Vector Spaces

Label the following statements as true or false.

(a) Every vector space contains a zero vector.
(b) A vector space may have more than one zero vector.
(c) In any vector space, $ax = bx$ implies that $a = b$.
(d) In any vector space, $ax = ay$ implies that $x = y$.

Subspaces

(a) If V is a vector space and W is a subset of V that is a vector space, then W is a subspace of V.
(b) The empty set is a subspace of every vector space.
(c) If V is a vector space other than the zero vector space, then V contains a subspace W such that $\mathsf{W} \neq \mathsf{V}$.
(d) The intersection of any two subsets of V is a subspace of V.

(e) An $n \times n$ diagonal matrix can never have more than n nonzero entries.
(f) The trace of a square matrix is the product of its diagonal entries.
(g) Let W be the xy-plane in R^3; that is, $\mathsf{W} = \{(a_1, a_2, 0) : a_1, a_2 \in R\}$. Then $\mathsf{W} = \mathsf{R}^2$.

Linear dependence and independence

(a) If S is a linearly dependent set, then each vector in S is a linear combination of other vectors in S.
(b) Any set containing the zero vector is linearly dependent.
(c) The empty set is linearly dependent.
(d) Subsets of linearly dependent sets are linearly dependent.
(e) Subsets of linearly independent sets are linearly independent.
(f) If $a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0$ and $x_1, x_2, \ldots, x_n$ are linearly independent, then all the scalars a_i are zero.

Bases and Dimensions

(b) Every vector space that is generated by a finite set has a basis.
(c) Every vector space has a finite basis.
(d) A vector space cannot have more than one basis.
(e) If a vector space has a finite basis, then the number of vectors in every basis is the same.

(h) Suppose that V is a finite-dimensional vector space, that S_1 is a linearly independent subset of V, and that S_2 is a subset of V that generates V. Then S_1 cannot contain more vectors than S_2.

(i) If S generates the vector space V, then every vector in V can be written as a linear combination of vectors in S in only one way.

(j) Every subspace of a finite-dimensional space is finite-dimensional.

(k) If V is a vector space having dimension n, then V has exactly one subspace with dimension 0 and exactly one subspace with dimension n.

(l) If V is a vector space having dimension n, and if S is a subset of V with n vectors, then S is linearly independent if and only if S spans V.

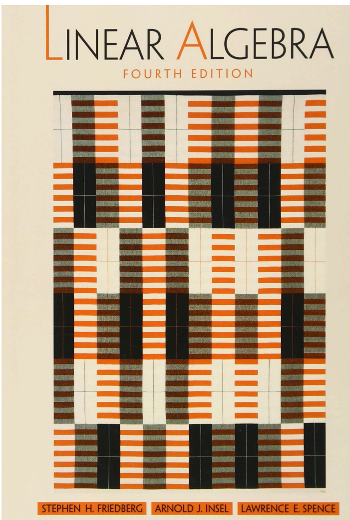
(a) T **(b)** F **(c)** F **(d)** F

(a) F (b) F (c) T (d) F (e) T (f) F (g) F

(a) F **(b)** T **(c)** F **(d)** F **(e)** T **(f)** T

(a) F **(b)** T **(c)** F **(d)** F **(e)** T

(h) T **(i)** F **(j)** T **(k)** T **(l)** T



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:(You should review linear algebra