Exercises 4

Exercise 4.1

What is the frequency of the photon emitted during the transition of the electron from the n = 5 shell to the n = 4 shell of a hydrogen atom?

Solution:

$$E_{photon} = -\Delta E = E_5 - E_4 = E_0\left(\frac{1}{n_4^{\ 2}} - \frac{1}{n_5^{\ 2}}\right)$$

$E_{photon} = E_0 \bullet \left(\frac{1}{16} - \frac{1}{25}\right) = \frac{9E_0}{400} \cong 4.90 \cdot 10^{-20}$ J, with $E_0 = 13.6$ eV

So, with the formula $E = h\nu$, we calculate $\nu \cong \frac{4.90 \cdot 10^{-20}}{6.626 \cdot 10^{-34}}$ Hz $\cong 7.40 \cdot 10^{13}$ Hz

Exercise 4.2

Calculate the wavelength and indicate the color of the second spectral line of the Balmer series ($n_1 = 2$).

Solution:

Balmer's series corresponds to the transitions from higher states to the n = 2 excited state for hydrogen. As above, according to the equation

$E_{photon} = E_4 - E_2 = E_0\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3E_0}{16} \cong 4.09 \cdot 10^{-19}$ J.

Therefore, with $E = h\nu$, we have $\nu \cong \frac{4.09 \cdot 10^{-19}}{6.626 \cdot 10^{-34}}$ Hz $\cong 6.17 \cdot 10^{14}$ Hz.

By the formula $c = \lambda\nu$, $\lambda \cong \frac{3.00 \cdot 10^{8}}{6.17 \cdot 10^{14}}$ m $\cong$ 486 nm. The spectral line is blue.

Exercise 4.3

Which transition of a hydrogen atom generates red light with a wavelength of 656.3 nm? (The Rydberg constant : $R = 3.290 \bullet 10^{15}$ Hz)

Solution:

By Rydberg's formula $\nu = \frac{c}{\lambda} = R\left(\frac{1}{n_1^{\ 2}} - \frac{1}{n_2^{\ 2}}\right) = R \cdot x$ we find that

$$x = \frac{c}{R\lambda} = \frac{2.998 \cdot 10^{8}}{3.290 \cdot 10^{15} \cdot 656.3 \cdot 10^{-9}} = 0.139$$

To calculate n_2, we transform the formula for x :

$$x = \frac{1}{n_1^{\ 2}} - \frac{1}{n_2^{\ 2}} \Rightarrow n_2 = \sqrt{\frac{1}{\frac{1}{n_1^{\ 2}} - x}}$$

The number under the root must be positive, so we must have $\frac{1}{n_1^2} - x \geq 0 \Leftrightarrow \frac{1}{n_1^2} \geq 0.139$. Since n_1 must be an integer, it is easy to verify that we must have $n_1 = \{1, 2\}$.

From here, we can see directly that the transitions to $n_1 = 1$ are part of Lyman's series, which emit UV and not the visible radiation of this exercise. We must therefore have $n_1 = 2$ and we can then calculate n_2 from the formula.

We can also check the 2 possibilities for n_1 in the equation to calculate n_2 (with $x = 0.139$). For $n_1 = 1$, we find $n_2 \cong 1.08$, which is not an integer. On the other hand, for $n_1 = 2$ we find $n_2 \cong 3.0015$ which we can round to $n_2 = 3$. The red light of the hydrogen atom is therefore emitted for the electronic transition from shell 3 to shell 2.

Exercise 4.4

Which subshell has 5 orbitals? How many orbitals does an l subshell have?

Solution:

The d subshell, having $l = 2$, has 5 orbitals $m_l = (-2, -1, 0, 1, 2)$. An l subshell has $2l + 1$ orbitals.

Exercise 4.5

Give the 4 quantum numbers of the electron of the hydrogen atom in its ground state.

Solution:

As the electron has 2 possible spins, the 4 quantum numbers can be either
$n = 1,\ l = 0,\ m_l = 0,\ m_s = 1/2$ or $n = 1,\ l = 0,\ m_l = 0,\ m_s = -1/2$.

Exercise 4.6

How many nodal areas does the orbital of an electron defined by the quantum numbers $\left(n, l, m_l, m_s\right) = (4, 2, -1, 1/2)$ have in total?

How many nodal surfaces of each type (angular/radial) does it have?

What is the type of this orbital?

Solution:

This orbital has n-1 = 3 nodal surfaces, $l = 2$ angular and $n - l - 1 = 4 - 2 - 1 = 1$ radial.

The value $l = 2$ tells us that it is a d orbital.

Exercise 4.7

How many nodal surfaces does a $5p$ orbital have in total?

And how many nodal surfaces does it have of each type?

Solution:

This orbital has 4 nodal surfaces, $l = 1$ angular and $n - l - 1 = 5 - 1 - 1 = 3$ radials.

Exercise 4.8

Determine the angular momentum of a s orbital and a p orbital.

Solution:
By the formula $L = \sqrt{l(l + 1)}\,\hbar$ we find, for a s orbital, $l = 0$ and therefore $L = 0$ J•s and for a p orbital, $l = 1$, therefore $L = \sqrt{2} \bullet \hbar \cong 1.5 \cdot 10^{-34}$ J•s.

Exercise 4.9
Give the ground state electron configuration of potassium, argon, arsenic, neon, and barium.

Solution:
$K : [Ar]4s^1,\ \ Ar : [Ar] = [Ne]3s^2 3p^6,\ \ As : [Ar]3d^{10}4s^2 4p^3,\ \ Ne : [Ne] = 1s^2 2s^2 2p^6,\ \ Ba : [Xe]6s^2$

Exercise 4.10
Give all the possible combinations for the 4 quantum numbers of the 8th electron of an atom in its ground state in absence of a magnetic field.

Solution:
According to the "Aufbau principle", *i.e.* Hund's rule and Pauli's principle, it is a $2p$ orbital, which corresponds to $n = 2,\ l = 1$.
For the ground state and in the absence of a magnetic field, the orbitals with $m_l = \{-1,\ 0,\ 1\}$ are degenerate as well as those with $m_s = \{-1/2,\ 1/2\}$. Therefore we have six possibilities: $(n,\ l,\ m_l,\ m_s) = \{(2,\ 1,\ -1,\ \pm 1/2),\ (2,\ 1,\ 0,\ \pm 1/2),\ (2,\ 1,\ 1,\ \pm 1/2)\}$.

Exercise 4.11
Give all the possible combinations for the 4 quantum numbers of the 19th electron of an atom in its ground state in absence of a magnetic field.

Solution: (The 4s orbital is lower in energy than the 3d orbital)
Based on the "Aufbau principle", we find that electron no. 19 is in the $4s$ orbital. We then find: $(n,\ l,\ m_l,\ m_s) = (4,\ 0,\ 0,\ \pm 1/2)$

Exercise 4.12
Which elements in the periodic table have an electron configuration of the type [noble gas] ns^2 ?

Solution:
These are all alkaline earths, that is, the second column of the periodic table.