 10^{-4}\,(m/s)} = 6.5\cdot 10^{-7}\,m$

iii) Surface renewal time : $\tau = \frac{D}{h^2} = \frac{3\cdot 10^{-10}\,(m^2/s)}{[4.6\cdot 10^{-4}\,(m/s)]^2} = 1.4\cdot 10^{-3}\,s = 1.4$ milliseconds