Week 12: Graded Exercise 2
Course: BIO-341 Dynamical systems in biology
Professor: *Julian Shillcock & Felix Naef*
SSV, BA5, 2025 (Graded exercise series)
All working by hand and calculations need to be shown

Consider the following two-dimensional dynamical model:

$$dx/dt = x\,(\,2 - x - y\,)$$

$$dy/dt = y\,(\,4\,x - x^2 - a\,)$$

where x and y are defined in the whole plane, and a is a constant.

1) Consider the case: a = 2. Find equations (or x, y values) for the nullclines. (**2 p*oints***)

2) Draw the nullclines in a phase portrait and label them clearly. (**4 *points***)

3) Mark the direction of the vector field along all parts of the nullclines in the phase portrait from part 2. (**4 points**)

4) Indicate the location of any fixed points on the phase portrait. (**2 *points***)

5) Find the Jacobian of the model in terms of x and y, and evaluate it at each of the fixed points; give their type and stability. (**6 *points***)

6) If you found one or more nodes or saddlepoints in part 5, find their eigenvalues/eigenvectors, and draw the slow/fast eigenvectors and/or stable/unstable manifolds in the phase portrait from part 2 (only draw short vectors) (**4 points**).

7) Redraw the phase portrait obtained (copy the nullclines, direction of the vector field on the nullclines, and on the axes and in other regions of the phase plane, etc, onto it); mark all fixed points and label them with their type/stability/eigenvectors/manifolds; add representative trajectories in all regions of the phase portrait. (**6 points**)

8) Now set a = 4, find the nullclines. Draw another phase portrait and add the nullclines and fixed points (only their location). How many fixed points are there now? Describe what has happened to the original fixed points. (**2 points**)