

Optical methods in chemistry  
or  
Photon tools for chemical sciences

Session 10:

# Course layout – contents overview and general structure

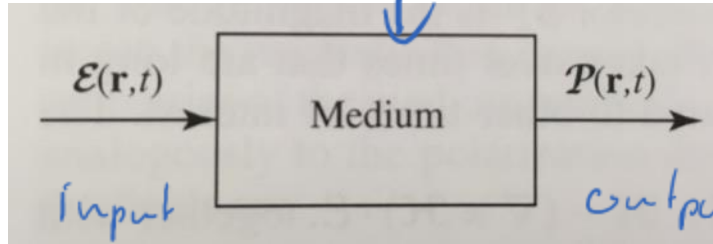
- Introduction and ray optics
- Wave optics
- Beams
- From cavities to lasers
- More lasers and optical tweezers
- From diffraction and Fourier optics
- Microscopy
- Spectroscopy
- Electromagnetic optics
- **Absorption, dispersion, and non-linear optics**
- Ultrafast lasers
- Introduction to x-rays
- X-ray diffraction and spectroscopy
- Summary

Today:

More materials properties, linear and non-linear

# Electromagnetic waves in dielectric media

General



interpretation  $\rightarrow$  output response

in response to an electric field  $\mathcal{E}$   
media creates a polarization

"instantaneous" "no  $\vec{r}$ " "no direction"

But stick with linear, nondispersive, homogenous, and isotropic media right now:

$$\mathcal{E} \rightarrow [\underline{M}] \rightarrow \mathcal{P}$$

simplify to

$$\mathcal{P} = \epsilon_0 \chi \mathcal{E}$$

electric susceptibility

$\chi$  scalar

$$\mathcal{E} \rightarrow [\underline{\chi}] \rightarrow \mathcal{P}$$

$$D = \epsilon \mathcal{E} \quad / \quad \epsilon = \epsilon_0 (1 + \chi)$$

$$\epsilon = \epsilon_0 (1 + \chi)$$

$\uparrow$  dielectric constant  
"electric permittivity"<sup>3</sup>

This leads to the following Maxwell and wave equations

$$\begin{aligned}\nabla \times \mathcal{H} &= \epsilon \frac{\partial \mathcal{E}}{\partial t} \\ \nabla \times \mathcal{E} &= -\mu \frac{\partial \mathcal{H}}{\partial t} \\ \nabla \cdot \mathcal{E} &= 0 \\ \nabla \cdot \mathcal{H} &= 0.\end{aligned}$$

$\epsilon$  - dielectric constant (electric permittivity)  
 $\mu$  - magnetic permeability

back to wave equation

$$\nabla^2 u - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad | \text{ in medium}$$

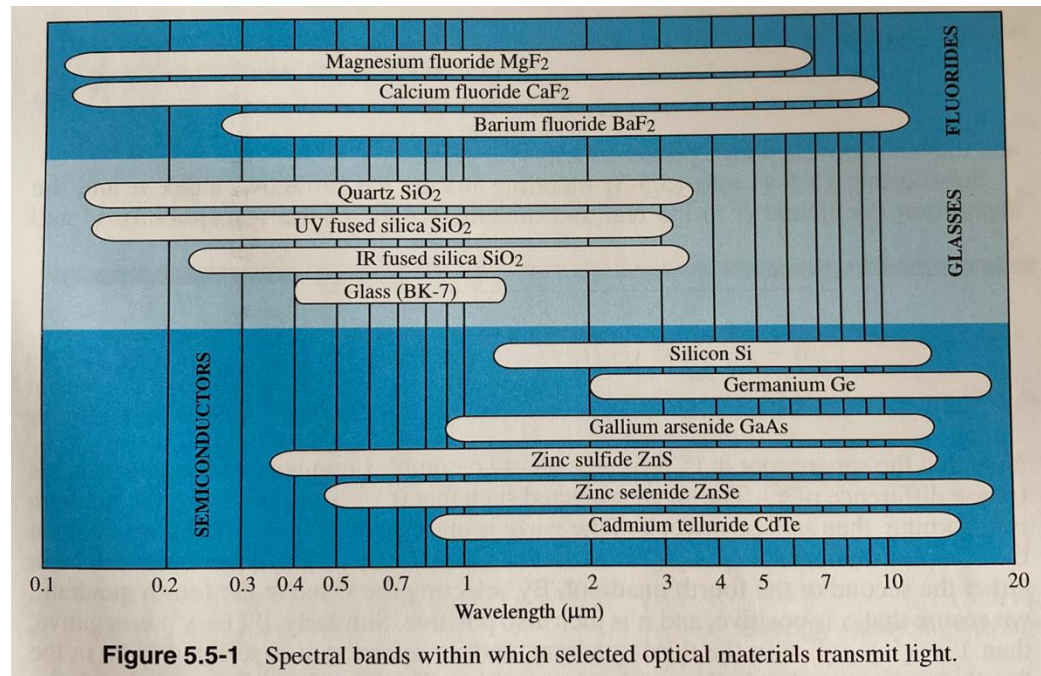
$$c = \frac{1}{\sqrt{\epsilon \mu}}$$

define  $n = \frac{c_0}{c} = \sqrt{\frac{\epsilon}{\epsilon_0} \frac{\mu}{\mu_0}} = \underline{\text{refractive index!}}$

non-magnetic materials

$$n = \sqrt{\frac{\epsilon}{\epsilon_0}} = \sqrt{1 + \chi}$$

# Generalized optical constant



Absorption: The imaginary part

# Complex refractive index

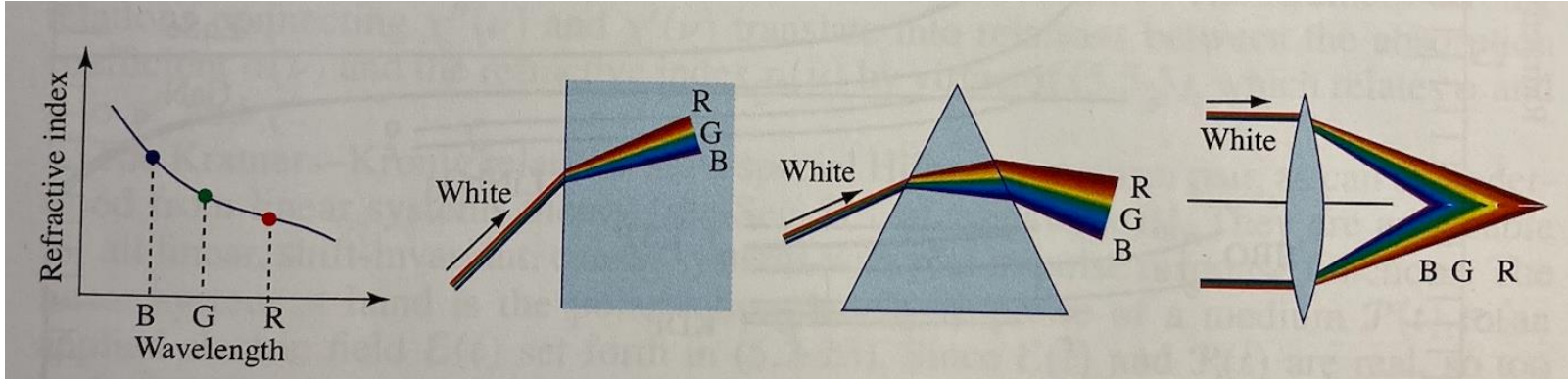
General:

$$n - j\frac{1}{2}\frac{\alpha}{k_0} = \sqrt{\epsilon/\epsilon_0} = \sqrt{1 + \chi' + j\chi''}.$$

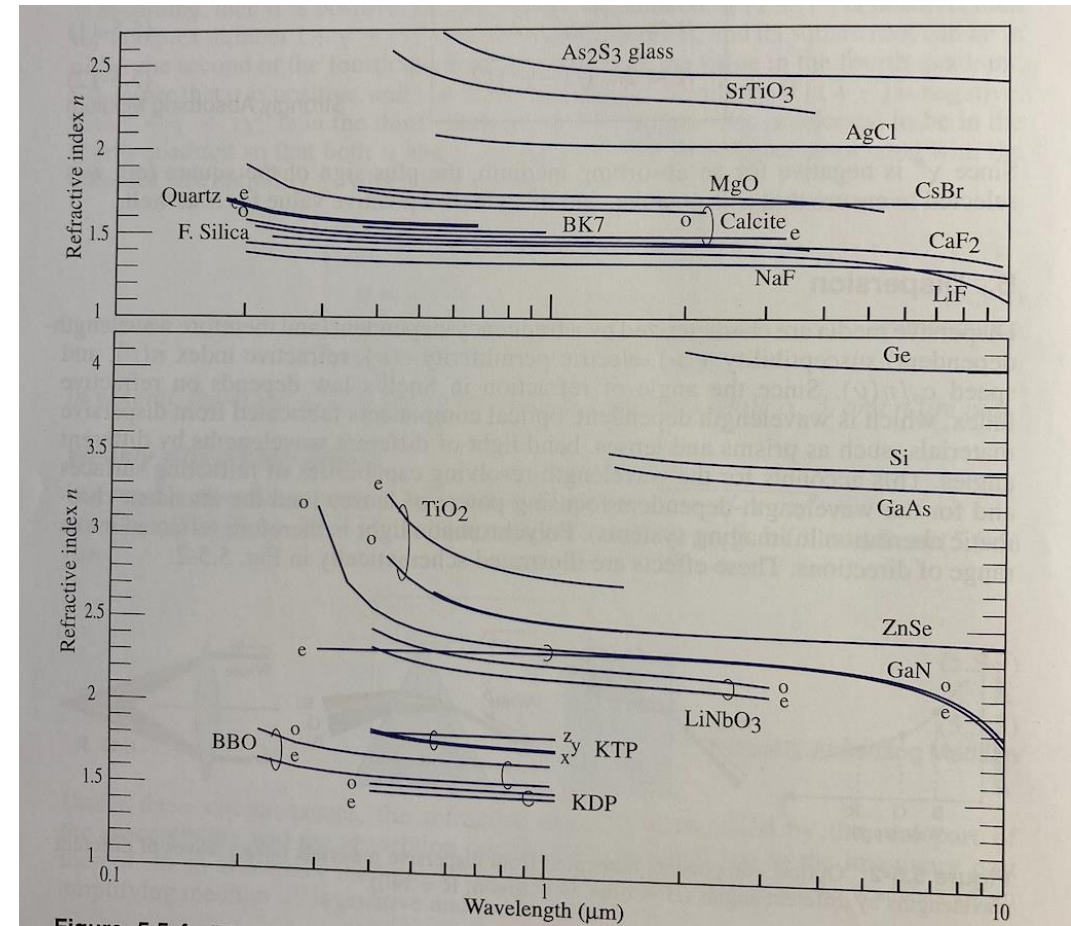
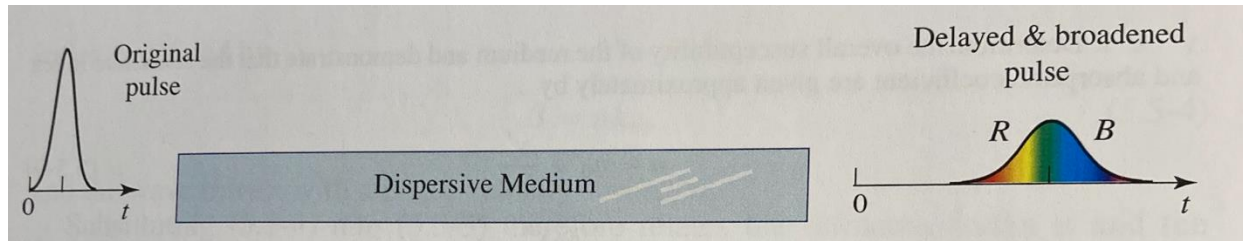
Weakly absorbing media:

$$n \approx \sqrt{1 + \chi'}$$
$$\alpha \approx -\frac{k_0}{n}\chi''.$$

# Dispersion: The real part



# A short pulse in a dispersive medium

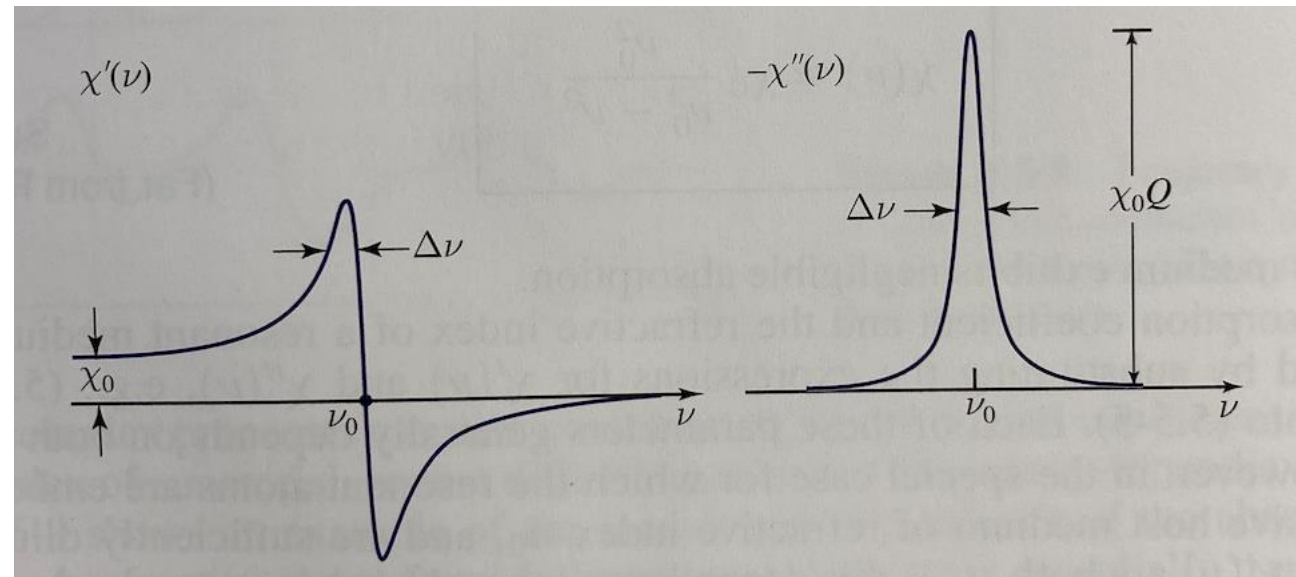


## The Kramers-Kronig Relation

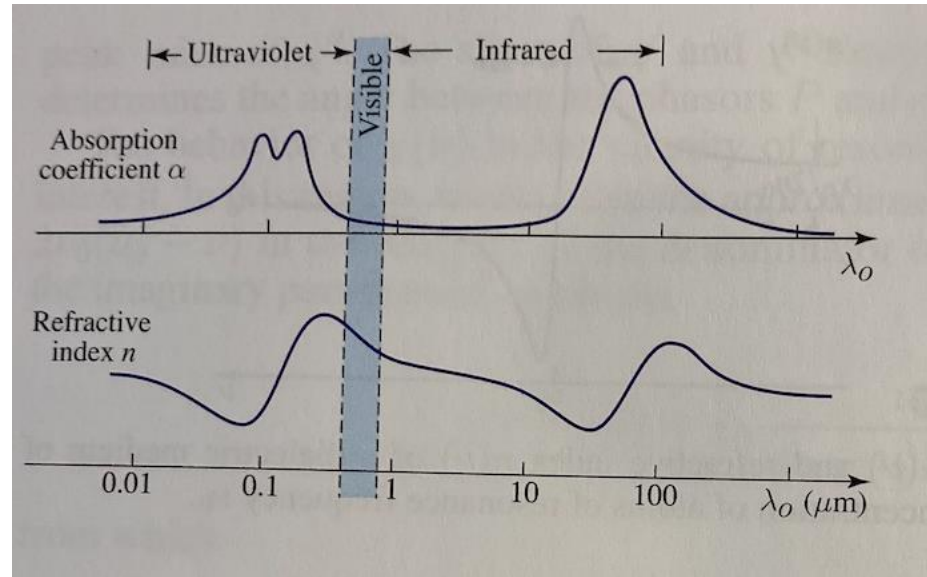
$$\chi'(\nu) = \frac{2}{\pi} \int_0^{\infty} \frac{s\chi''(s)}{s^2 - \nu^2} ds$$

$$\chi''(\nu) = \frac{2}{\pi} \int_0^{\infty} \frac{\nu\chi'(s)}{\nu^2 - s^2} ds.$$

## Resonances and refractive index

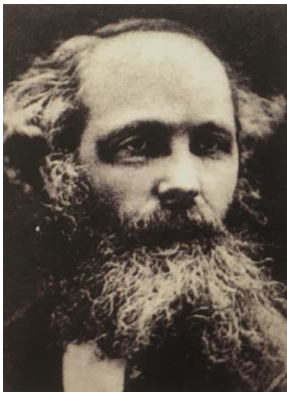


# A real optical (transparent) material



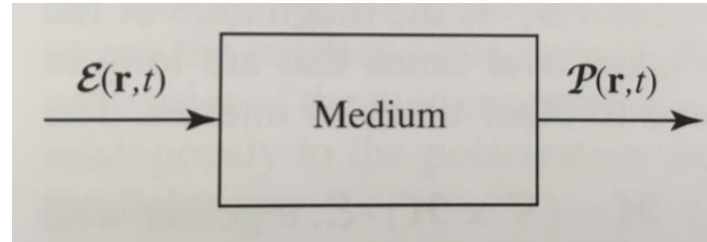
From linear to non-linear

# Back to formal description of light as EM wave (from Session 9)



James Maxwell  
1831 - 1879

$$\begin{aligned}\nabla \times \mathcal{H} &= \epsilon_0 \frac{\partial \mathcal{E}}{\partial t} \\ \nabla \times \mathcal{E} &= -\mu_0 \frac{\partial \mathcal{H}}{\partial t} \\ \nabla \cdot \mathcal{E} &= 0 \\ \nabla \cdot \mathcal{H} &= 0,\end{aligned}$$



But stick with linear, nondispersive, homogenous, and isotropic media right now:

$$\mathcal{P} = \epsilon_0 \chi \mathcal{E},$$

$$\begin{aligned}\nabla \times \mathcal{H} &= \epsilon \frac{\partial \mathcal{E}}{\partial t} \\ \nabla \times \mathcal{E} &= -\mu \frac{\partial \mathcal{H}}{\partial t} \\ \nabla \cdot \mathcal{E} &= 0 \\ \nabla \cdot \mathcal{H} &= 0.\end{aligned}$$

## Generalization of susceptibility $\chi$ (still linear)

- Inhomogenous media
- Anisotrope media
- Dispersive media

General:

Interpretation: Dynamic relationship between  $E$  and  $P$

- $E$  induces bound electrons in material to oscillate
- Time-dependent Polarization density  $P(t)$
- Time-delay between  $E(t)$  and  $P(t)$

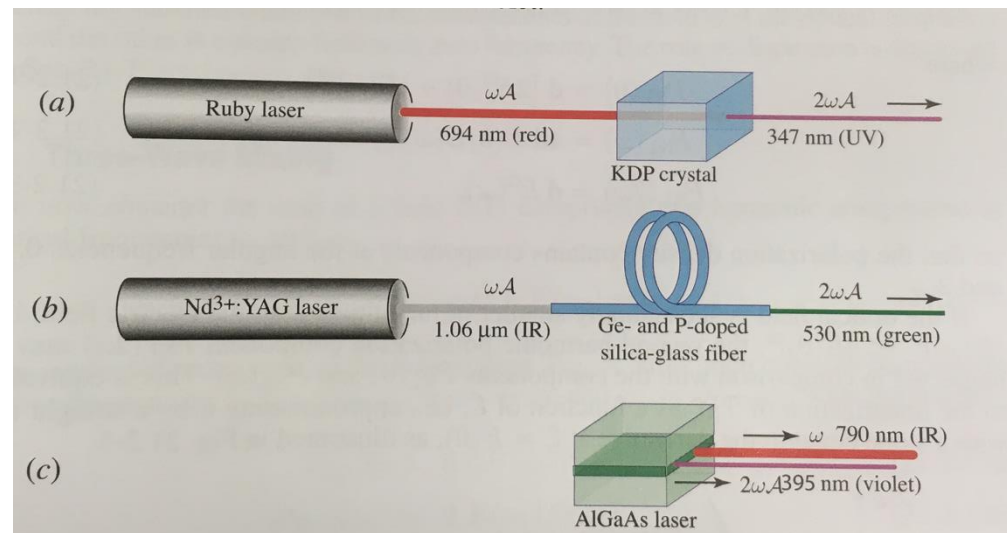
# Non-linear optical media

Handwaving:

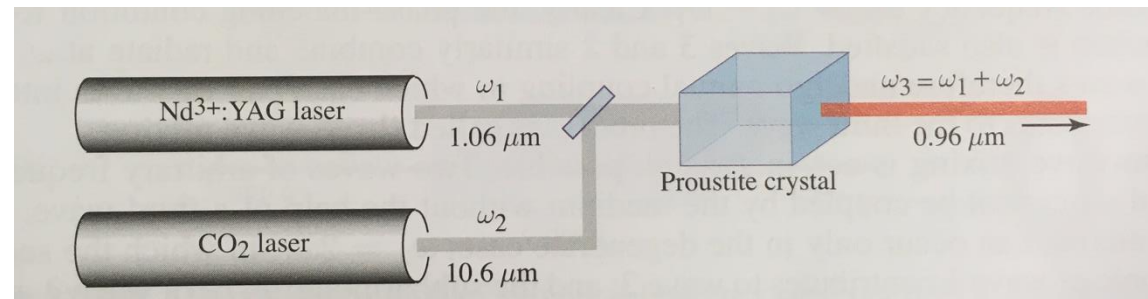
- Linear: restoring force of light induced fields linear (“Hookes law applies”)
- Non-linear: Light induced fields comparable to inter-atomic fields in crystal (“no more linear forces”)
- (Note: fields still weak compared to intra-atomic fields – that is a later topic)

# Expanded description of relationship between Polarization density $P$ and Electric field $E$

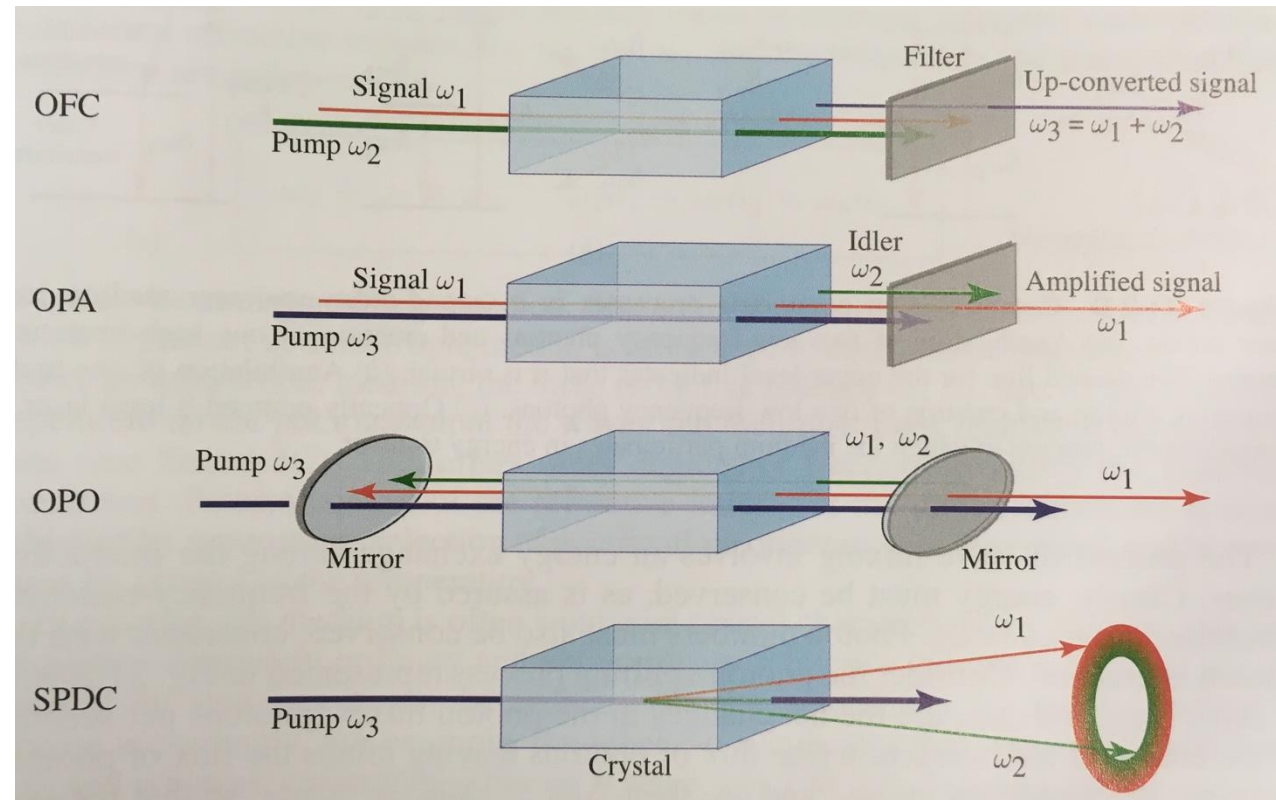
# Second order non-linear optics example: Second harmonic generation



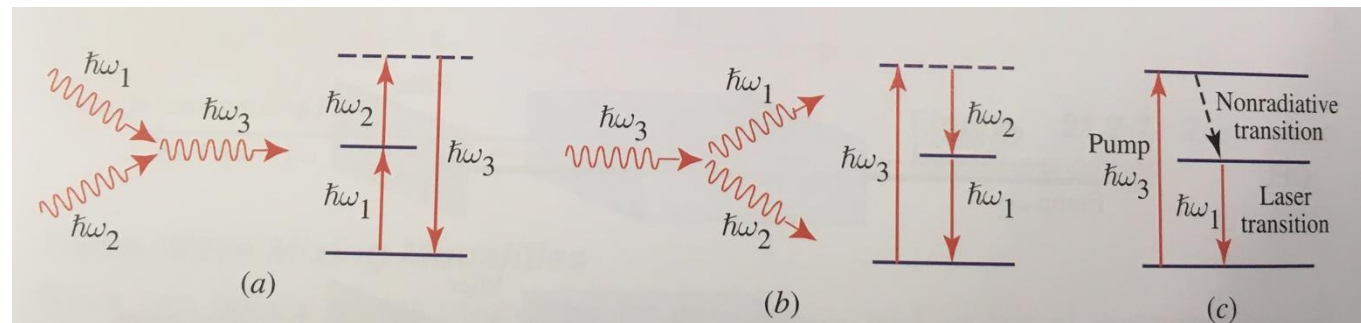
## Second order non-linear optics example: Sum frequency generation



## Second order non-linear optics example: Optical parametric devices

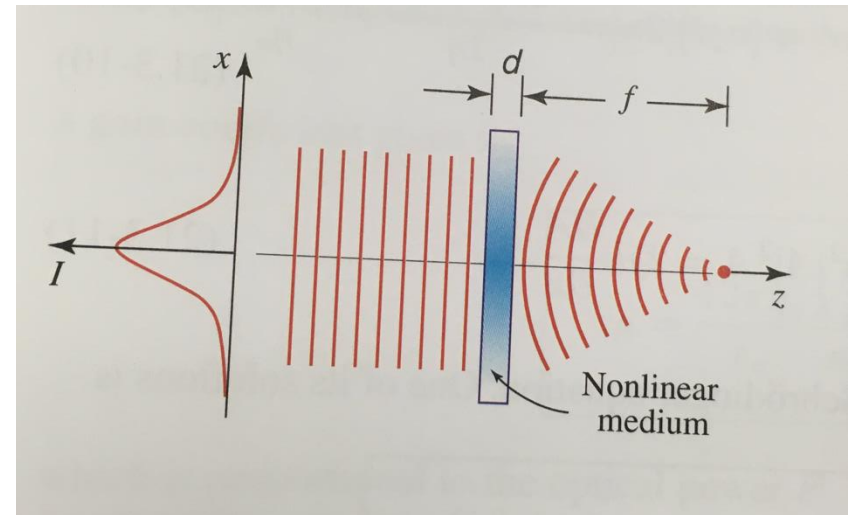


# Second order non-linear optics example: Description as photon interaction process



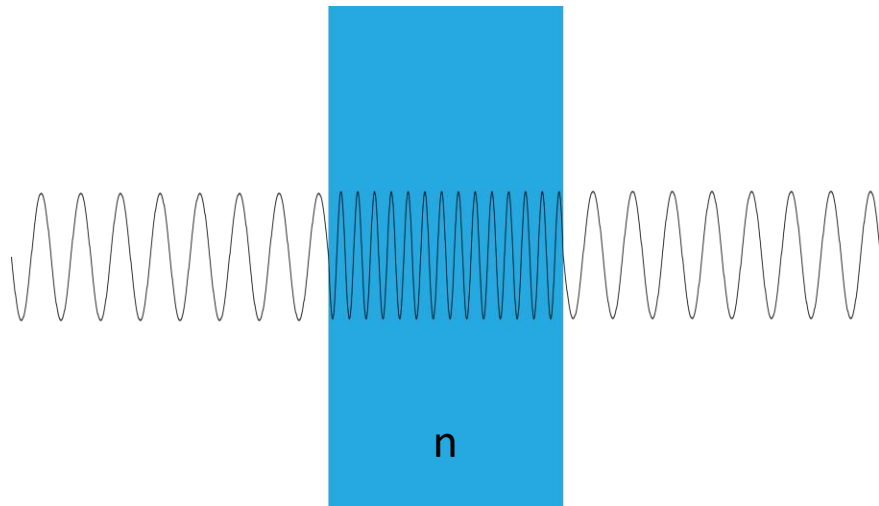
## Third order non-linear optics example: Third-harmonic generation

## Third order non-linear optics example: Optical Kerr Effect and Self-Focusing

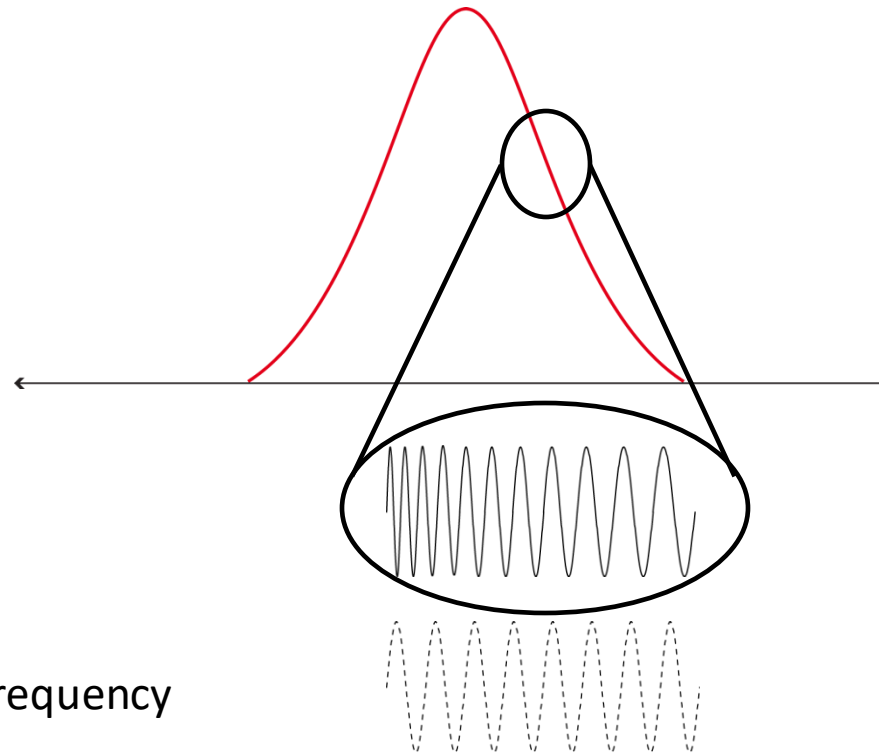


# Third order non-linear optics example: Self-phase modulation

normal refractive index



time-dependent refractive index



$$\omega(t) = -\frac{d\phi(t)}{dt} ; \text{instantaneous frequency}$$

- Conceptually – consider a plane monochromatic wave
- Time-dependent index leads to time-dependent frequency
- New frequency components are generated

The end.