

## Exercises 3

### Exercise 3.1

A particle is confined in a linear box of length  $L$  surrounded by walls of infinite potential. The ground state of this system is described by the following wave function:

$$\Psi_1(x) = \sqrt{\frac{2}{L}} \times \sin \left( \frac{\pi x}{L} \right)$$

- What is the probability of finding the particle at a given position  $x$ ?
- At which position is the maximum probability density?
- What is the total probability of finding the particle in the box?
- If  $L = 10$  nm, what is the probability that the particle is between 4:95 and 5:05 nm?

Note: Exercise 3.1 will be solved on the board during the exercise session this Friday, September 26, 2025.

### Exercise 3.2

The total energy of the particle in the box can be calculated as

$$E_{tot} = E_{kin} + E_{pot},$$

where the kinetic energy is given by

$$E_{kin} = \frac{1}{2}mv^2.$$

Write down an expression for the total energy of the particle in the box, using the de Broglie relationship ( $p = mv = \frac{h}{\lambda}$ ) and the fact that the wavelength must satisfy  $\lambda = \frac{2L}{n}$ .

What is the main implication of this equation?

### Exercise 3.3

Using the particle-in-a-box model for the hydrogen atom and treating the atom as an electron in a one-dimensional box of length 150 pm, predict the wavelength of radiation emitted when the electron falls from the level with  $n = 5$  to that with  $n = 4$ .

Repeat the calculation for the transition from  $n = 4$  to  $n = 3$ .

### Exercise 3.4

The energy levels of a particle of mass  $m$  in a two-dimensional square box of size  $L$  are given by  $\frac{(n_1^2+n_2^2)h^2}{8mL^2}$ .

Do any of these levels have the same energy?

If so, find the values of the quantum number  $n_1$  et  $n_2$  for the first three cases.

### Exercise 3.5

Refer to the previous exercise. If one side of the box is twice as long as the other, the energy levels are given by:

$$\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2}\right) \cdot \frac{h^2}{8m},$$

with  $L_2 = 2L_1$ .

Do any of these levels have the same energy?

If so, find the values of the quantum numbers  $n_1$  and  $n_2$  for the two lowest levels having identical energies.