Exercise_08

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0.0.1 Course: BIO-341 *Dynamical systems in biology*

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[3]:
```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib_inline.backend_inline
from ipywidgets import interact
from scipy.integrate import odeint
from IPython.display import set_matplotlib_formats
matplotlib_inline.backend_inline.set_matplotlib_formats('png', 'pdf')
```

1 One-dimensional bifurcations (pencil and paper)

1.1 Question 1

For each of the 1D vector fields A – G:

A) $\frac{dx}{dt} = r - x^2$

B) $\frac{dx}{dt} = rx - x^2$

C) $\frac{dx}{dt} = rx - 4x^3$

D) $\frac{dx}{dt} = rx + 4x^3$

E) $\frac{dx}{dt} = rx - sinh(x)$

F) $\frac{dx}{dt} = x + \frac{rx}{1+x^2}$

1.1 Sketch the vector field for the three cases r < 0, r $= 0$, r $>$ (labelled by their value of r), mark the fixed point(s) and their stability, and indicate the direction of flow of the vector field on the diagram.

1.2 Sketch a few typical trajectories for the three cases, r < 0, r $= 0$, r > 0.

1.3 Draw the bifurcation diagram and label the branches as stable or unstable (use a solid line for stable branches and a dashed line for unstable branches).

1.4 Identify the type of bifurcation in the bifurcation diagram.

1.2 Question 2

Construct a vector field that depends on a real parameter r, $dx/dt = f(r, x)$, and that has no fixed points for $r < 0$, and four fixed points for $r > 0$. Sketch the vector field in the two cases, indicate the direction of flow, and label any fixed points with their stability.

1.3 Question 3

Draw the bifurcation diagram for the system in Question 2.