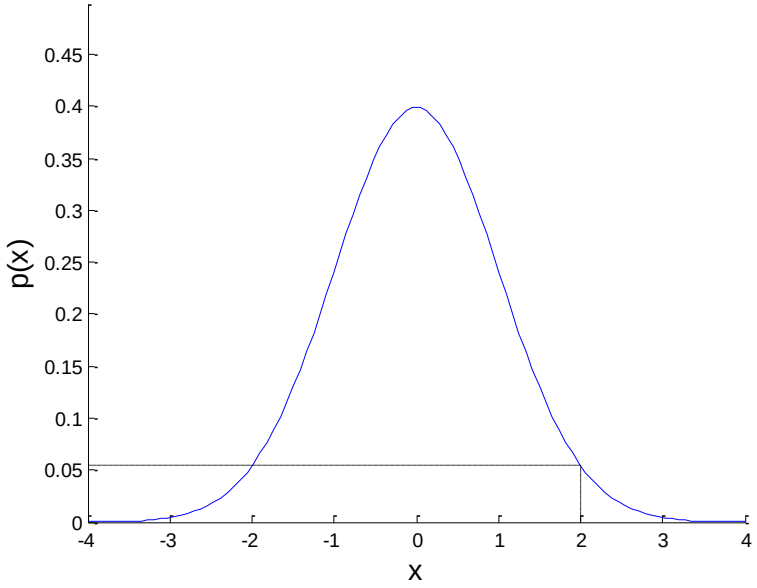
Exercise 1

- What is the likelihood of drawing a 2 from a random normal distribution with a mean of zero and a standard deviation of one?

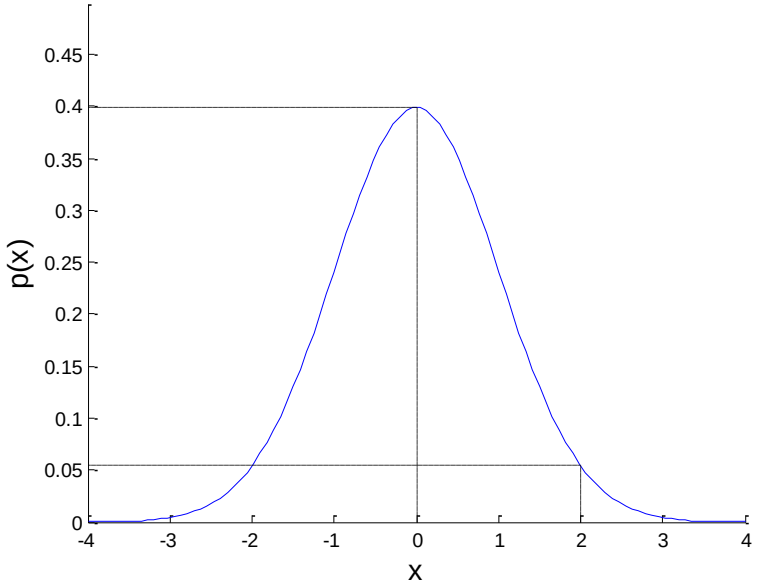


Exercise 1

- $P(x=2|x\sim N(0,1)) = 0.0540$

Exercise 2

- What is the likehood of drawing a 2 and a 0 from a normal distribution with a mean of zero and a standard deviation of one?



Exercise 2

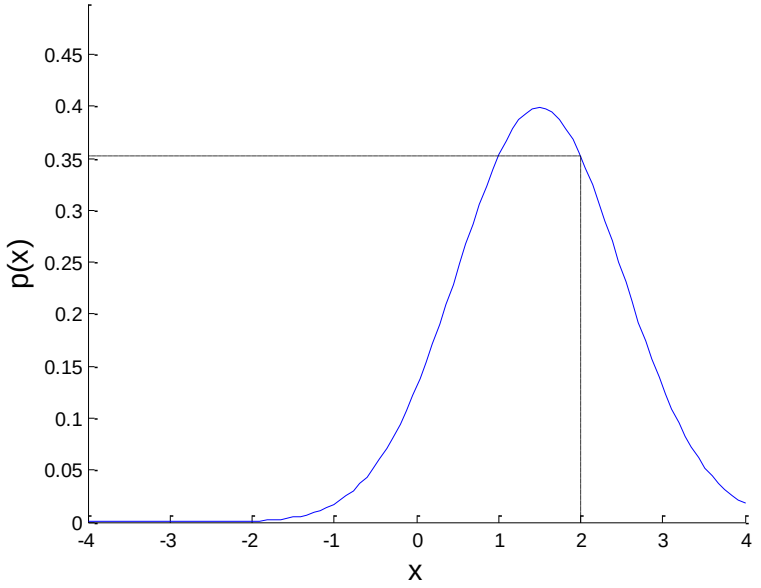
$P(x1=2,x2=0|x\sim N(0,1)) = P(x1=2|x\sim N(0,1))*P(x2=0|x\sim N(0,1))$

$= 0.0540*0.3989$

$= 0.0215$

Exercise 3

- What is the likelihood of drawing a 2 from a normal distribution with a mean of 1.5 and a standard deviation of 1?

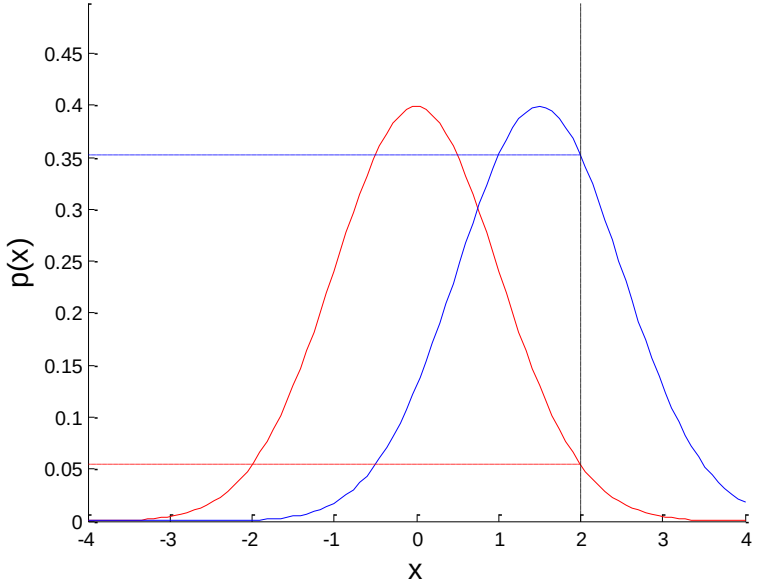


Exercise 3

- $P(x=2|x\sim N(1.5,1)) = 0.3521$

Exercise 4

- I toss a coin, where the probability of heads is 0.5 and the probability of tails is 0.5.
- If the coin comes up heads, I draw a number from a normal distribution with a mean of zero and a standard deviation of 1.
- If the coin comes up tails, I draw a number from a normal distribution with a mean of 1.5 and a standard deviation of 1.
- I toss the coin and, based on the coin toss results, I select the appropriate distribution and draw the number 2.
- What are the odds that I tossed tails relative to the odds that I tossed heads given that I drew a 2?



Exercise 4

$$\frac{P(Tails|x=2)}{P(Heads|x=2)} = \frac{P(x=2|Tails)P(Tails)}{P(x=2|Heads)P(Heads)}$$

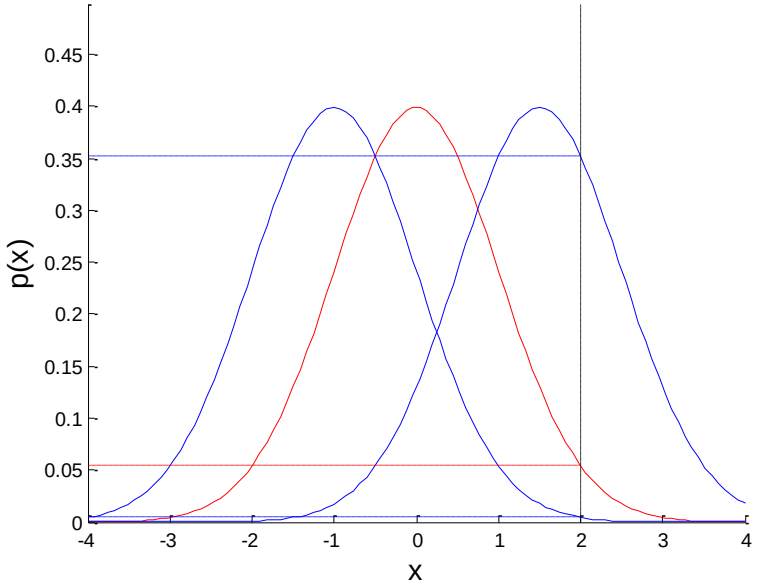
$$= \frac{0.3521 \cdot 0.5}{0.0540 \cdot 0.5}$$

$$= 6.5208$$

The odds are 6.5208:1 that I tossed tails.

Exercise 5

- I toss a coin, where the probability of heads is 0.5 and the probability of tails is 0.5.
- If the coin comes up heads, I draw a number from a normal distribution with a mean of zero and a standard deviation of one.
- If the coin comes up tails, I draw a number from either a normal distribution with a mean of 1.5 and a standard deviation of 1, or from a normal distribution with a mean of -1 and a standard deviation of 1, with equal probability.
- I toss the coin and, based on the coin toss results, I select the appropriate distribution and draw the number 2.
- What are the odds that I tossed tails relative to the odds that I tossed heads given that I drew a 2?



Exercise 5

$$\frac{P(Tails|x=2)}{P(Heads|x=2)} = \frac{P(x=2|Tails)P(Tails)}{P(x=2|Heads)P(Heads)}$$

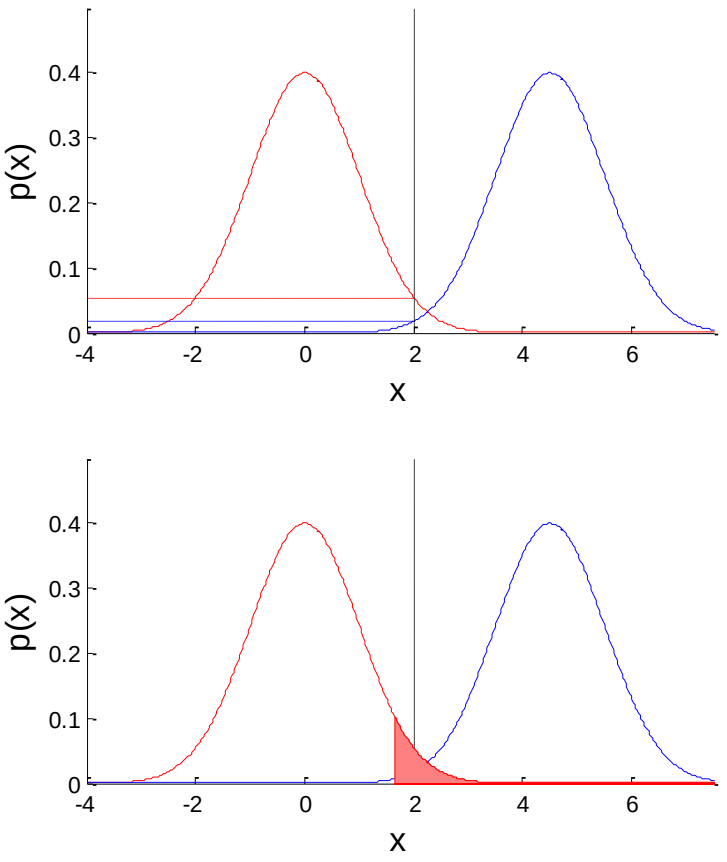
$$= \frac{\left[P\left(x=2|x\sim\aleph(1.5,1)\right)\cdot P\left(x\sim\aleph(1.5,1)\right) + P\left(x=2|x\sim\aleph(-1,1)\right)\cdot P\left(x\sim\aleph(-1,1)\right)\right]P(Tails)}{P\left(x=2|x\sim\aleph(0,1)\right)P(Heads)}$$

$$= \frac{[0.3521\cdot 0.5 + 0.0044\cdot 0.5]\cdot 0.5}{0.0540\cdot 0.5}$$

$$= 3.3015$$

Exercise 6

- I toss a coin, where the probability of heads is 0.5 and the probability of tails is 0.5.
- If the coin comes up heads, I draw a number from a normal distribution with a mean of zero and a standard deviation of one.
- If the coin comes up tails, I draw a number from a normal distribution with a mean of 4.5 and a standard deviation of 1.
- I toss the coin and, based on the coin toss results, I select the appropriate distribution and draw the number 2.
- What are the odds that I tossed tails relative to the odds that I tossed heads given that I drew a 2?
- What is the area under the normal distribution for heads at or beyond a value of 2?
- Is this area less than 0.05?
- Does the fact that the sampled value of 2 lies in a region that is less than 5% of the area under the curve mean that it is more likely to have arisen from the tails distribution?



Exercise 6

$$\frac{P(Tails|x=2)}{P(Heads|x=2)} = \frac{P(x=2|Tails)P(Tails)}{P(x=2|Heads)P(Heads)}$$

$$= \frac{0.0175 \cdot 0.5}{0.0540 \cdot 0.5}$$

$$= 0.3241$$

The odds are 0.3241:1 that I tossed tails.
The area under the heads curve at or beyond 2 is 0.0228.
This area is less than 0.05, but it does not mean that the data are more likely to have arisen from the tails distribution.

Exercise 7

- A newly-wed couple is trying to have a baby. They try really hard on their first night together and, after waiting the appropriate amount of time, they conduct a pregnancy test.
- The test comes out positive. What is the probability of pregnancy?

Exercise 7

Not Pregnant → Pregnancy Test

- 5/100 → Positive
- 95/100 → Negative

Sex

- 199/200 → Not Pregnant
- 1/200 → Pregnant

Pregnant → Pregnancy Test

- 99/100 → Positive
- 1/100 → Negative

Exercise 7

Sex

P(Not Pregnant) = 199/200

Not Pregnant

P(Positive|Not Pregnant) = 5/100

Pregnancy Test

P(Negative|Not Pregnant) = 95/100

Positive

Negative

P(Pregnant) = 1/200

Pregnant

P(Positive|Pregnant) = 99/100

Pregnancy Test

P(Negative|Pregnant) = 1/100

Positive

Negative

Exercise 7: Bayes' Theorem

$$P(pregnant \mid positive) = \frac{P(positive \mid pregnant)P(pregnant)}{P(positive)}$$

$$= \frac{\frac{99}{100} \times \frac{1}{200}}{\frac{99}{100} \times \frac{1}{200} + \frac{5}{100} \times \frac{199}{200}}$$

$$= 0.0905$$

Therefore, given the positive test result, there is a 9.05% chance of pregnancy.