Introduction to Chemical Engineering

Problem Sheet 6 – Week 7 – October 31, 2025

<u>**Goal**</u>:

The aim of this session is to deepen your understanding of the energy balance equation for non-reactive processes and introduce energy balance for reactive systems.

Problem 1: Energy Balance, Reactive System

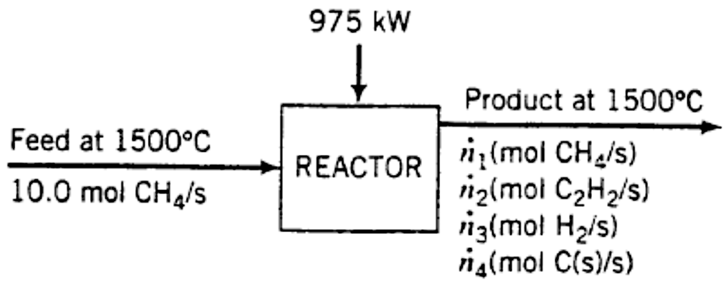
You are checking the performance of a reactor in which acetylene is produced from methane in the reaction:

$$2 \quad CH_4(g) \rightarrow C_2H_2(g) + 3\,H_2(g)$$

An undesired side reaction is the decomposition of acetylene:

$$C_2H_2(g) \rightarrow 2\,C(s) + H_2(g)$$

Methane is fed to the reactor at 1500°C at a rate of 10.0 mol CH_4/s. Heat is transferred to the reactor at a rate of 975 kW. The product temperature is 1500°C and the fractional conversion of methane is 0.600. A flowchart of the process and an enthalpy table are shown below.



References: C(s), H_2(g), at 25°C, 1 atm

Substance	ṅ_in (mol/s)	Ĥ_in (kJ/mol)	ṅ_out (mol/s)	Ĥ_out (kJ/mol)
CH_4	10.0	41.65	$\dot{n}_1$	$\hat{H}_1$
C_2H_2	—	—	$\dot{n}_2$	$\hat{H}_2$
H_2	—	—	$\dot{n}_3$	$\hat{H}_3$
C	—	—	$\dot{n}_4$	$\hat{H}_4$

(a) Using the heat capacities given below for enthalpy calculations, write and solve material balances and an energy balance to determine the product component flow rates and the yield of acetylene (mol C_2H_2 produced / mol CH_4 consumed).

- CH_4(g): Cp = 0.079 kJ/(mol·°C)
- C_2H_2(g): Cp = 0.052 kJ/(mol·°C)
- H_2(g): Cp = 0.031 kJ/(mol·°C)
- C(s): Cp = 0.022 kJ/(mol·°C)

For example, the specific enthalpy of methane at 1500°C relative to methane at 25°C is:

$$[0.079\text{ kJ/(mol·°C)}] \times (1500°C - 25°C) = 116.5\text{ kJ/mol.}$$

(b) The reactor efficiency may be defined as the ratio (actual acetylene yield / acetylene yield with no side reaction). What is the reactor efficiency for this process?

Problem 2: Energy Balance, Non-reactive System

A liquid stream containing 60.0 wt% ethane and 40.0% *n*-butane is to be heated from 150 K to 200 K at a pressure of 5 bar. Calculate the required heat input per kilogram of the mixture, neglecting potential and kinetic energy changes, using tabulated enthalpy data for C_2H_6 and C_4H_{10} and assuming that mixture component enthalpies are those of the pure species at the same temperature.

Problem 3: Energy Balance, Non-reactive Systems

Ralph Rackstraw, your next-door neighbor, surprised his wife last January by having a hot tub installed in their back yard while she was away on a business trip. It surprised her, all right, but instead of being pleased she was horrified. "Have you lost your mind, Ralph?" she sputtered. "It will cost a fortune to keep this thing hot." "Don't be silly, Josephine," he replied. "It can't cost more than pennies a day, even in the dead of winter." "No way—and when did you become such an expert, anyway?" "I guarantee it will cost nothing—and I don't see your Ph.D. certificate on the kitchen wall either." They argued for awhile and then, remembering your chemical engineering education, came to ask you to settle it for them. You asked a few questions, made several observations, converted everything to metric units, and arrived at the following data, all corresponding to an average outside air temperature of about 5°C.

- The tub holds 1230 liters of water.
- Rackstraw normally keeps the tub temperature at 29°C, raises it to 40°C when he plans to use it, keeps it at 40°C for about one hour, and drops it back to 29°C when he is finished.
- During heating, it takes about three hours for the water temperature to rise from 29°C to 40°C. When the heat is shut off, it takes eight hours for the water temperature to drop back to 29°C.
- Electricity costs 10 cents per kilowatt-hour.

Taking the heat capacity of the tub contents to be that of pure liquid water and neglecting evaporation, answer the following questions.

(a) What is the average rate of heat loss (kW) from the tub to the outside air? (Hint: Consider the period when the tub temperature is dropping from 40°C to 29°C.)
(b) At what average rate (kW) does the tub heater deliver energy to the water when raising the water temperature? What is the total quantity of electricity (kW·h) that the heater must deliver during this period? [Consider the result of part (a) when performing the calculation.]
(c) (These answers should settle the argument.) Consider a day in which the tub is used once. Use the results of parts (a) and (b) to estimate the cost ($) of heating the tub from 29°C to 40°C and the cost ($) of keeping the tub at a constant temperature. (There is no cost for the period in which T is dropping.) What is the total daily cost of running the tub? Assume the rate of heat loss is independent of the tub temperature.

(d) The tub lid, which is an insulator, is removed when the tub is in use. Explain how this fact would probably affect your cost estimates in part (c).