BIOENG 455 PRACTISE TEST 1

29/9/25

1) $\left(p + \frac{a N^2}{V^2}\right)\left(V - Nb\right) = N k_B T$

The units of two quantities added (or substracted) must be the same.

$\therefore \left[\frac{aN^2}{V^2}\right] = [p] = \frac{\text{FORCE}}{\text{AREA}} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

$\Rightarrow [a] = ML^{-1}T^{-2} \times L^6$ NB $[N] = \phi$

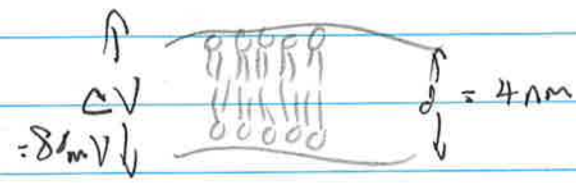
$\underline{\underline{[a] = ML^5T^{-2}}}$

Similarly for b:

$[Nb] = [V]$

$\therefore \underline{\underline{[b] = L^3}}$

2) $E = \frac{\Delta V}{d}$



$= \frac{80\,mV}{4\,nm}$

$\underline{\underline{= 20\ 10^6\ V/m}}$ cp. Lightning ~ $3\ 10^6$ V/m!

3 We have a polymer:

$N = 20{,}000$ monomers

$a = 1\,nm$

$l_k = 3\,nm$

By definition: $l_k = \dfrac{\langle R_{ee}^2 \rangle}{L_0}$

$$n_k = \frac{L_0}{l_k}$$

a) So, $L_0 = 20{,}000 \times 1\,nm = 2\,10^4\,nm$

$$\Rightarrow n_k = \frac{2\,10^4\,nm}{3\,nm} = 6667 \text{ kuhn lengths}$$

b) $\langle R_{ee}^2 \rangle = l_k \times L_0 = 3\,nm \times 2\,10^4\,nm = 6\,10^4\,nm^2$

$$\therefore \sqrt{\langle R_{ee}^2 \rangle} = 245\,nm$$

4) Equipartition Thm. see Lecture 3, slide 9.

5) 

$$p = \frac{V}{a_0 l}$$

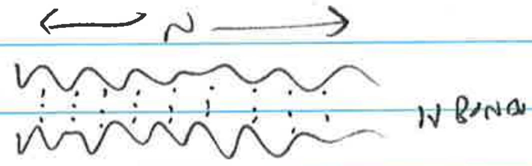
V = lipid volume

a_0 = cross-sectional area

l = max. tail length

Removing one tail would reduce the volume (V), and to a first approximation leave a_0 and l unaltered, so p would decrease.

6)



$N = 10^6$ base pairs

$a = 0.3$ nm

$l_k = 300$ nm

$\Delta E = 5 k_B T$

$$\langle R_{ee}^2 \rangle = l_k . L_0 = 300\,nm \times 10^6 \times 0.3\,nm$$

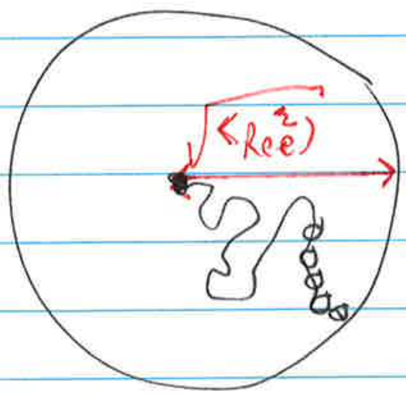
$$= 9.10^7\,nm^2$$

$$\therefore \sqrt{\langle R_{ee}^2 \rangle} = 9487\,nm \sim 9.5\,\mu m$$

Energy $= \sum$ H-Bonds $= 10^6 \times 5 k_B T$

$$= 5\,10^6 k_B T$$

7)



N monomers of size a.

$$V_{mon} = N a^3 \frac{4\pi}{3}$$

$$V_{space} = \tfrac{4}{3}\pi\left(\sqrt{\langle R_{ee}^2 \rangle}\right)^3$$

$$\therefore \frac{V_{mono}}{V_{space}} = \frac{Na^3}{(Na^2)^{3/2}}$$

$$= \frac{N}{N^{3/2}} = \frac{1}{\sqrt{N}}$$

Given $N = 200$, $a = 1$ nm (we don't actually need a)

$$\frac{V_{mono}}{V_{space}} = \frac{1}{\sqrt{200}} = 0.07 \quad \sim 7\%$$