Exercise 6: Population dynamics model with a limit cycle

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Course: BIO-341 *Dynamical systems in biology*

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Note that this document is primarily aimed at being consulted as a Jupyter notebook, the PDF rendering being not optimal.

[5]:
```
#import important libraries
import numpy as np
import matplotlib.pyplot as plt
from ipywidgets import interact
from scipy.integrate import odeint
from IPython.display import set_matplotlib_formats
from matplotlib.markers import MarkerStyle
set_matplotlib_formats('png', 'pdf')
```

1 Population dynamics model with a limit cycle

In the course, we studied predator-prey models with a stable spiral, which means that the two populations settle to coexist after the oscillatory transient decay.

Here, we study the following model:

$$\frac{dN}{dt} = a\,N(K-N) - c\,P\frac{N}{N+R} \quad (1)$$

$$\frac{dP}{dt} = b\,P\,(sN-P) \quad (2)$$

where the predators P might represent C. Elegans worms feeding on E. Coli bacteria N (typically used as food in laboratory dishes). All parameters should be taken positive (> 0).

1) Explain the different parameters in the model (a,b, c, R, K, s). What are their units?

Hint: Make the connection with the logistic growth model.

2) Calculate and plot the nullclines for the following values of the parameters:

a = b = 0.01

c = 1.

K = 200

R = 50

s = 5

Plot the fixed points and qualitatively describe their meanings in terms of populations of predators and preys

3) Plot the stability of the fixed point with both N,P $\neq$ 0 in the R-K plane (for K, R in [5, 500])

4) Fix K, and use the same values of the parameters for a,b,c and s as above. Choose 3 values of R for which you have respectively a stable f.p., a stable spiral and a limit cycle.

Simulate the trajectories (of N and P) using a python solver (ex: odeint). Plot the trajectories in function of time and the phase portrait (P in function of N) (use subplots) for the three cases with different initial conditions.

In the case of the limit cycle, what is the stability of the fixed point?

[41]:
```
# paramters:
a = 0.01
b = 0.01
c = 1
K = 200
s = 5
tspan = np.linspace(0,100, 10000)
```

5) Describe in words the behavior of the trajectories in terms of the number of predators and preys for each case.

6. (OPTIONAL): Check your answers using euler's method

Euler's method is the simplest way to solve a differential equation numerically. In order to approximate the solution of :

$$\dot{x} = F(x(t))\,,\, x(t_0) = x_0$$

We can write one step of the method as :

$$x(t\,+\,dt) \simeq x(t)\,+\,dt\,F(x(t))$$

for a specific timestep size dt.

a) Implement your own Euler method using Python to solve numerically the following differential equation:

b) Simulate the same trajectories than in 4) using the Euler's method.