

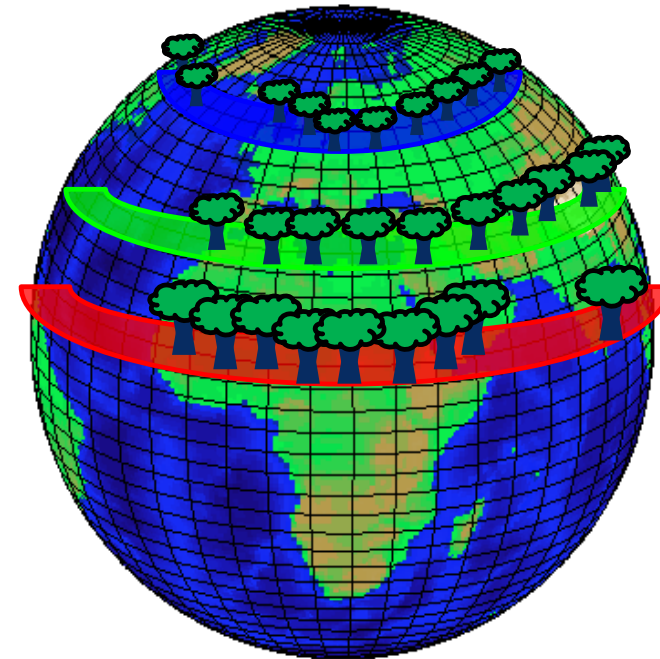
# UNDERSTANDING STATISTICS & EXPERIMENTAL DESIGN

1. Basic Probability Theory
2. Signal Detection Theory (SDT)
3. SDT and Statistics I and II
4. Statistics in a nutshell
5. Multiple Testing
6. ANOVA
7. Experimental Design & Statistics
8. Correlations & PCA
9. Meta-Statistics: Basics
10. Meta-Statistics: Too good to be true
11. Meta-Statistics: How big a problem is publication bias?
12. Meta-Statistics: What do we do now?

# Statistics & Data Reduction

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- Suppose we want to compare tree heights over the three regions shown on the right in the red, green, and blue bands.
- Are the tree heights all the same, or does at least one region contain trees whose heights differ from those in the other regions?



# How to avoid false discovery

During  $m$  independent statistic test with  $\alpha$  significance level, the probability of at least one false discovery should be

$$1 - (1 - \alpha)^m < 0.05$$

$$\alpha = 1 - (1 - 0.05)^{1/m} \approx \frac{0.05}{m}$$

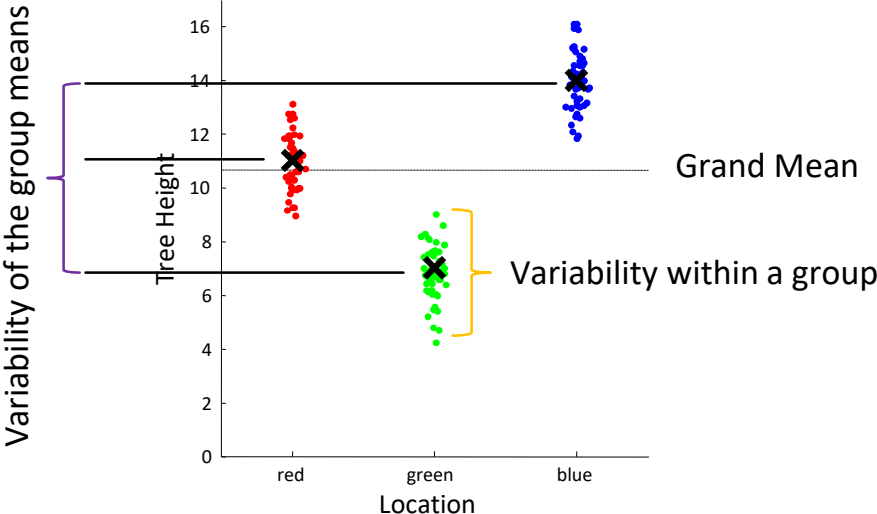
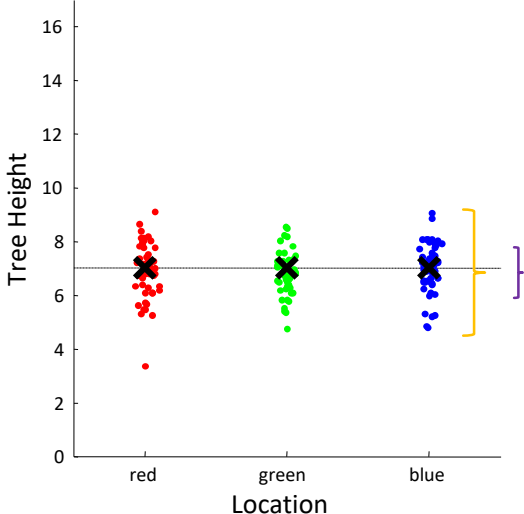
Bonferroni correction: during  $m$  independent statistic tests only those results are significant, for which

$$p < \frac{0.05}{m}$$

$$F = \frac{\text{Variability between treatments}}{\text{Variability within treatments}}$$

$$F = \frac{\text{Small Number}}{\text{Large Number}} = \text{Small Number}$$

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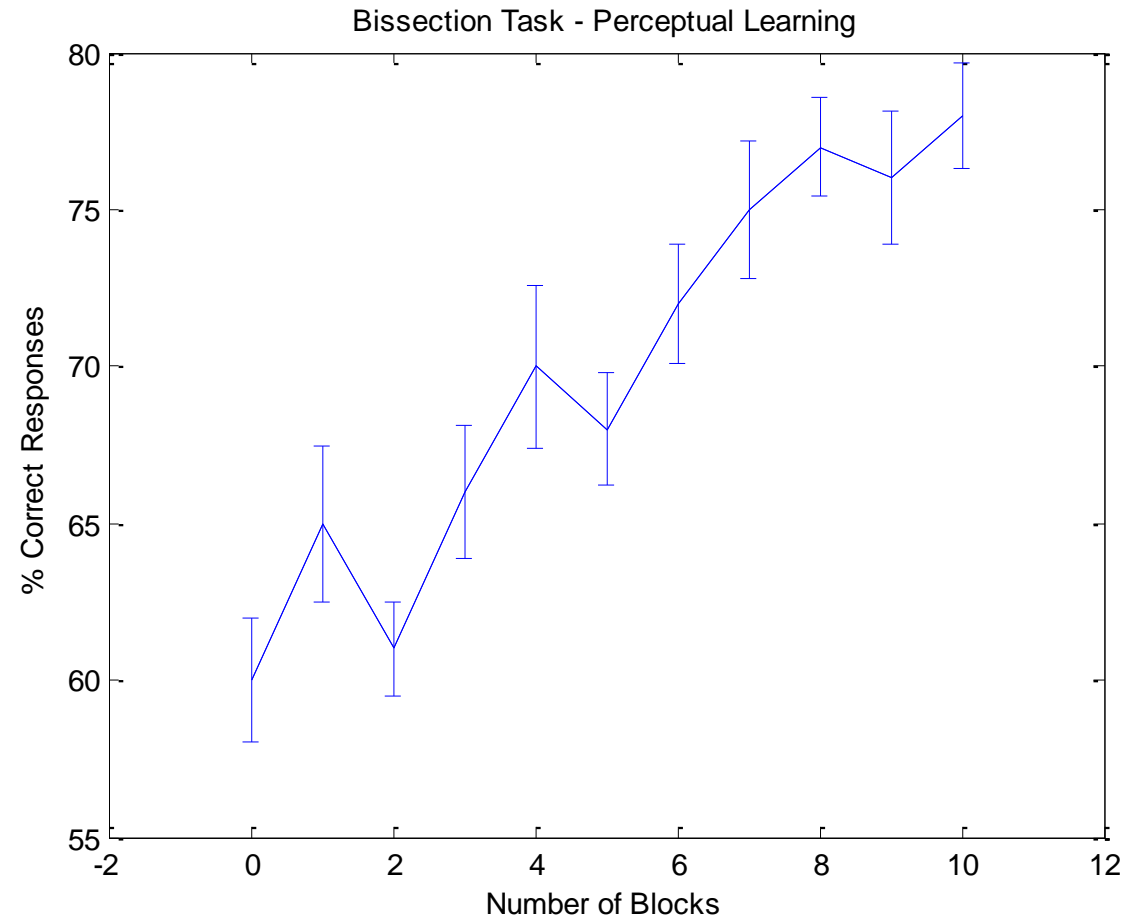


# Reduce data I

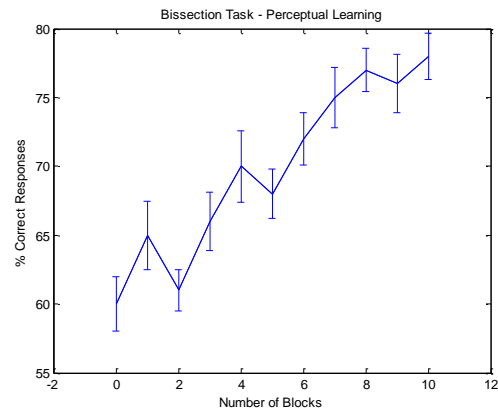
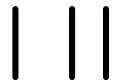
## Model Fitting

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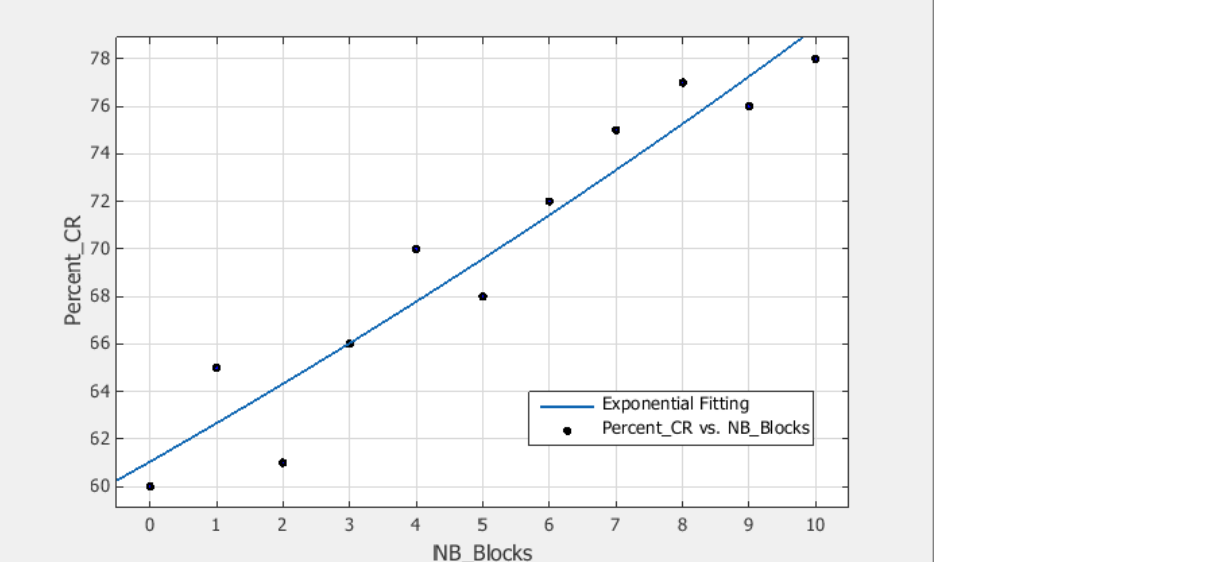
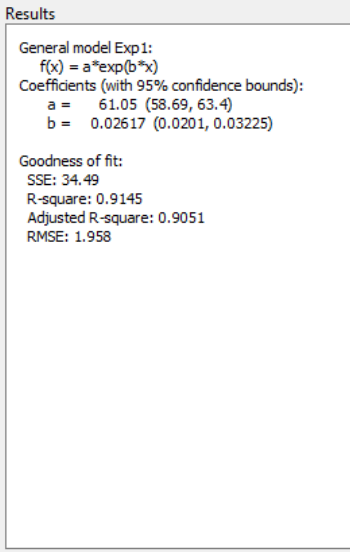
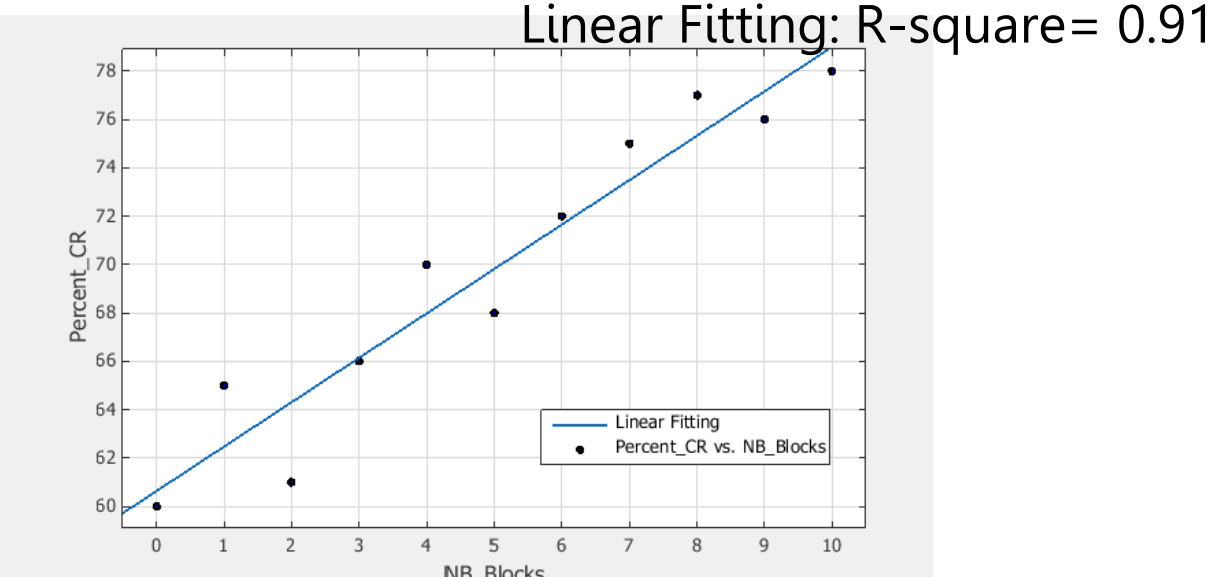
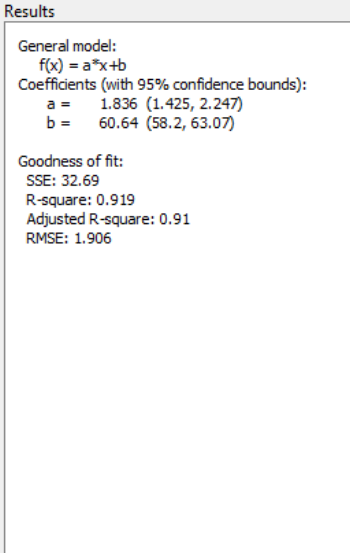
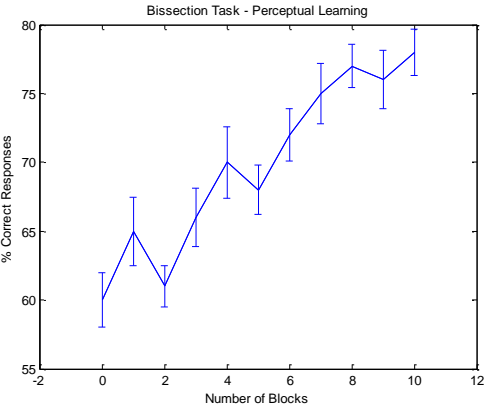
## Bisection Task



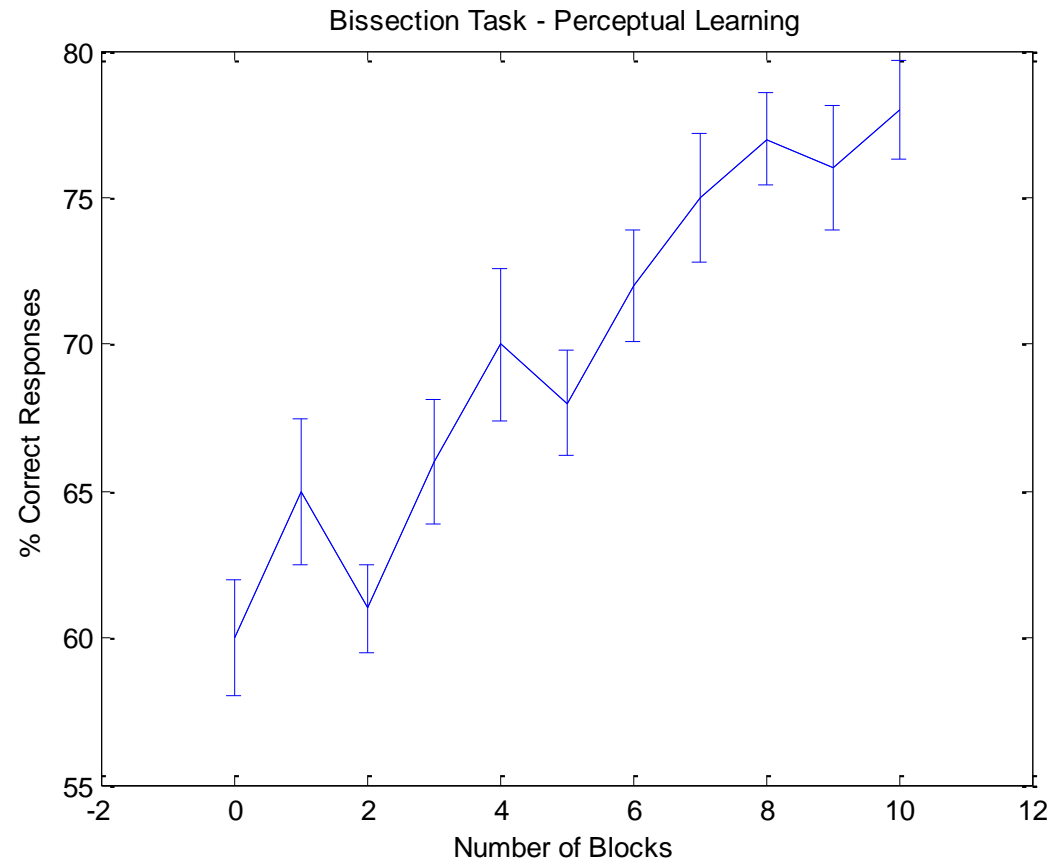
## Bisection Task



## Bisection Task



## Bisection Task



## Two sample $t$ -test

$$t_n = \frac{|\bar{x}_{North} - \bar{x}_{South}|}{\hat{\sigma}/\sqrt{n}}$$

## One sample $t$ -test

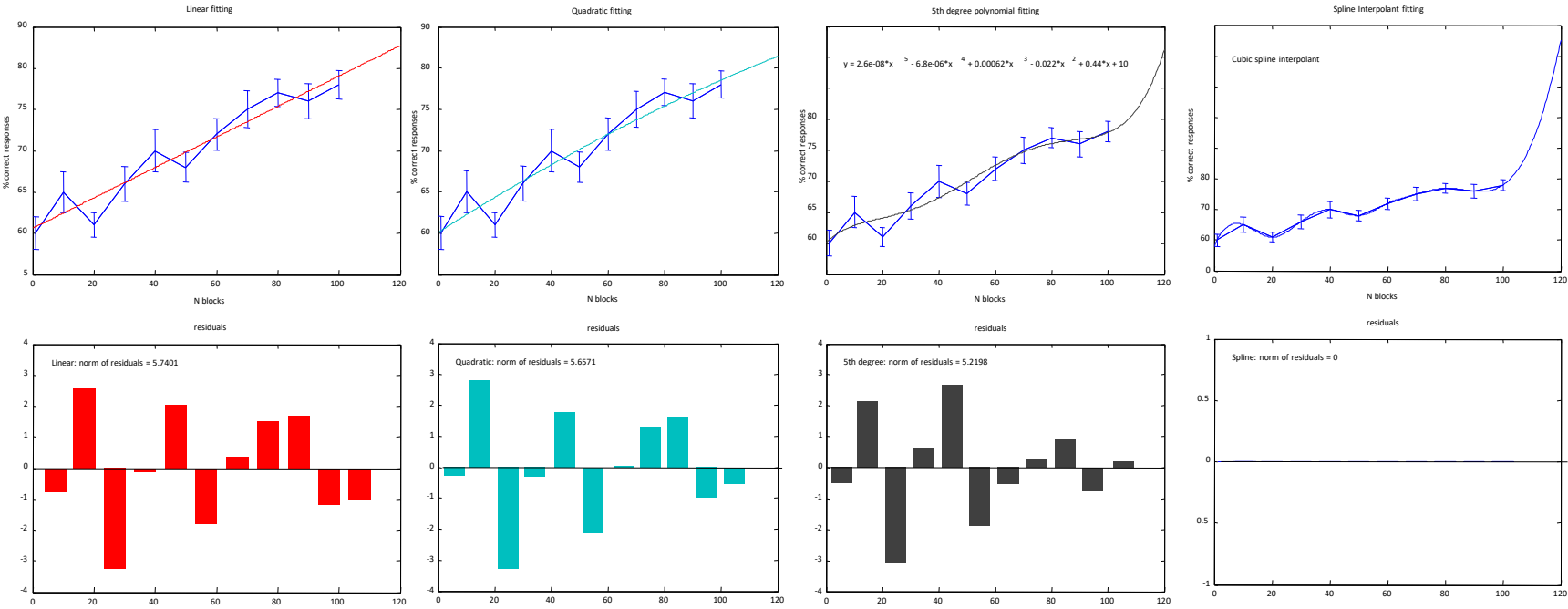
$$t = \frac{|\bar{x} - \mu|}{\hat{\sigma}/\sqrt{n}}$$

## Repeated measures $t$ -test

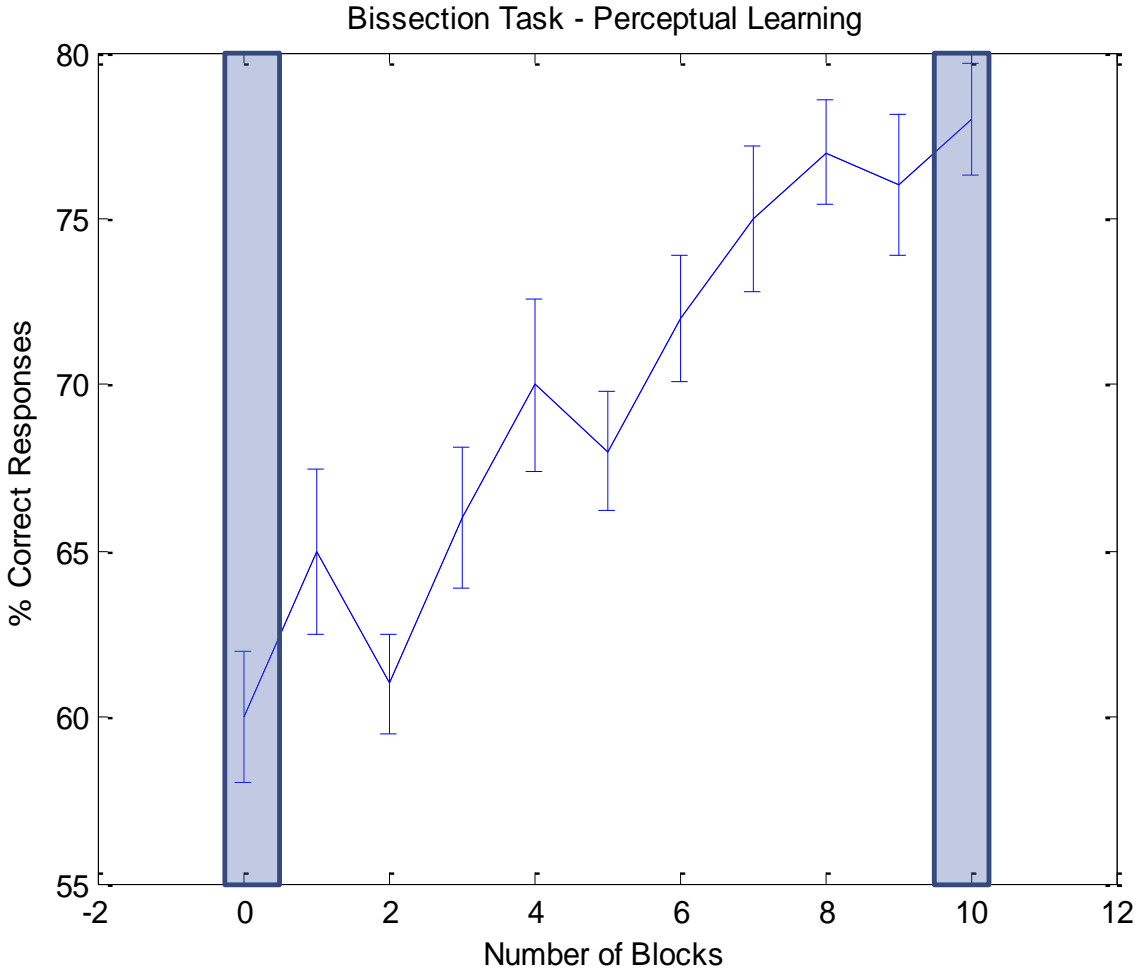
The two sample  $t$ -test is in fact a one-way  $t$ -test with  $\mu = 0$

$$y = 0.18 * x + 11$$

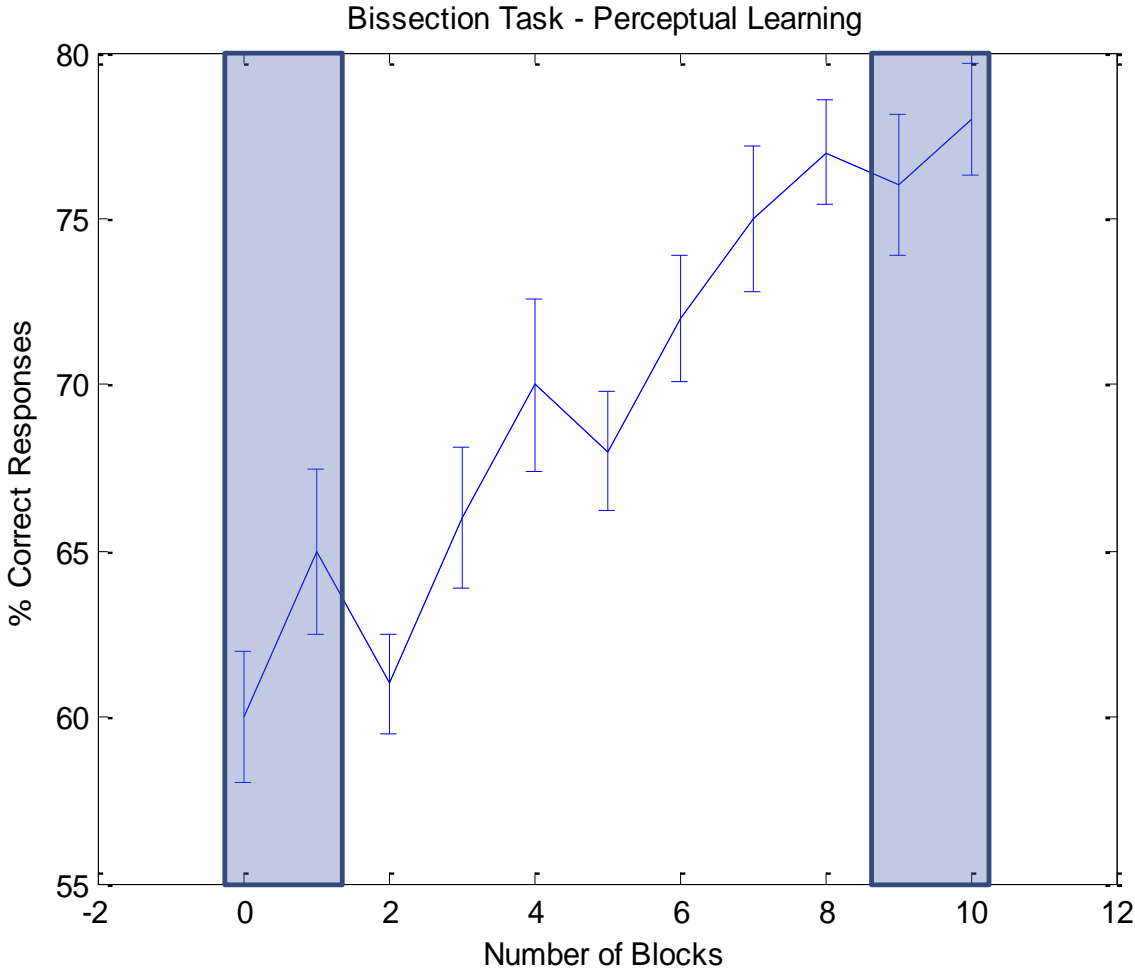
$$y = - 0.00034 * x^2 + 0.22 * x + 10$$



## Bisection Task



## Bisection Task



## Two sample $t$ -test

$$t_n = \frac{|\bar{x}_{North} - \bar{x}_{South}|}{\hat{\sigma}/\sqrt{n}}$$

## One sample $t$ -test

$$t = \frac{|\bar{x} - \mu|}{\hat{\sigma}/\sqrt{n}}$$

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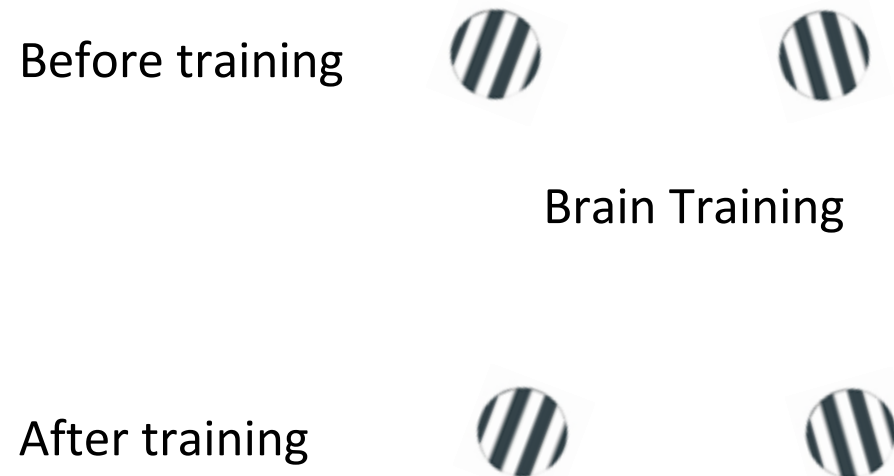
# Reduce data IIa: Method of constant stimuli (MCS)

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A brain enhancing program proposes to increase not only cognition but also perception. We like to test this proposal using an orientation discrimination task before and after training.

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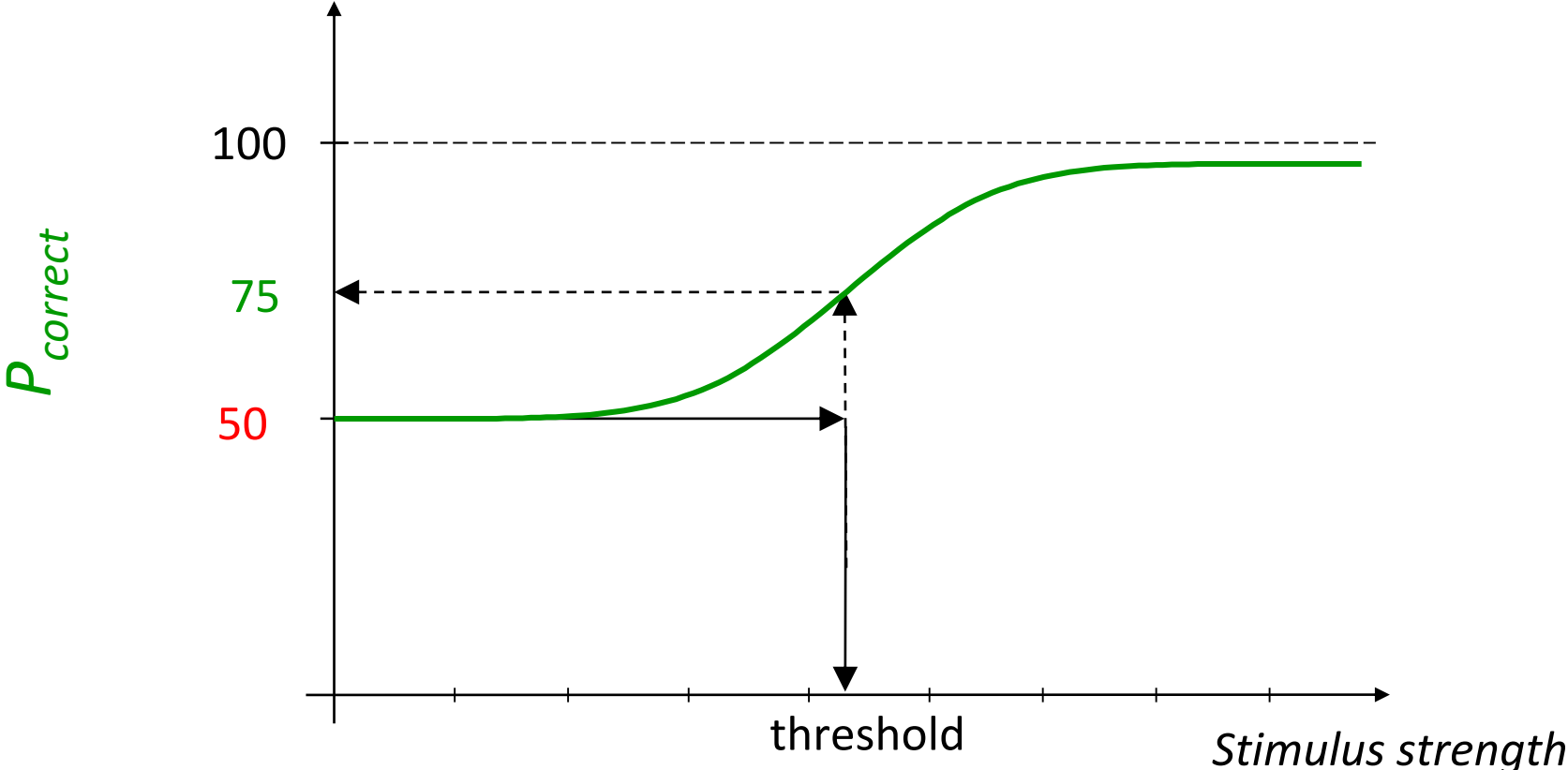
We need to think about floor and ceiling effects and therefore offer a range of stimuli:



Train: Brain enhancing



Compute ANOVA?



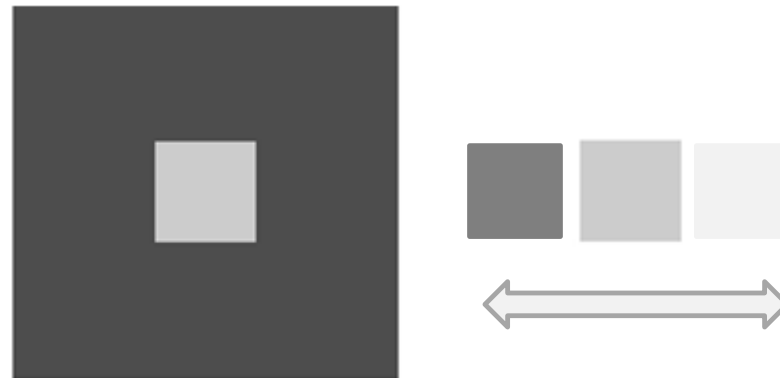
# Reduce data IIc: Adjustment Tasks

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## Orientation adjustment



## Color adjustment



Adapted from: [http://www.frontiersin.org/files/Articles/9032/fpsyg-02-00119-HTML/image\\_m/fpsyg-02-00119-g001.jpg](http://www.frontiersin.org/files/Articles/9032/fpsyg-02-00119-HTML/image_m/fpsyg-02-00119-g001.jpg)

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## Reduce data collection II

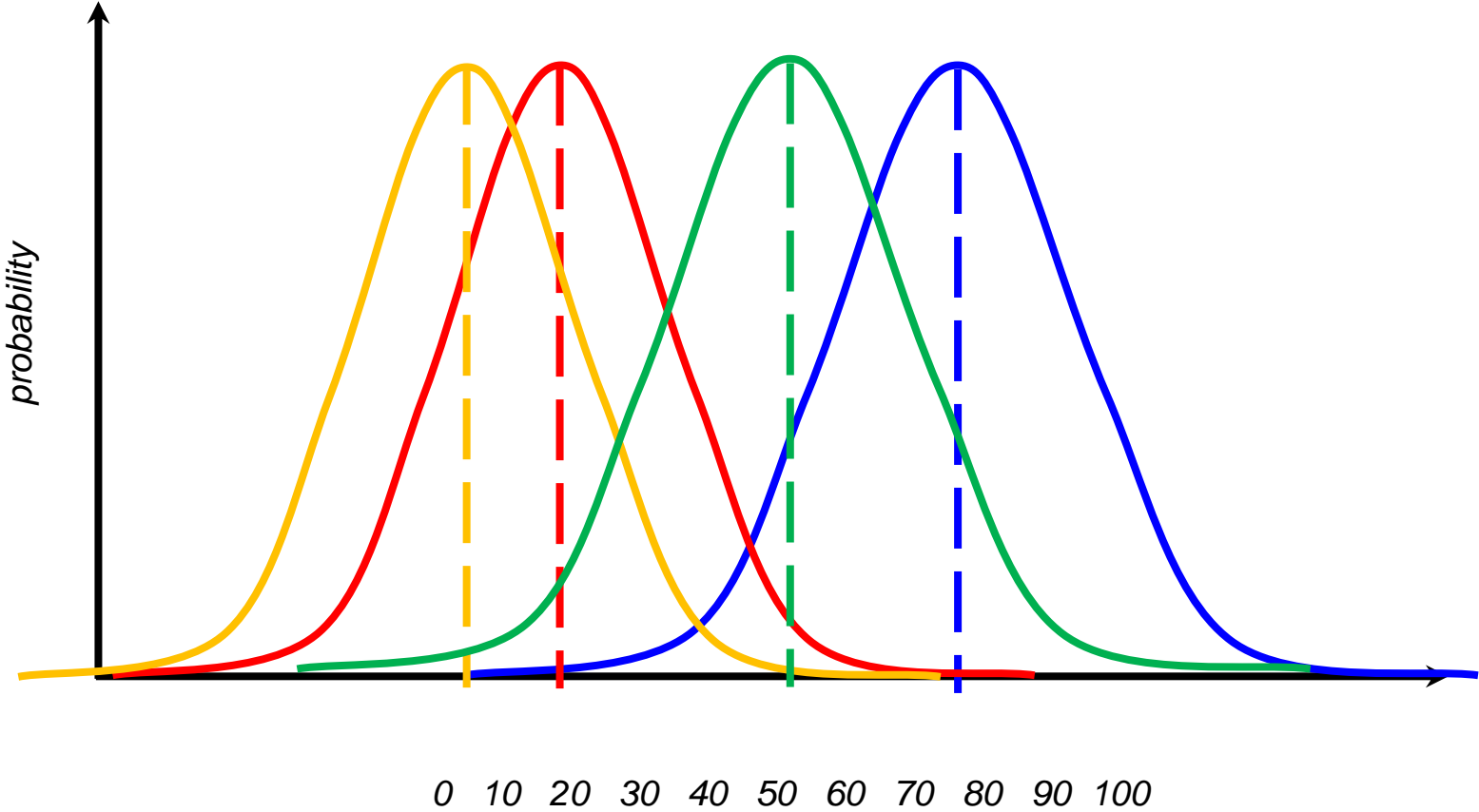
### Adjustment Tasks

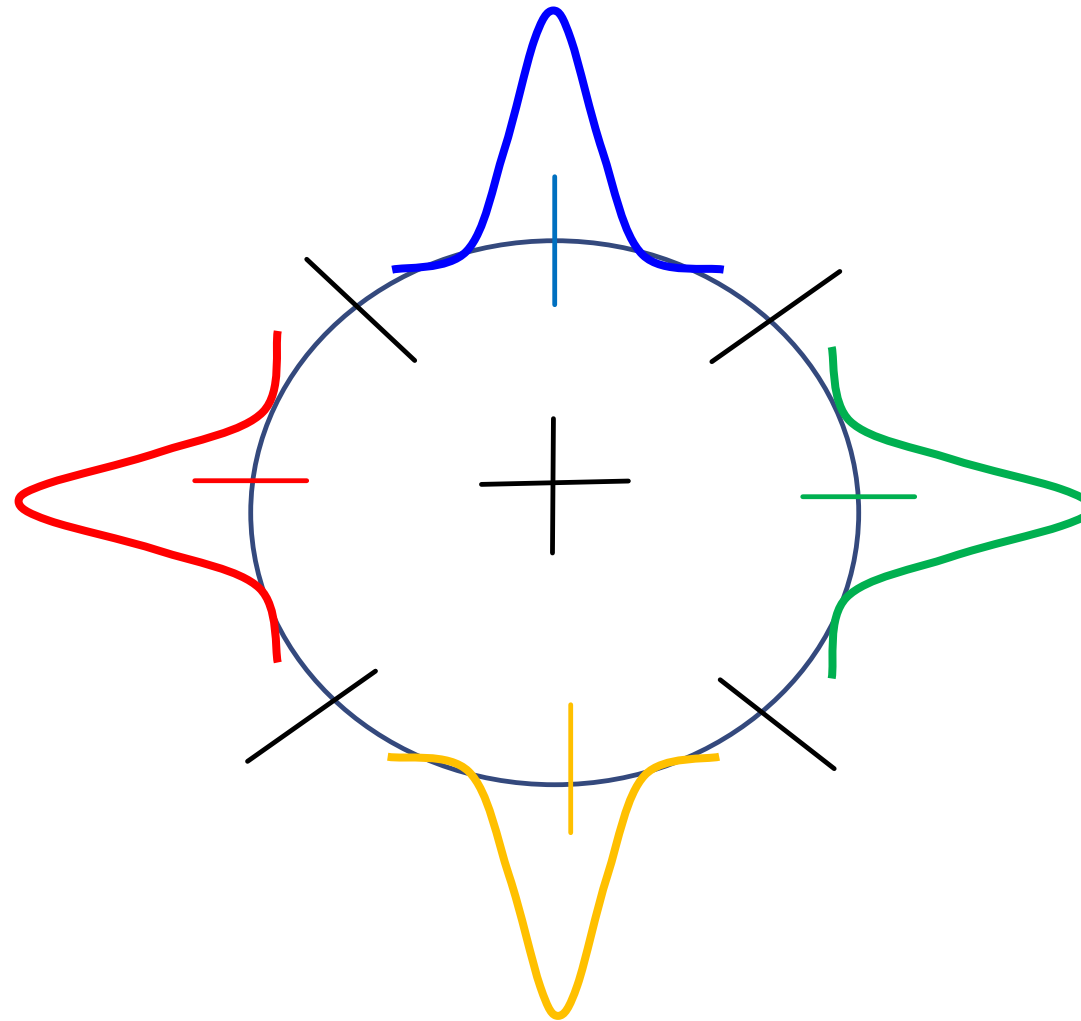
Main Problem: outlier detection

# Why not identification task?

A brain enhancing program proposes to increase not only cognition but also perception. We like to test this proposal using an orientation discrimination task before and after training.







# Reduce data collection III

## Number of trials

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Resources are always limited in science

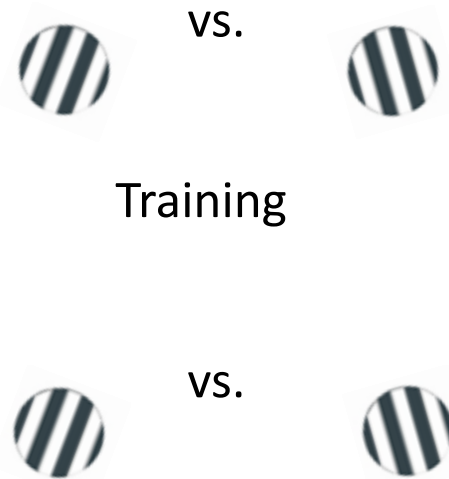
For example, you can screen only

- 1000 cells with an anti-body
- Ask 1000 people in a poll
- Collect 5 stones from moon
- Check 10% of a product for quality control because of budget, time etc. limitations

**Q: How to spend trials best?**

- 
- It is more useful to study one animal for 1000 hours than to study 1000 animals for one hour” — B. F. Skinner (quoted in Kerlinger and Lee, 1999)

A brain enhancing program proposes to increase not only cognition but also perception. We like to test this proposal using an orientation discrimination task, tested before and after training.



We determine performance in terms of correct responses. We can measure only 1.000 trials (line clockwise/counter-clockwise?) because whatever reason.

**Q:** How many participants (N) with how many trials (M)?

$$n = N \times M = 1.000$$

Suppose you have  $N$  subjects who each run  $m$  trials. The score for the  $i$ th trial for the  $j$ th subject is defined by

$$X_{ij} = \mu + \alpha_j + e_{ij} \quad (1)$$

where  $\mu$  is the overall grand population mean across subjects,  $\alpha_j$  is the deviation from the grand mean for subject  $j$ , and  $e_{ij}$  is the noise for a particular trial for a particular subject.

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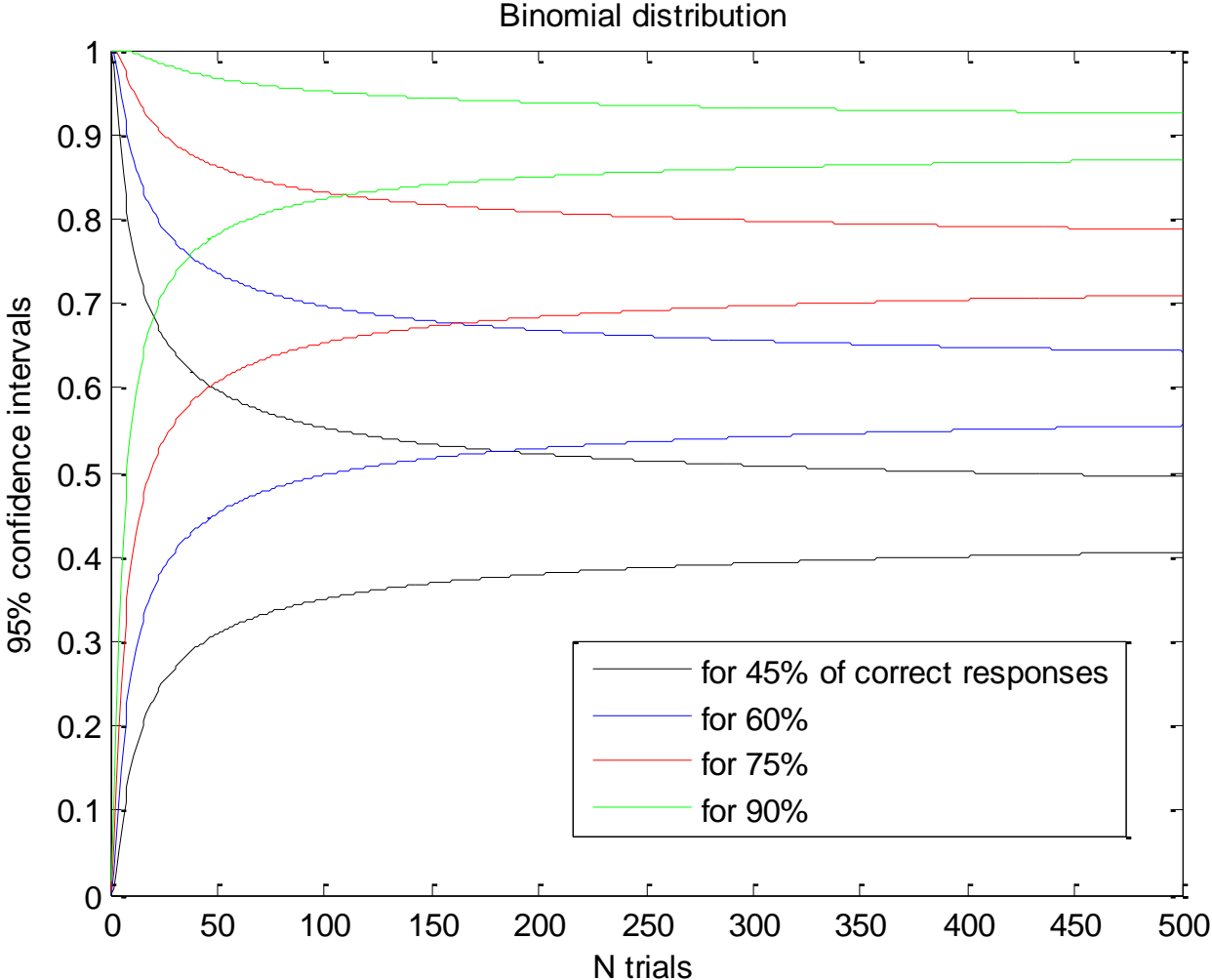
where  $\mu$  is the overall grand population mean across subjects,  $\alpha_j$  is the deviation from the grand mean for subject  $j$ , and  $e_{ij}$  is the noise for a particular trial for a particular subject.

$$\sigma_{\bar{X}} = \sqrt{\frac{\sigma^2}{N}} = \sqrt{\frac{\sigma_{\alpha}^2}{N} + \frac{\sigma_{\epsilon}^2}{MN}}$$

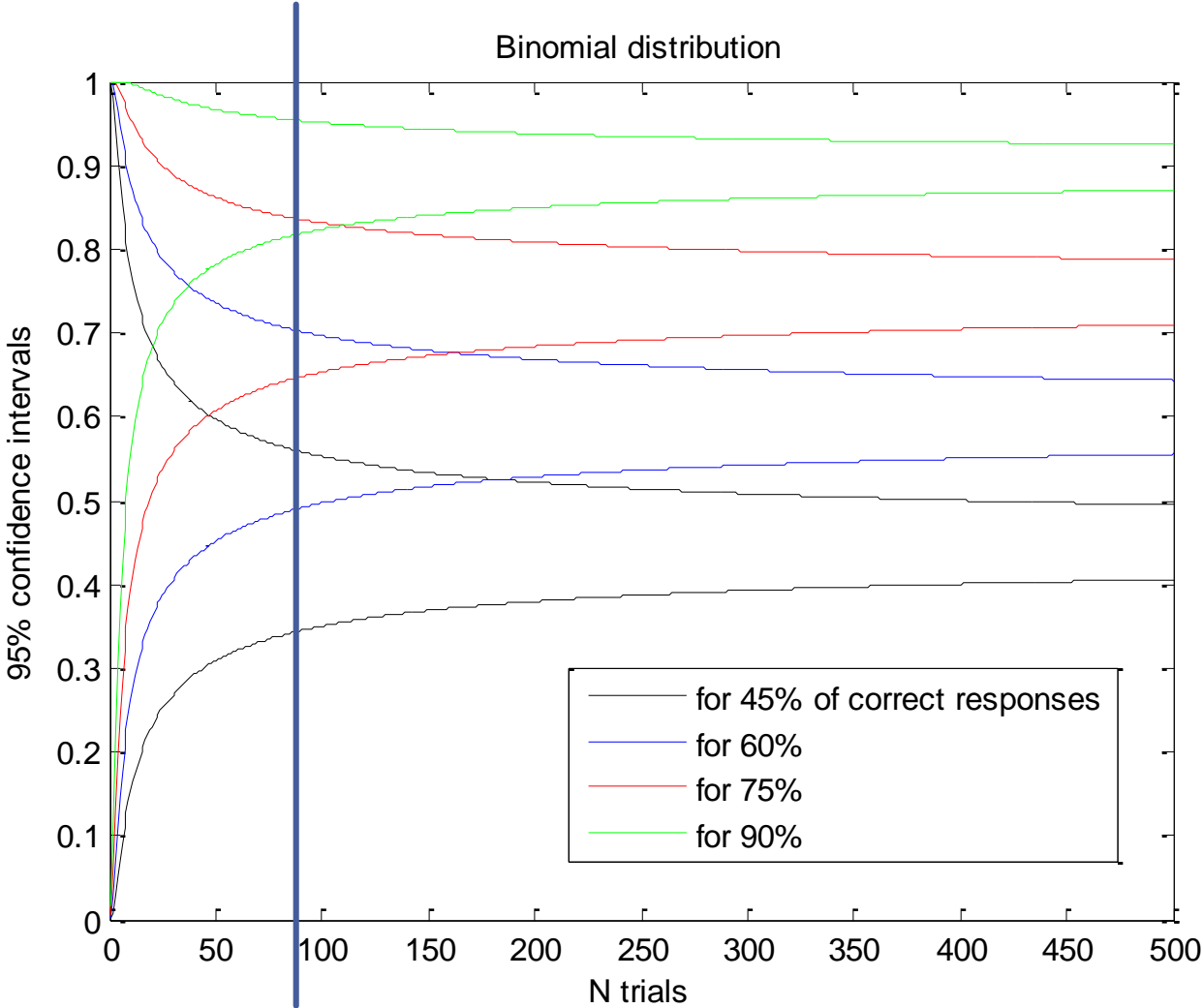
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$\sigma_\epsilon$	$N$	$M$	Estimated Power
0.25	100	1	1
0.25	50	2	0.93
0.25	20	5	0.57
0.25	10	10	0.29
0.25	2	50	0.06

$\sigma_\epsilon$	$N$	$M$	Estimated Power					
0.25	100	1	1					
0.25	50	2	0.93	2	100	1	0.61	
0.25	20	5	0.57	2	50	2	0.52	
0.25	10	10	0.29	2	20	5	0.36	
0.25	2	50	0.06	2	10	10	0.23	
0.5	100	1	0.99	2	2	50	0.06	
0.5	50	2	0.91	4	100	1	0.23	
0.5	20	5	0.56	4	50	2	0.21	
0.5	10	10	0.29	4	20	5	0.18	
0.5	2	50	0.06	4	10	10	0.15	
1	100	1	0.94	4	2	50	0.06	
1	50	2	0.81					
1	20	5	0.49					
1	10	10	0.27					
1	2	50	0.06					



# Percent correct & Binomial Distribution



## Take Home Messages

1. Keep your design simple: consider compressing raw data into intermediate variables, which then are subjected to statistical analysis.
2. Compute a power analysis before you do your experiment to check whether there is a real chance that it may show an existing effect.
3. Keep your design simple: if a theory presupposes both significant and null results your power may be strongly reduced.

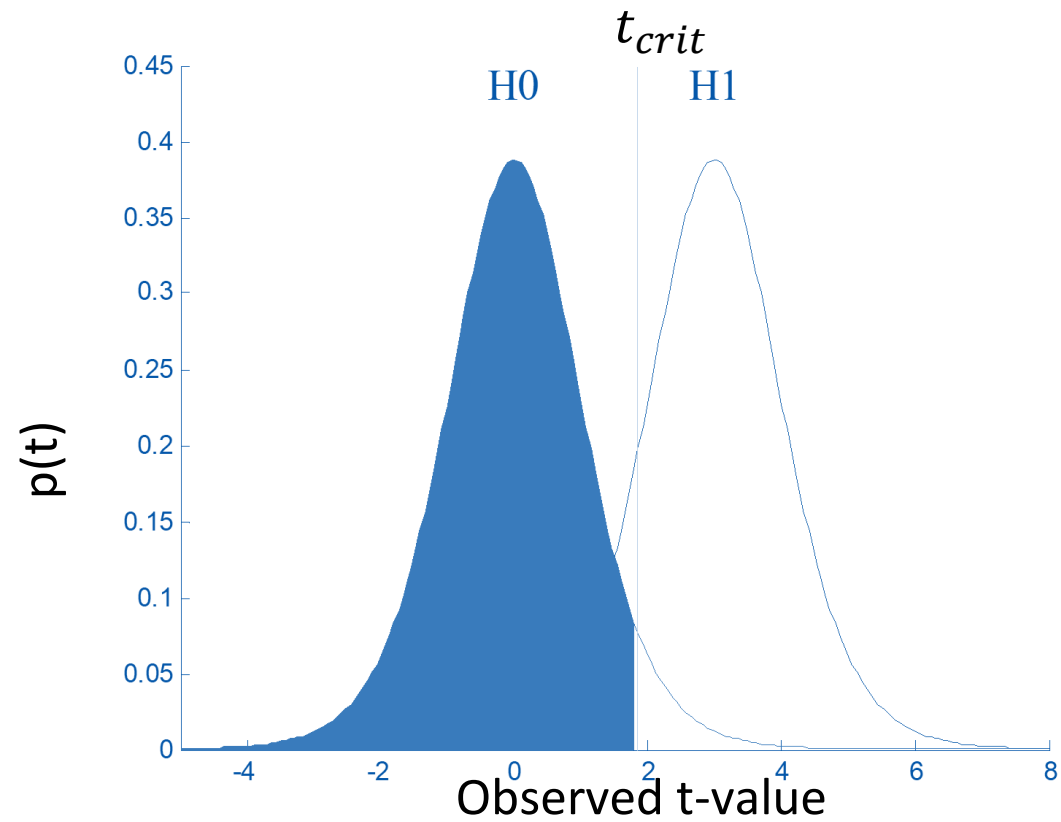
# END Class 7

# Power Analysis

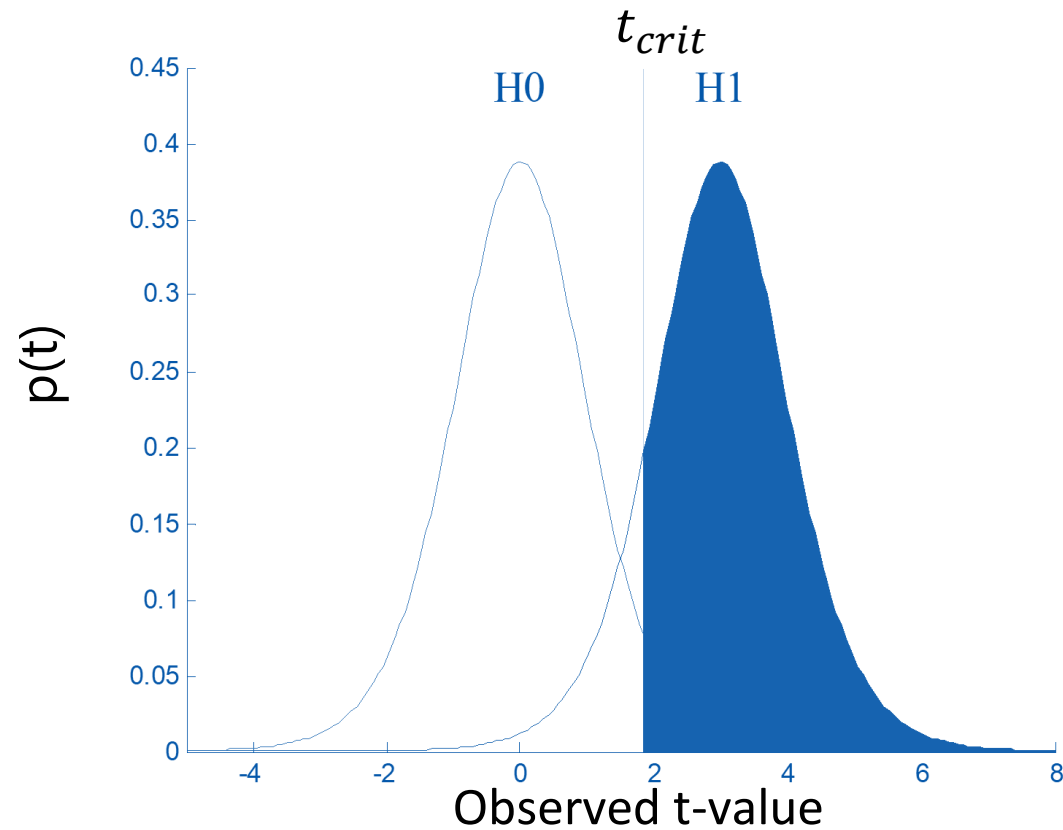
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	$H_0$ is false	$H_0$ is true
Decide there is a significant difference	Hit	False Alarm (Type I error)
Do not decide there is a significant difference	Miss (Type II error)	Correct Rejection

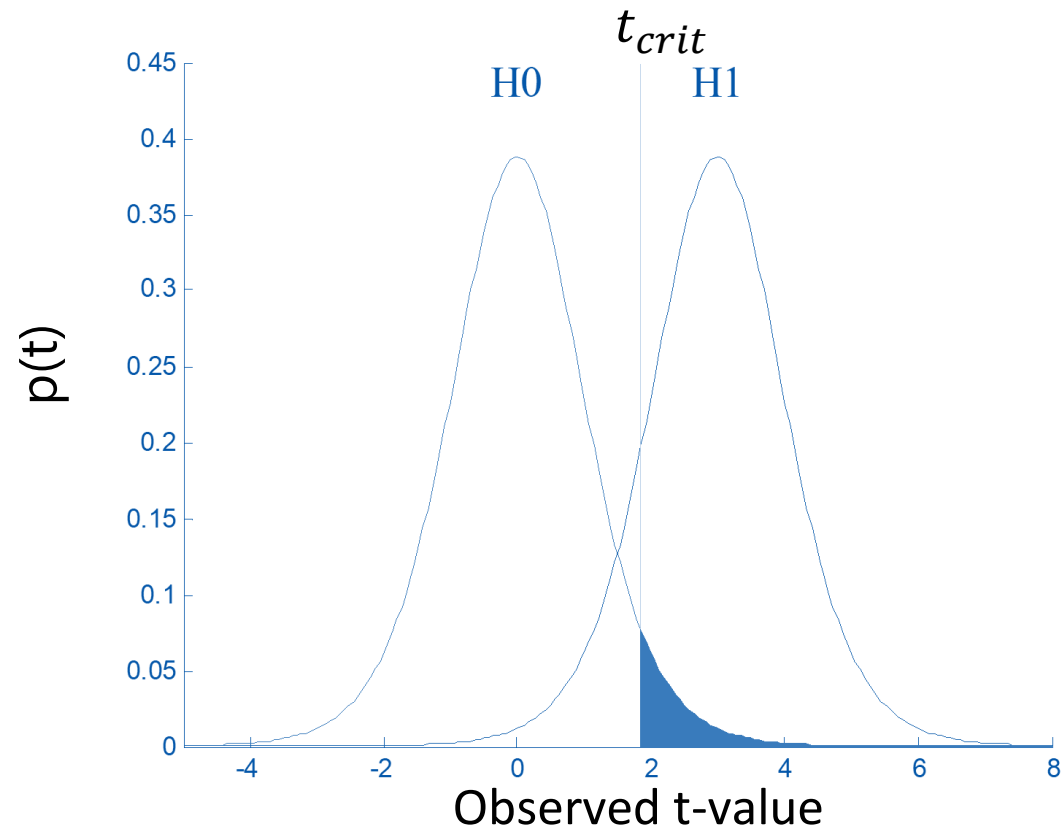
	$H_0$ is false	$H_0$ is true
Decide there is a significant difference	Hit <b>Power</b>	False Alarm (Type I error)
Do not decide there is a significant difference	Miss (Type II error)	Correct Rejection



	H0 True	H1 True
We Say H0 True	$1-\alpha$	$\beta$
We Say H1 True	$\alpha$	Power ( $1-\beta$ )



	H0 True	H1 True
We Say H0 True	$1-\alpha$	B
We Say H1 True	$\alpha$	<b>Power (<math>1-\beta</math>)</b>



	H0 True	H1 True
We Say H0 True	$1-\alpha$	B
We Say H1 True	$\alpha$	Power ( $1-\beta$ )