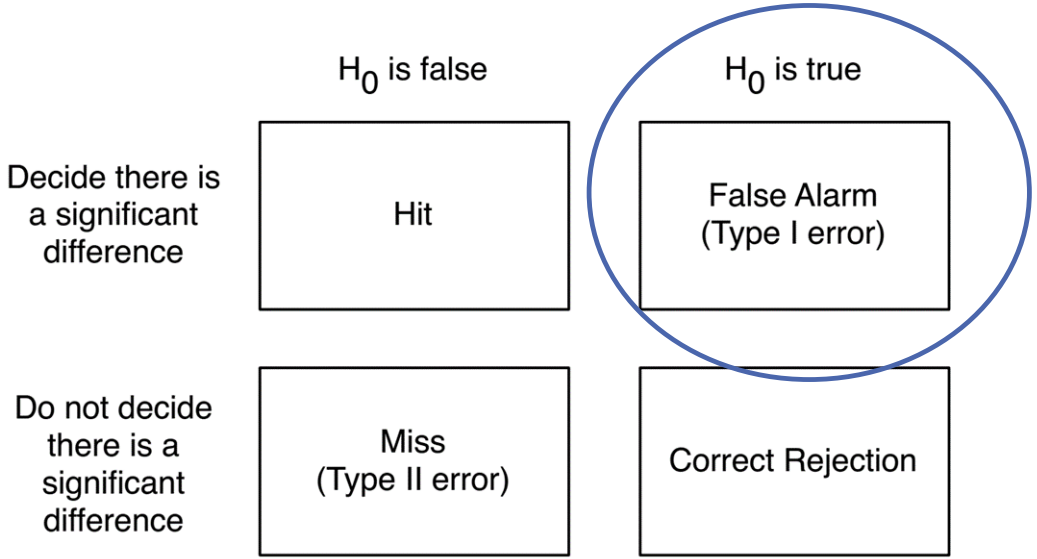
UNDERSTANDING STATISTICS & EXPERIMENTAL DESIGN

Content

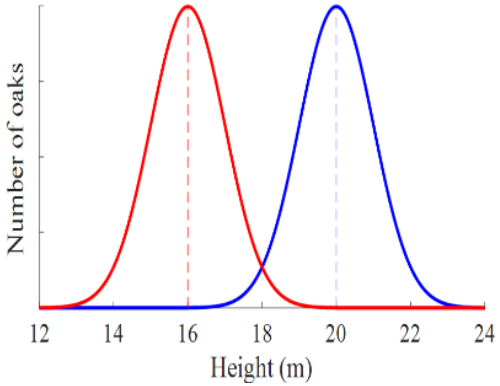
Power & Power analysis (book ch.7)

Undersampling Causes Errors



Summary of the transformations

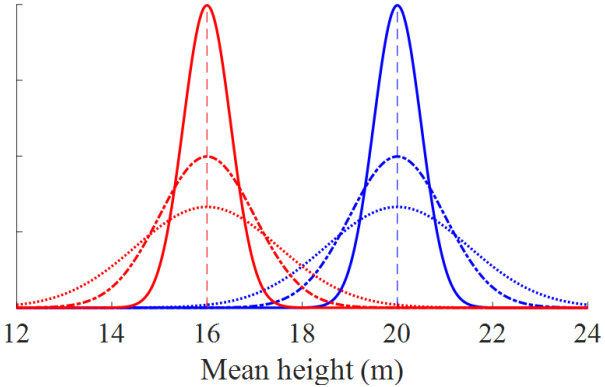
Population distributions



$$t = \frac{(\bar{x}_{North} - \bar{x}_{South})}{\frac{s}{\sqrt{n/2}}}$$

Sampling distributions

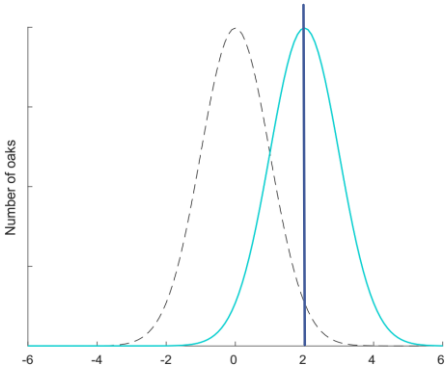
$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$



$$t = d * \sqrt{n/2}$$

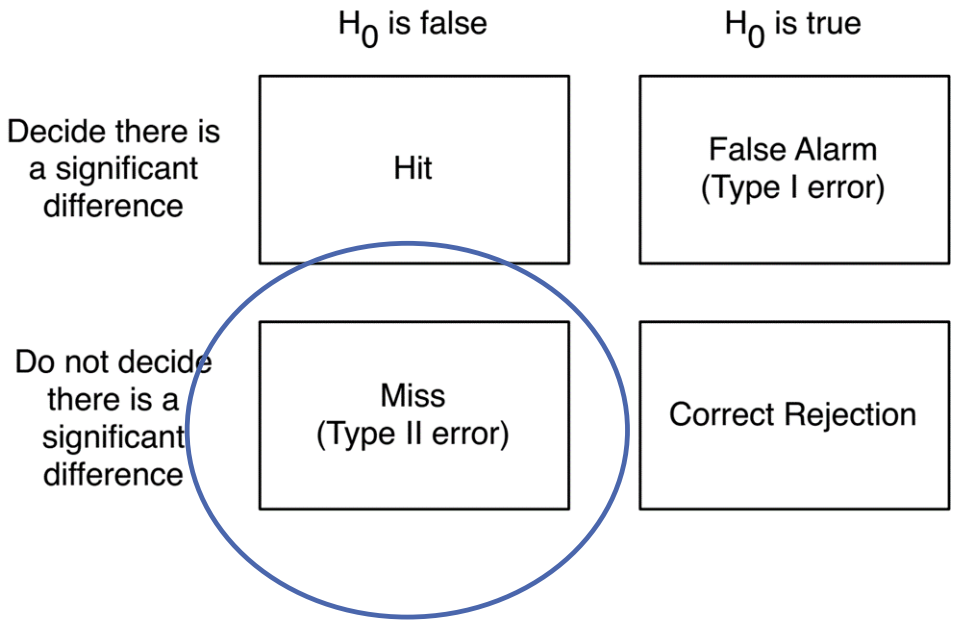
Null + Difference distribution

$$s_{\bar{x}_{North} - \bar{x}_{South}} = s\sqrt{\frac{2}{n}}$$

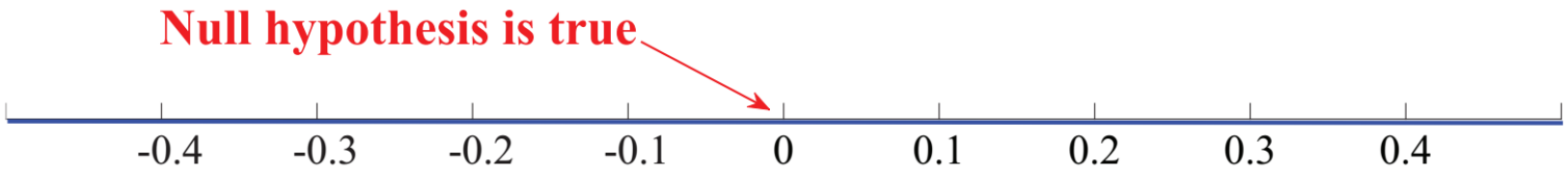


$$p = (data|H_0)$$

Undersampling Causes Errors



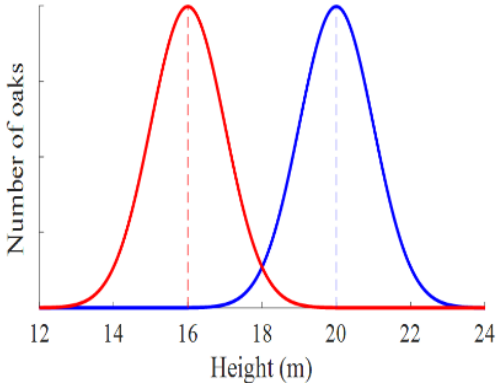
Implication IId: The Null and the Alternative hypothesis



$$t = \frac{\bar{x}_1 - \bar{x}_2}{\hat{\sigma}} * \sqrt{n}$$

Summary of the transformations

Population distributions



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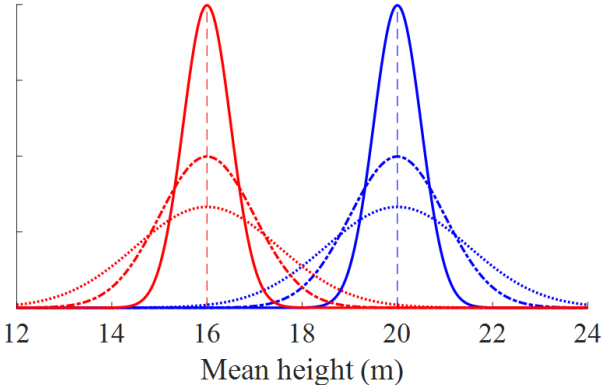

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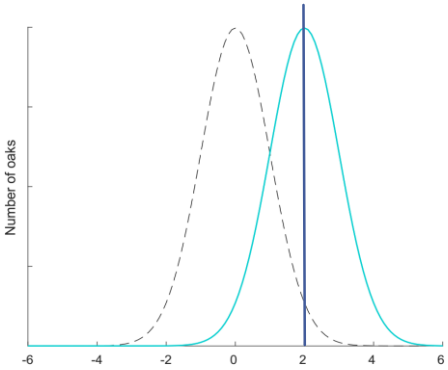


$$p = (data|H_0)$$

Calculating statistical power depends on

1. The sample size
2. The level of statistical significance required
3. The minimum size of effect that it is reasonable to expect
4. The test you are using

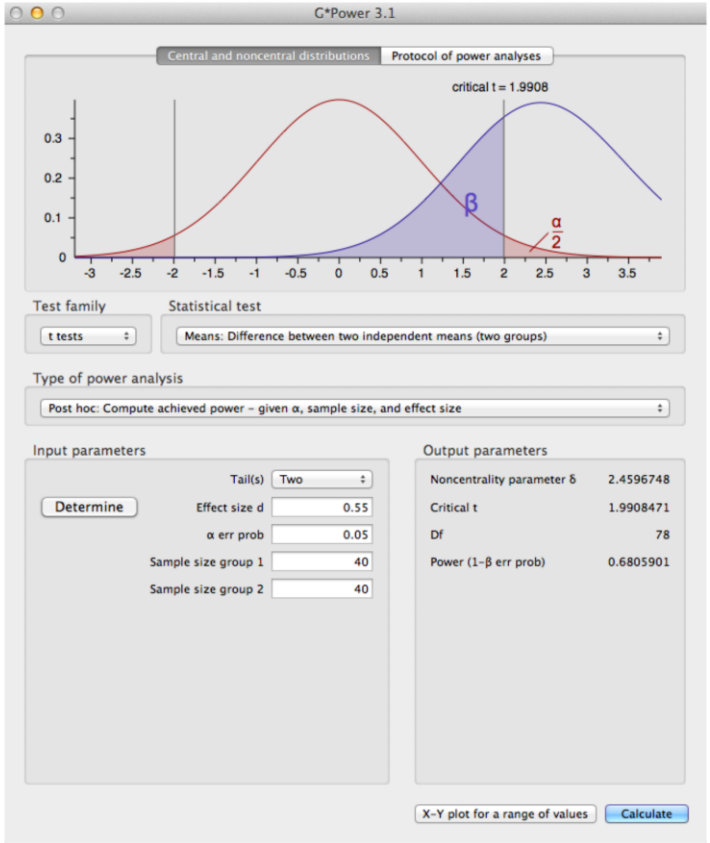
Conventions And Decisions About Statistical Power

Acceptable risk of a Type II error is often set at 1 in 5, i.e., a probability of 0.2 (β).

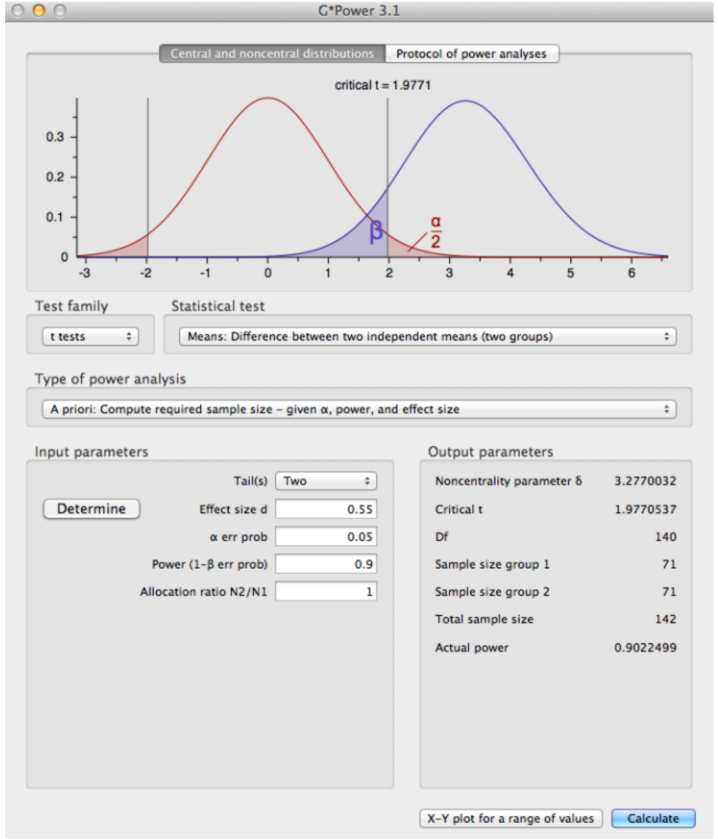
The conventionally uncontroversial value for “adequate” statistical power is therefore set at 1 - 0.2 = 0.8.

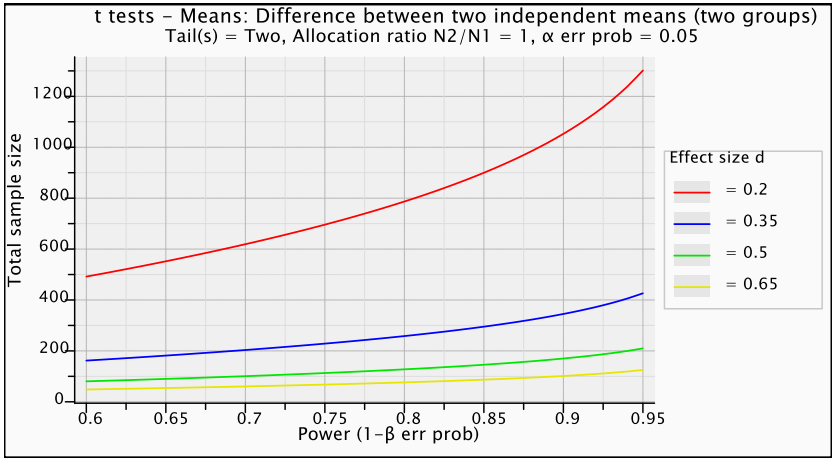
A *retrospective power analysis* is used in order to know whether the studies you are interpreting were well enough designed.

If you fail to reject the null hypothesis you might want to know what chance you had of finding a significant result – defending the failure

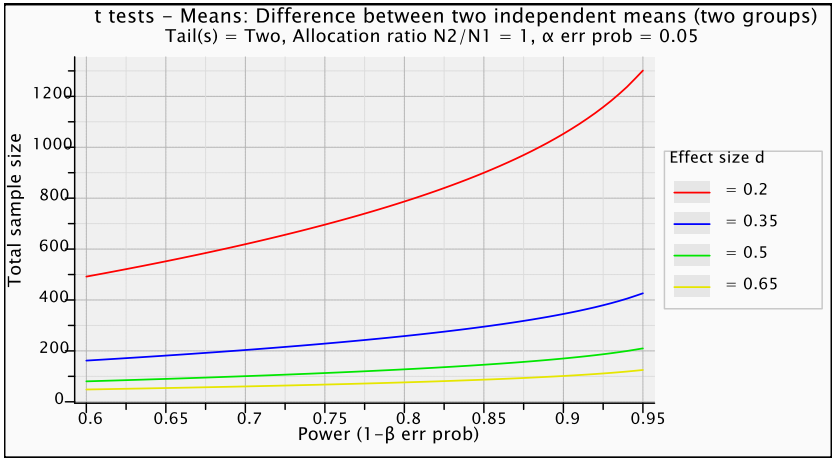

G*Power 3.1
Central and noncentral distributions
Protocol of power analyses
critical t = 1.9908
0.3
0.2
0.1
0
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
β
α/2
Test family
t tests
Statistical test
Means: Difference between two independent means (two groups)
Type of power analysis
Post hoc: Compute achieved power – given α, sample size, and effect size
Input parameters
Tail(s) Two
Determine
Effect size d 0.55
α err prob 0.05
Sample size group 1 40
Sample size group 2 40
Output parameters
Noncentrality parameter δ 2.4596748
Critical t 1.9908471
Df 78
Power (1–β err prob) 0.6805901
X-Y plot for a range of values
Calculate

A *prospective power analysis* is used before collecting data, to consider *design sensitivity*


G*Power 3.1
Central and noncentral distributions
Protocol of power analyses
critical t = 1.9771
0.3
0.2
0.1
0
-3
-2
-1
0
1
2
3
4
5
6
β
α/2
Test family
t tests
Statistical test
Means: Difference between two independent means (two groups)
Type of power analysis
A priori: Compute required sample size – given α, power, and effect size
Input parameters
Tail(s)
Two
Determine
Effect size d
0.55
α err prob
0.05
Power (1–β err prob)
0.9
Allocation ratio N2/N1
1
Output parameters
Noncentrality parameter δ
3.2770032
Critical t
1.9770537
Df
140
Sample size group 1
71
Sample size group 2
71
Total sample size
142
Actual power
0.9022499
X-Y plot for a range of values
Calculate


t tests – Means: Difference between two independent means (two groups)
Tail(s) = Two, Allocation ratio N2/N1 = 1, α err prob = 0.05
Total sample size
1200
1000
800
600
400
200
0
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
Power (1–β err prob)
Effect size d
= 0.2
= 0.35
= 0.5
= 0.65

Why are there so many significant results with so low power, both low n and d'?



The classic approach: terminology & metrics

Some words about Metrics

- A **metric** on a set X is a function (called the distance function or simply **distance**)
- $d : X \times X \rightarrow R+$ (where R+ is the set of non-negative real numbers). For all x, y, z in X, this function is required to satisfy the following conditions:

- $d(x, y) \geq 0$ (non-negativity)

- $d(x, y) = 0$ if and only if $x = y$ (identity of indiscernibles. Note that condition 1 and 2 together produce positive definiteness)

- $d(x, y) = d(y, x)$ (symmetry)

- $d(x, z) \leq d(x, y) + d(y, z)$ (subadditivity / triangle inequality).

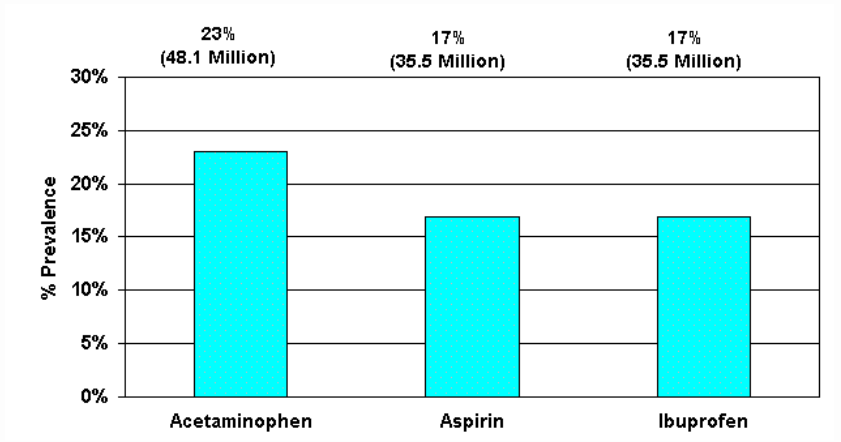
Some words about Metrics

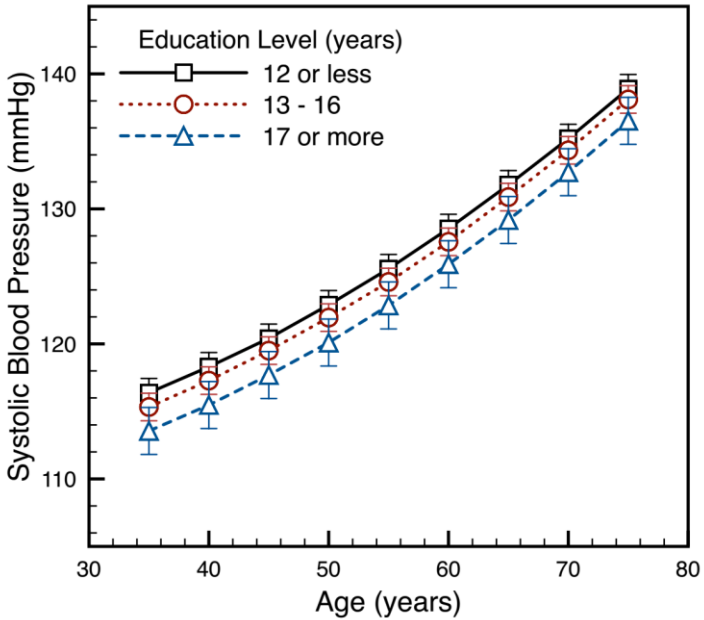
Metrics: Scaling

- Nominal: no order (countries, genes, therapies)
- Ordinal: rank order (military ranks)
- Interval: addition makes sense (no 0), ratios make no sense (eg temperature)
- Ratio: addition & division make sense (with 0, eg body weight)

Some words about Metrics

Metrics: Scaling





Some words about Metrics

Design

- Experimental design: split up observers randomly into two groups
- Cohorts: groups are defined by “natural” labels, e.g. patients vs. controls

The classic approach: Null hypothesis testing

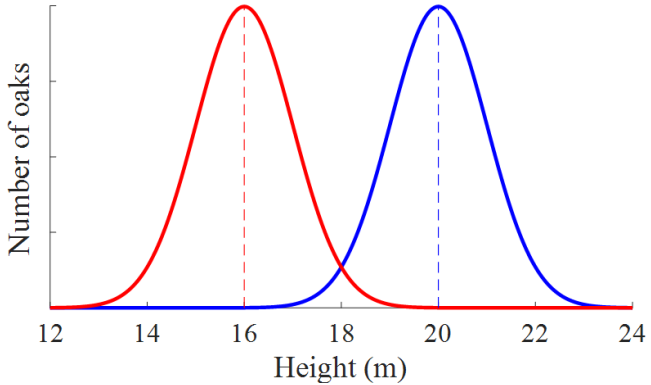
The t-test

The Null Hypothesis Significance Testing (NHST) approach

- Formulate H_1(alternative-hypothesis, the one matching your strategy)
- Choose a statistical model, that represents the alternative hypothesis
- Formulate H_0 (null-hypothesis)
- Assume that H_0 is true (normally H_0 is that there is no effect)
- Compute *p* value
- If $p<0.05$, reject H_0 and accept H_1

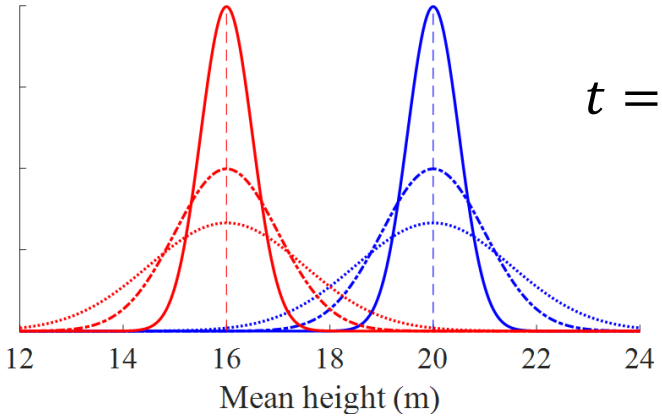
The t-test

Population distribution



Family (n)

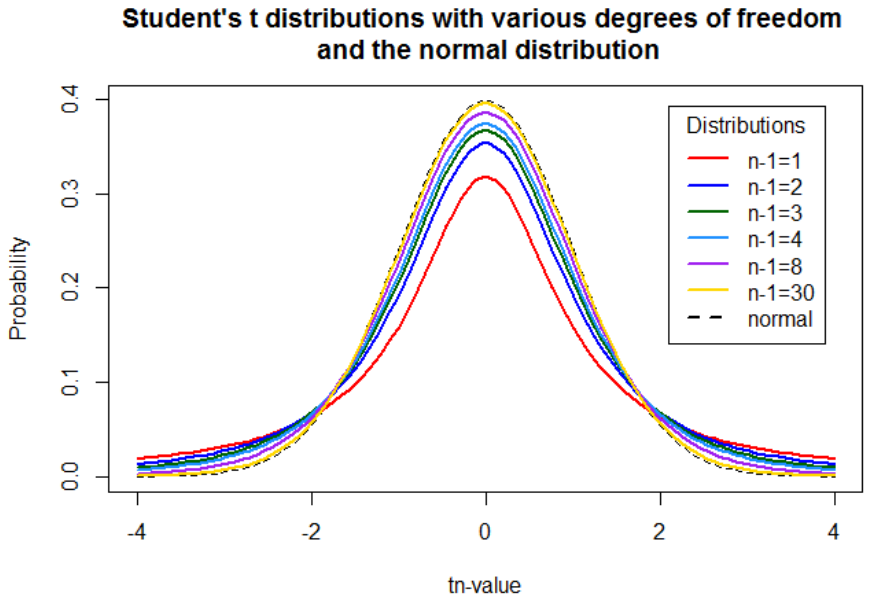
Sampling distributions

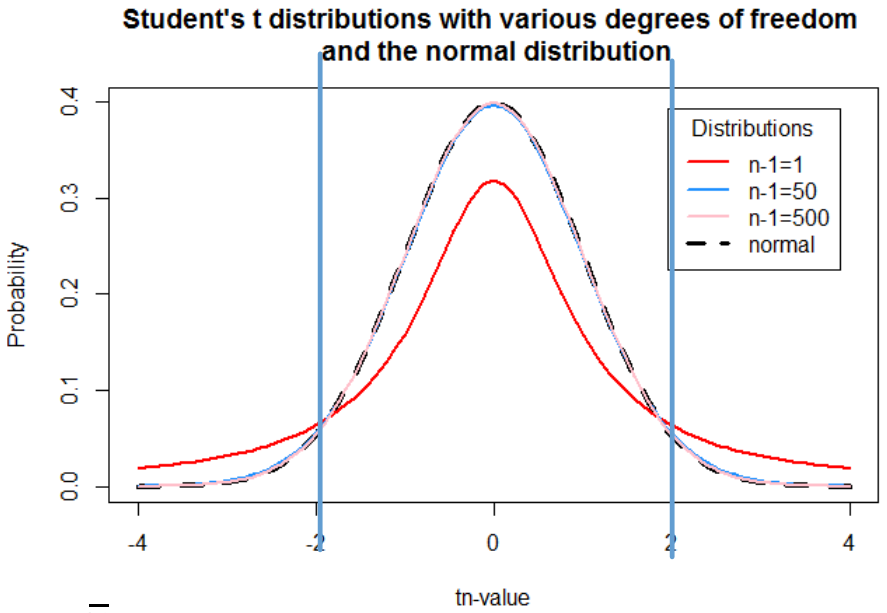


$$t = \frac{\bar{x}_{North} - \bar{x}_{South}}{s/\sqrt{n/2}}$$

The t-test

The t distributions for the Null hypothesis

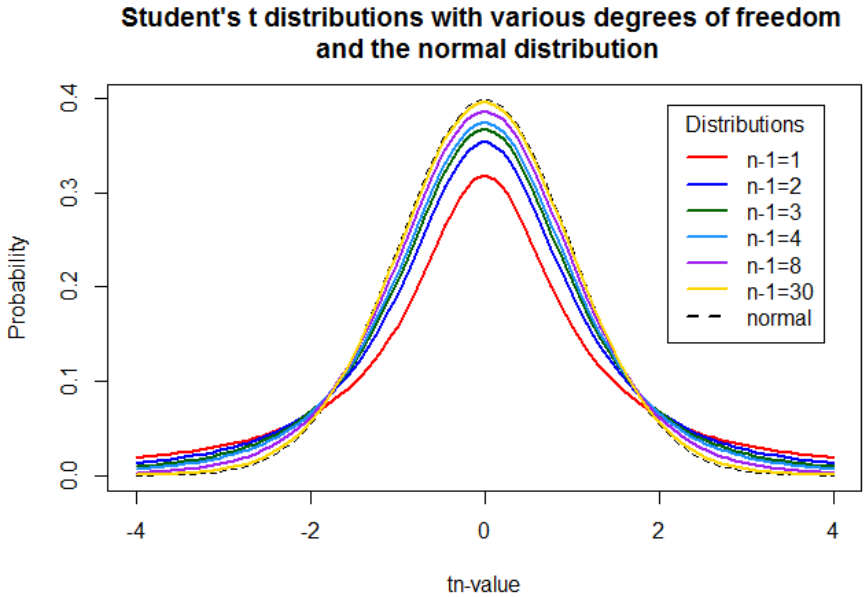




$$t = \frac{\bar{x}_{North} - \bar{x}_{South}}{s/\sqrt{n/2}}$$

The t-test

The t distributions



When the population variance is unknown (the usual case) and sample size is small (n<100, the usual case), the t-distribution is used. The *t* distribution is a short, fat relative of the normal. The shape of *t* depends on n. As *n* becomes infinitely large, *t* becomes normal.

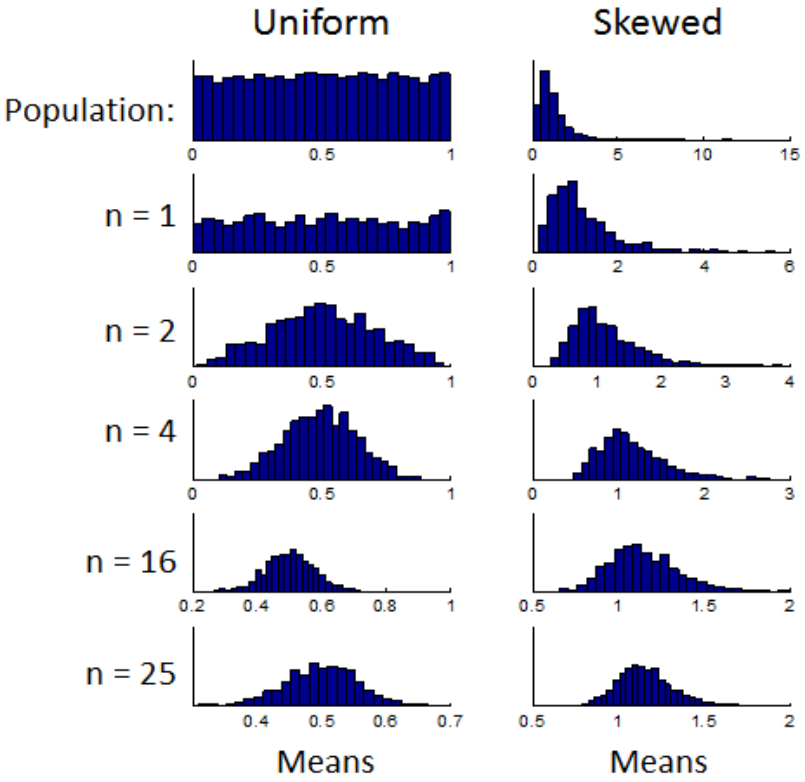
Central Limit Theorem (CLT)

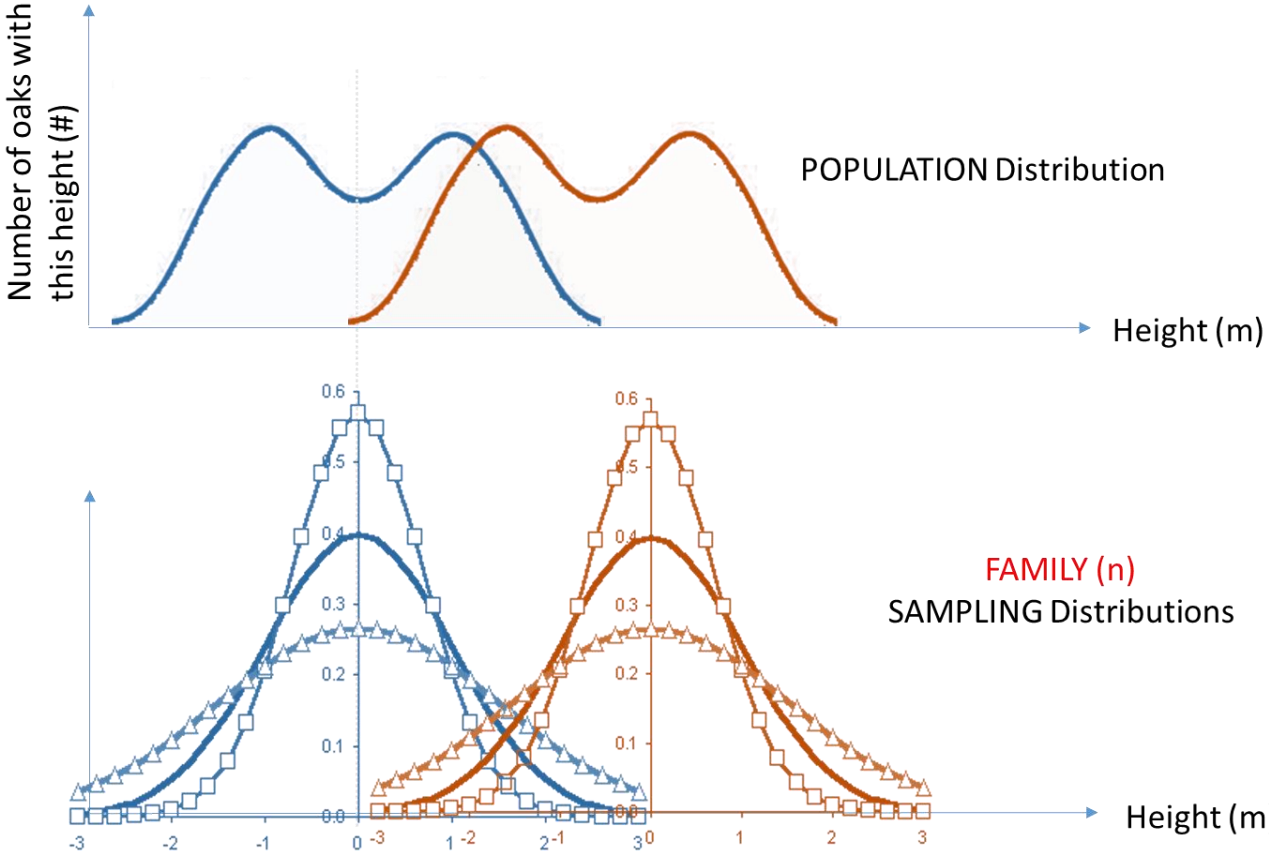
For n → inf:

- $\bar{x} \rightarrow \mu$, with $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- $s_{\bar{x}} \rightarrow 0$, with $\frac{s}{\sqrt{n}}$ (std error of the mean: SEM)
- Sampling distribution: Gaussian: N(μ,*SEM*)

Central Limit Theorem (CLT)

- The population distribution, i.e., raw scores, may have any arbitrary shape.
- However, the distribution of the sample means tends to be bell-shaped, regardless of how raw scores are distributed.
- This is important because certain statistical tests assume that data are normally distributed (i.e. Gaussian, or “bell-shaped”).




Number of oaks with this height (#)
POPULATION Distribution
Height (m)
FAMILY (n)
SAMPLING Distributions
Height (m)

The classic approach: types of t-tests

Types of t-tests

- Independent samples *t-test:* 2 independent groups; eg trees in the above example, participants randomly assigned to a group
- Independent samples *t-test (Welsh test):* 2 independent groups, with different variances and sample sizes
- One sample *t-test*: 1 group is compared with a standardized distribtion, eg IQ test
- Repeated measures *t-test:* 1 group, two conditions, eg same participants in a pre/post test, or related participants, such husband-wife, brother-sister
- One- and two-sided t-tests

Types of t-tests

- Two sample t-test

$$t = \frac{\bar{x}_{North} - \bar{x}_{South}}{s/\sqrt{n/2}}$$

- One sample t-test

$$t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}}$$

- Repeated measures t-test: a one sample t-test with $\mu_0 = 0$

Example

Brother	Sister	Diff	$(D-\bar{D})^2$
5	7	2	1
7	8	1	0
3	3	0	1
$x_1 = 5$	$x_2 = 6$	$\bar{D} = 1$	

$$s_D = \sqrt{\frac{\sum(D-\bar{D})^2}{3-1}} = 1$$

$$t = \frac{\bar{x} - \mu}{s_D/\sqrt{3}} = \frac{1-0}{1/\sqrt{3}}$$

Types of t-tests

- The independent two sample t-test (*Welsh test*)

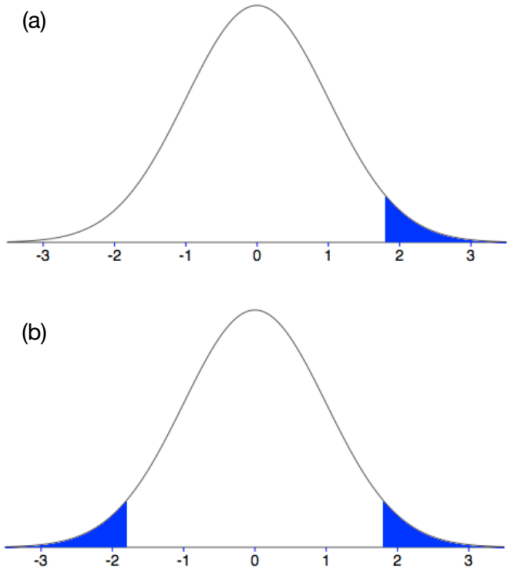
$$t = \frac{\bar{x}_{North} - \bar{x}_{South}}{\sqrt{\frac{{s_{north}}^2}{n_{north}} + \frac{{s_{south}}^2}{n_{south}}}} \quad \text{when } n_{north} \neq n_{south}, s_{north} \neq s_{south}$$

Types of t-tests

One and two sided t-tests

The two tailed t-test assumes that there is a difference between, for example, therapy A and B

The one tailed t-test assumes that therapy A is better than B (or therapy B is better than A) and is usually not justified and looks fishy; advantage: higher power

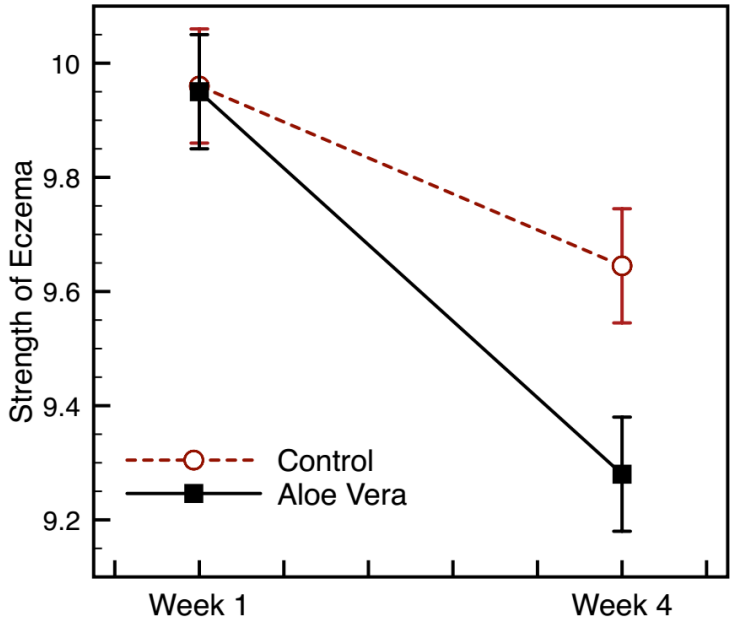


What test to choose?

Implication IIIb: A difference of significance is not a significant difference!

This is the same situation whenever a control group is used. It makes usually very little sense to state: "there was an effect in condition A but not B".

Never ever compare a significant result with a Null result…….



Koch C, Dölle S, Metzger M, et al. Docosahexaenoic acid (DHA) supplementation in atopic eczema: a randomized, double-blind, controlled trial. Br J Dermatol 2008;158:786-792.

Assumptions

Assumptions

- Normality (for n<100)
- Equal Variance & sample size
- Independent variable is ratio scaled
- Independent samples
- Identical distribution

Assumptions

	$n_1 = n_2 = 5$		$n_1 = 5,\ n_2 = 25$	
	$\sigma_2 = 1$	$\sigma_2 = 5$	$\sigma_2 = 1$	$\sigma_2 = 5$
$\sigma_1 = 1$	0.050	0.074	0.052	0.000
$\sigma_1 = 5$	0.073	0.051	0.383	0.047

Table 1: Type I error rates for 10,000 simulated t-tests with different population standard deviations and sample sizes.

http://shinyapps.org/apps/p-hacker/

Assumptions

- Normality (for n<100)
- Equal Variance & sample size
- Dependent variable is ratio scaled
- Independent samples
- Identical distribution

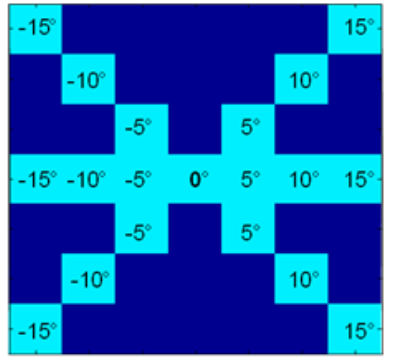
Assumptions

Assumptions of all statistical tests: iid:

identically and independently distributed

- Independent samples
- Identical Distribution

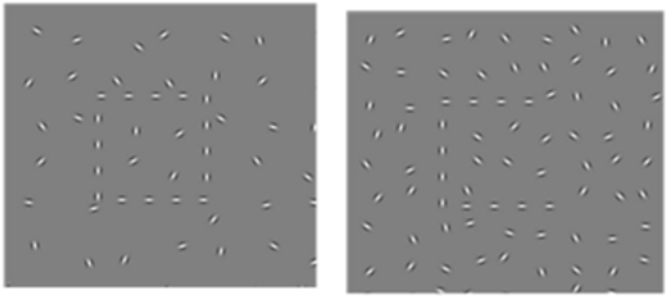
Example



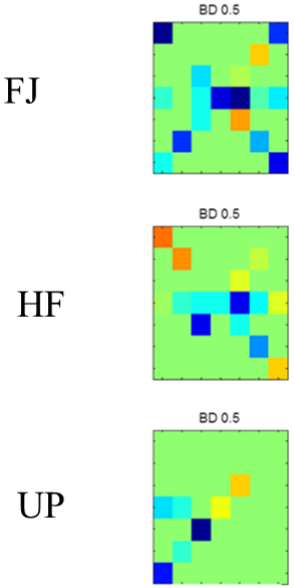
UP (optic tract)

FJ (occipital-parietal-temporal)

HF (occipital)



Task: Is there a square?



One Sample T-Test

	t	df	p
FA rate	-2.499	29.00	0.018

Example

	Cardiovascular Disease	Cancer	Accident	Other
Reference Groups				
US Population Ages 55–64, (*n* = 338, 127)	27%	34%	5%	35%
Non-Flight Astronauts, (*n* = 35)	9%*	29%	53%*	9%*
Astronaut Groups				
All Flight Astronauts, (*n* = 42)	17%	31%	43%*	10%*
Low Earth Orbit Astronauts, (*n* = 35)	11%*	31%	49%*	9%*
Apollo Lunar Astronauts, (*n* = 7)	43%†‡	29%	14%^	14%

From: Apollo Lunar Astronauts Show Higher Cardiovascular Disease Mortality: Possible Deep Space Radiation Effects on the Vascular Endothelium

Confidence Intervals

Calculating Confidence Intervals

$$\mu = \bar{x} \pm t * \frac{s}{\sqrt{n}}$$

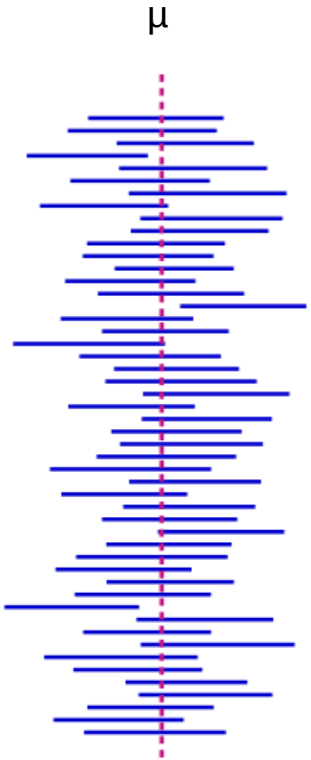
$\frac{s}{\sqrt{n}}$ = standard error of the sample mean

$\bar{x}$ = sample mean

t determines how sure you want to be that μ lies in that interval, e.g. for a 95% CI, *t*(.975, df=inf) = 1.96

Confidence Intervals

- Each blue line depicts the Confidence Interval of one individual sample (e.g., one experiment with n observations, or one patch of forest with n trees), centered around the respective sample mean $\bar{x}$
- Since the CI is centered on $\bar{x}$ the CI will have equally long 'legs' to both sides.
- This length, like $\bar{x}$, is likely to vary from one sample to another due to chance.
- Yet, the sample means $\bar{x}$ 1 … z will be randomly, but normally distributed around μ.



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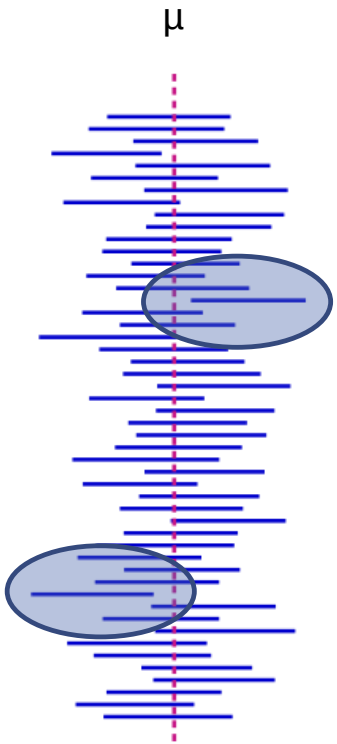

- If Null hypothesis is true and if you continue taking samples (experiment) over and over again until the end of days, you will find that α (e.g. 5%) of these CIs do not contain μ

Confidence Intervals for the evaluations of hypotheses

How is this useful? Example:

Some friend told you that Chinese men are 150cm tall on average. You don't believe him and travel to China and measure 100 men at random. This sample gives you an average height of 170cm and 10cm variance.

Claim: $\mu_0 = 150\text{cm}$

Observation: $\bar{x} = 170\text{cm}$ s = 10cm n = 100

Choice: $\alpha = 0.05$ $\mu_1 = \bar{x} \pm t * \frac{s}{\sqrt{n}} = 170 \pm 1.96 * \frac{10}{\sqrt{100}} = 170 \pm 1.96$

95% CI for μ = [168.04; 171.96]

150 is not included in the 95% CI. In fact, it doesn't even come close

Why null hypothesis significance testing is unnecessary

Example:

You're a researcher and working on a highly debated issue: It has been suggested, that playing action video games can help dyslexics children read better.

You review the literature and indeed you find ten studies that investigated the phenomenon. You evaluate the studies and they are all methodologically sound and set up in the same way.

Example

Reported on the right are the pre-test / post-test differences and test statistics (t, p) per study.

What do you conclude?

A. The evidence is equivocal, we need more research.

B. All of the mean differences show a positive effect of the intervention, therefore, we have consistent evidence that the treatment works.

C. Five of the studies show a significant result ($p < .05$), but the other 5 do not. Therefore, the studies are inconclusive: some suggest that the intervention is better than alpha, but others suggest there's no difference. The fact that half of the studies showed no significant effect means that the treatment is not (on balance) more successful in reducing symptoms than the control.

Study	Difference between Means	t	p
Study 1	4.193	3.229	0.002*
Study 2	2.082	1.743	0.086
Study 3	1.546	1.336	0.187
Study 4	1.509	0.890	0.384
Study 5	3.991	2.894	0.006*
Study 6	4.141	3.551	0.001*
Study 7	4.323	3.745	0.000*
Study 8	2.035	1.479	0.155
Study 9	6.246	4.889	0.000*
Study 10	0.863	0.565	0.577

Example

Reported on the right are the pre-test / post-test differences, test statistics (t, p), **and confidence intervals** per study.

What do you conclude?

A. The evidence is equivocal, we need more research.

B. All of the mean differences show a positive effect of the intervention, therefore, we have consistent evidence that the treatment works.

C. Five of the studies show a significant result ($p < .05$), but the other 5 do not. Therefore, the studies are inconclusive: some suggest that the intervention is better than alpha, but others suggest there's no difference. The fact that half of the studies showed no significant effect means that the treatment is not (on balance) more successful in reducing symptoms than the control.

Study	Difference between Means	t	p
Study 1	4.193	3.229	0.002*
Study 2	2.082	1.743	0.086
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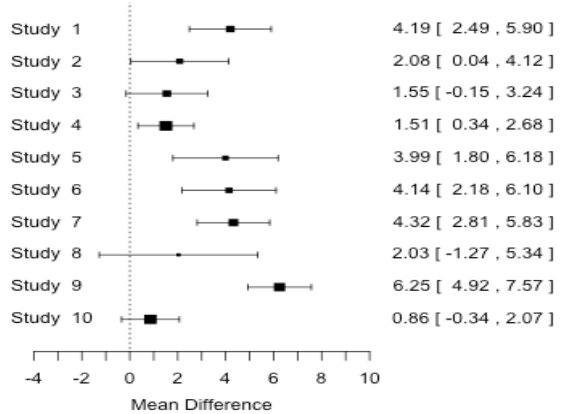

Example

What added value do the *p* values have over the CI information?

Exactly *none,* that's why they're unnecessary!

What added value do the CIs have over plain p values?

They contain additional information that is crucial: it changes your conclusion!

What does that tell us?

p values *alone* can be deceiving!

Dichotomous true / not-true decisions based on whether p < .05 *are not very smart*.

Study	Difference between Means	t	p
Study 1	4.193	3.229	0.002*
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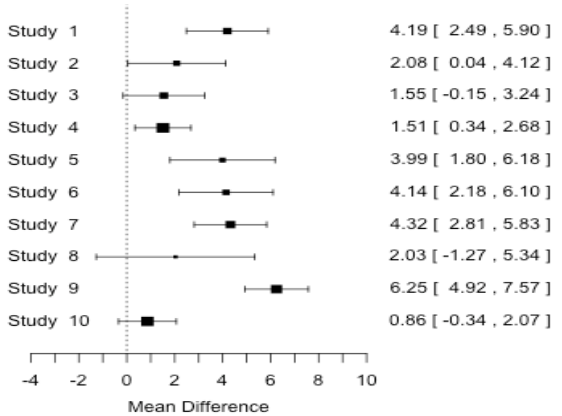
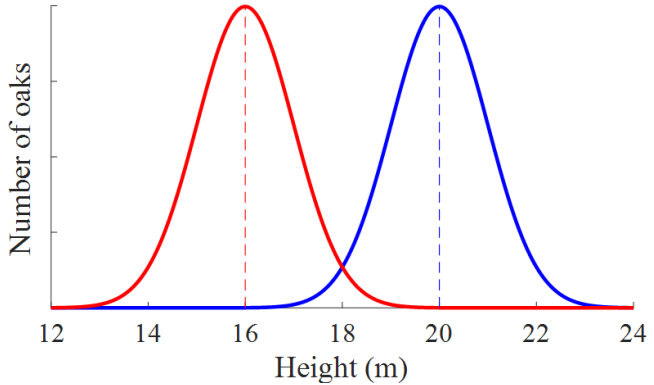


Non-Parametric Tests

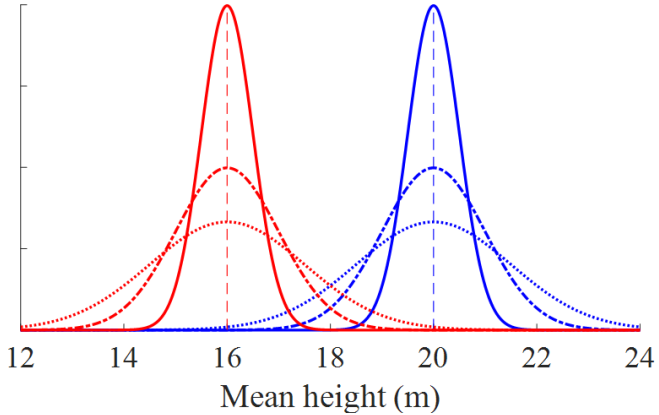
Parametric tests

Population distribution

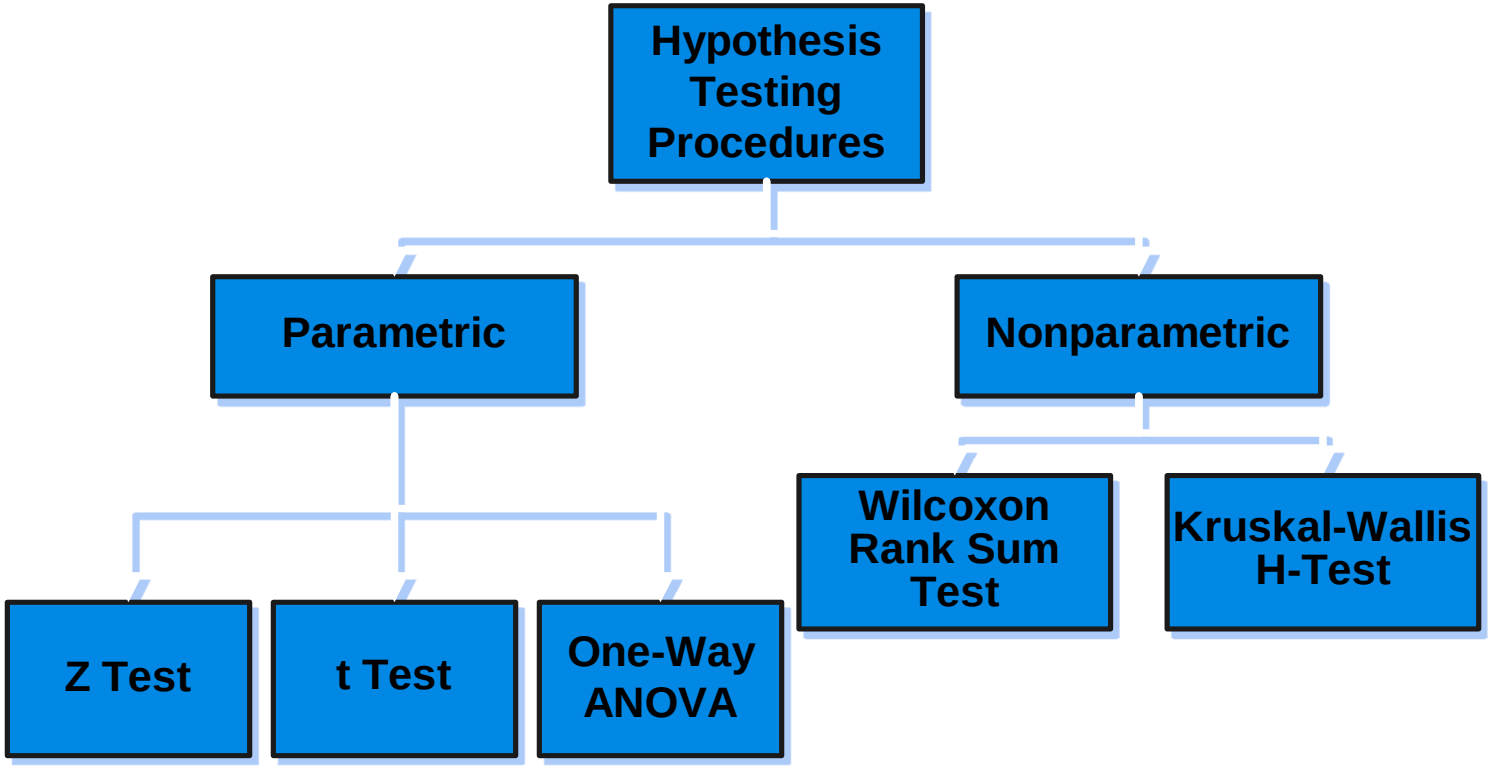


Family (n)

Sampling distributions



Hypothesis Testing Procedures



Popular Nonparametric Tests

1. Sign Test corresponds to one sample test

2. Man-Whitney U-test corresponds to independent t-test

3. Wilcoxon Signed rank corresponds to repeated measures t-test

Sign Test Example

You're an analyst for Chef-Boy-R-Dee. You've asked 7 people to rate a new ravioli on a 5-point scale (1 = terrible,…, 5 = excellent) The ratings are**:**

2 5 3 4 1 4 5

At the .05 level, is there evidence that the median rating is **at least 3**?

Sign Test Solution

- H_0: $\eta = 3$
- H_1: $\eta < 3$
- $\alpha = 0.05$
- Test statistic:

  S = 2 (Ratings 1&2 are < $\eta = 3$: **2, 5, 3, 4, 1, 4, 5**)

  Is observing 2 or less a small prob event?

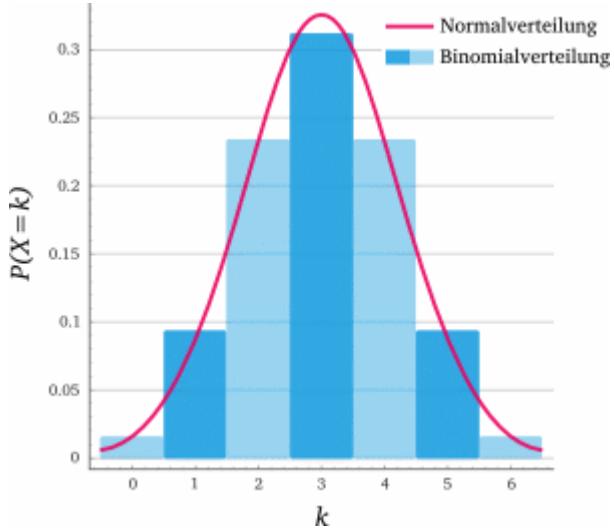
  p-value: p(x>2) = 1-p(x≤1) = 0.9297

  (Binomial table, n = 7, p = 0.50)

  Decision: Do not reject at $\alpha = 0.05$

  Conclusion: There is No evidence for Median < 3

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$



Example

Example:
The table below shows the hours of relief provided by two analgesic drugs in 12 patients suffering from arthritis. Is there any evidence that one drug provides longer relief than the other?

Case	Drug A	Drug B	Case	Drug A	Drug B
1	2.0	3.5	7	14.9	16.7
2	3.6	5.7	8	6.6	6.0
3	2.6	2.9	9	2.3	3.8
4	2.6	2.4	10	2.0	4.0
5	7.3	9.9	11	6.8	9.1
6	3.4	3.3	12	8.5	20.9

Solution:
In this case our null hypothesis is that the median difference is zero. Our actual differences (Drug B - Drug A) are:

$$+1.5, +2.1, +0.3, -0.2, +2.6, -0.1, +1.8, -0.6, +1.5, +2.0, +2.3, +12.4$$

Our actual median difference is 1.65 hours. We have $r^+ = 9, r^- = 3, n = 12$, $r = \max(r^-, r^+) = 9$. Therefore our two-sided p-value (from binomial tables) is $p = 0.146$. We would conclude that there is no evidence for a difference between the two treatments. Note that the Wilcoxon Signed Rank Sum Test would also be appropriate in this case and is a more powerful test because it takes account of the magnitude of the differences as well as the sign.

Rosie Shier. 2004

Example

The observations in a sample of size n are $x_1, x_2, \ldots x_n$

The null hypothesis is that the population median is equal to some value M. Suppose that r+ of the observations are greater than M and r− are smaller than M. Values of x which are exactly equal to M are ignored; the sum r+ + r− may therefore be less than n — we will denote it by n'.

Under the null hypothesis we would expect half the x's to be above the median and half below. Therefore, under the null hypothesis both r+ and r− follow a binomial distribution with p =0.5 and n = n'.

The test procedure is as follows:

1. Choose r = max(r−, r+).

2. Use tables of the binomial distribution to find the probability of observing a value of r or higher assuming p = 0.5 and n = n'. If the test is one-sided, this is your p-value.

3. If the test is a two-sided test, double the probability obtained in (2) to obtain the p-value.

Rosie Shier. 2004

Advantages of Nonparametric Tests

1. Used With All Scales
2. Easier to Compute
3. Make Fewer Assumptions
4. Need Not Involve, Population Parameters
5. Results May Be as Exact as Parametric Procedures

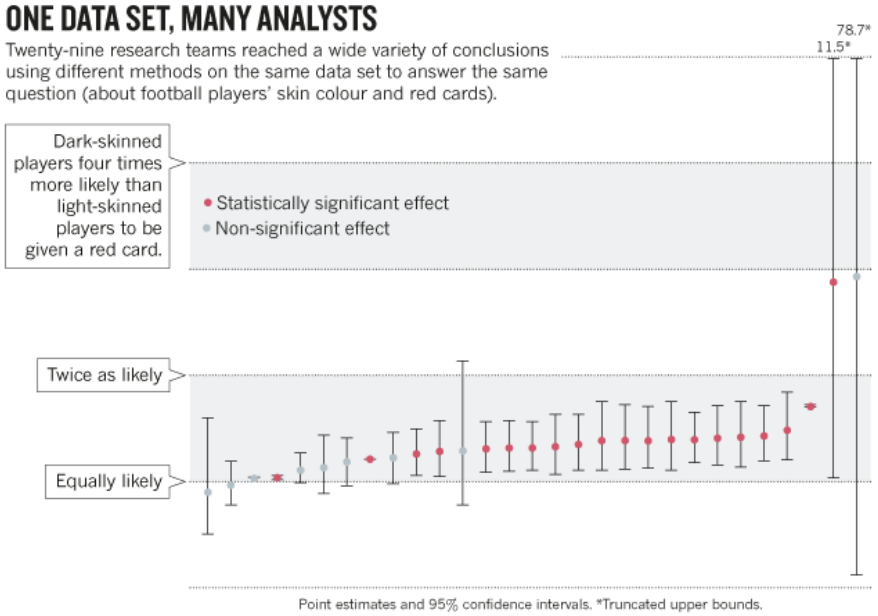
Disadvantages of Nonparametric Tests

1. May Waste Information Parametric model more efficient if data permit
2. Difficult to Compute by hand for Large Samples
3. Tables Not Widely Available

What test to choose?

Choosing statistical tests and expectation

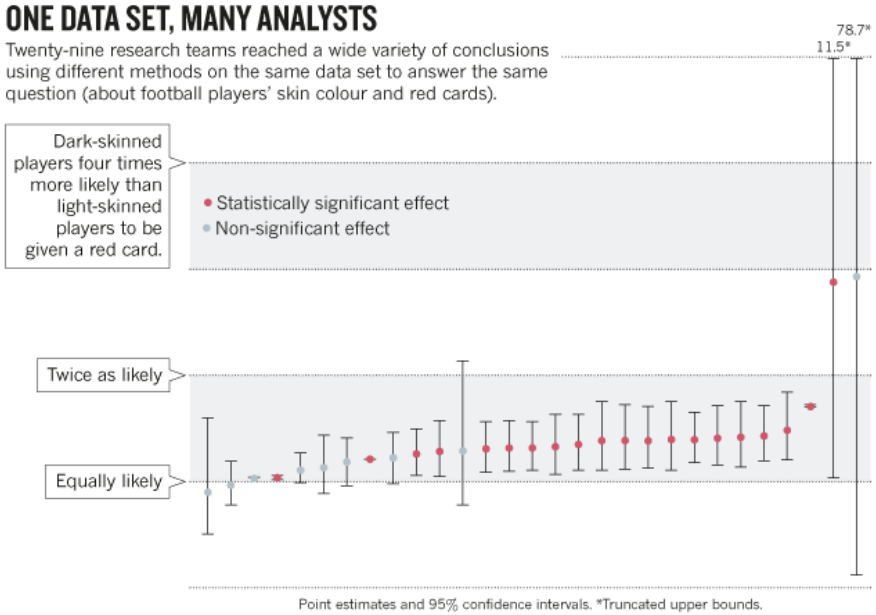
20/29 found effect, 1 even opposite effect



Bias by choosing statistical test and expectation

Solution: inverse labels

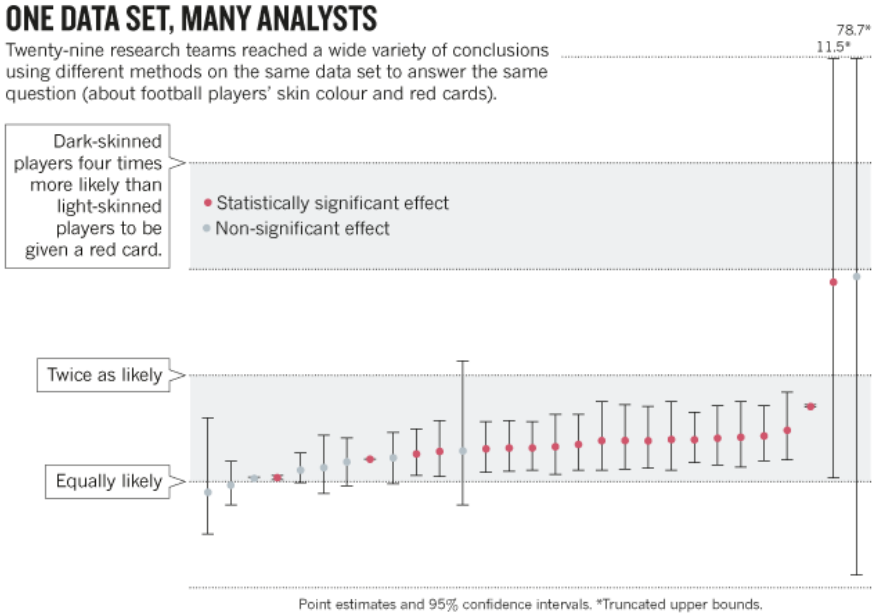
20/29 found effect, 1 even opposite effect



Bias by choosing statistical test and expectation

Obviously, different tests, different results

Choice of tests depends on expectation: Bias



Good & Bad Statistical Practice

Good Practice

- Specific research question and output variable(s)
- Choose statistical test beforehand
- Choose alpha (0.05) and beta level (0.2)
- Choose effect size and compute sample size n
- Run the experiment without any changes
- Compute statistics
- No post-hoc hypothesis testing
- Assumptions: iid, stats model?
- Always: Effect size? Ceiling, Floor effects?

Bad Practice

- Measure as many variable as possible
- See what happens: interesting hot spots?
- Create Post-hoc hypothesis
- Remove “outliers”- depending on post-hoc hypothesis
- Try various statistical tests

  …….until significant

Actually this is all fine, if you treat your experiment as an exploratory experiment- and repeat…….

Example of an exam question

Discuss the robustness of a *t* hypothesis test to various violations. Consider the impact of violating population normality, homogeneity of population variance, and fixed sample size. How do these violations affect control of Type I error? What can be done to reduce the impact of these violations?

Take home messages

Take Home Messages

1. For a t-test, make sure your data are iid distributed and the y-axis is a ratio-scale.
2. Data should be Gaussian distributed or n should be large.

END Class 4