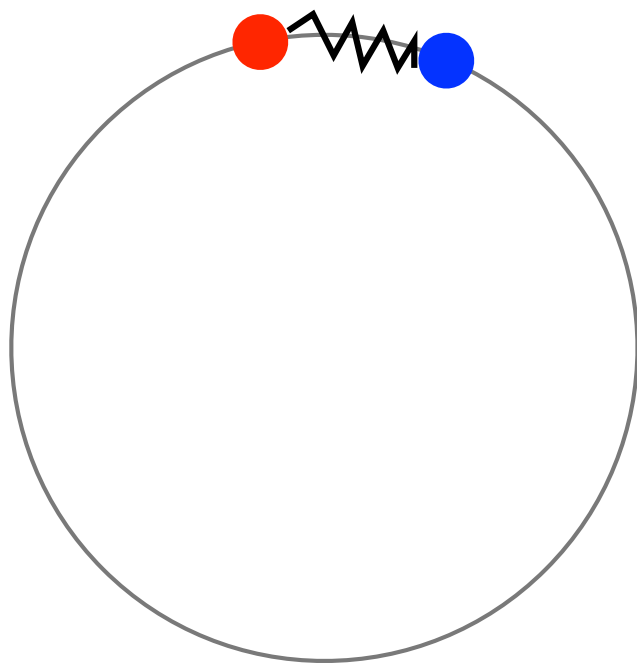


Phase oscillators & entrainment

Two coupling models

Model I (6.1)

- $\dot{\alpha} = \Omega$
- $\dot{\theta} = \omega + K \sin(\alpha - \theta)$



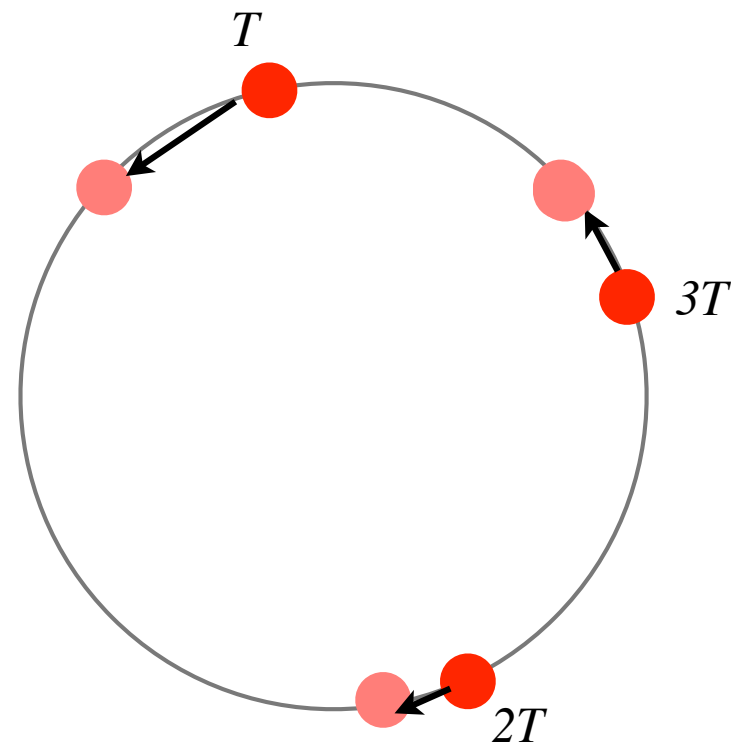
spring

Model II (6.2)

- $\dot{\theta} = \omega + \sum_n \delta(t - nT)g(\theta(t))$

$$\theta_n = \theta_{n-1} + \omega T + g(\theta_{n-1})$$

$$g(\theta) = e \sin(\theta)$$



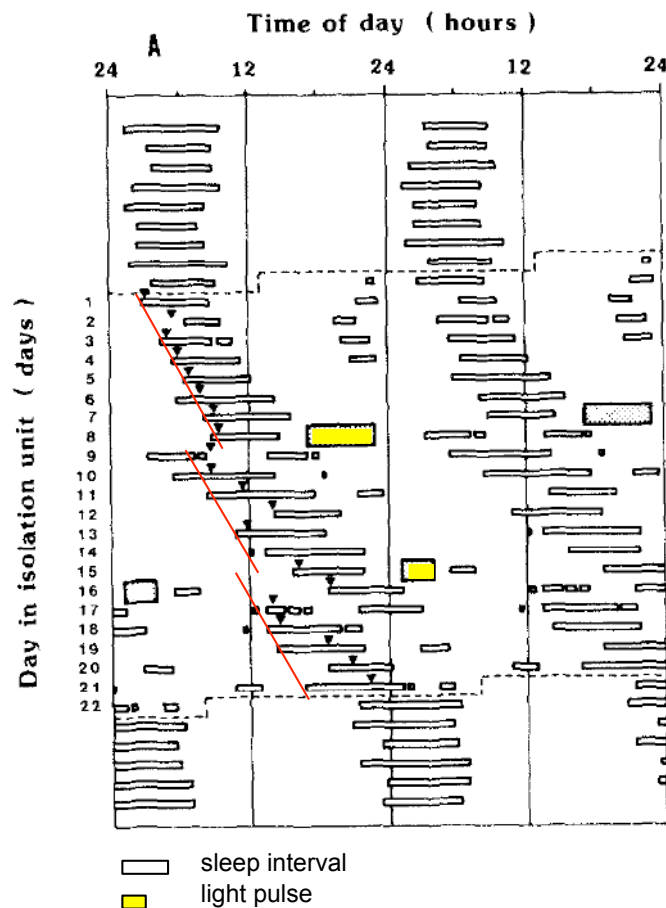
instantaneous kicks

A phase response curve to single bright light pulses in human subjects

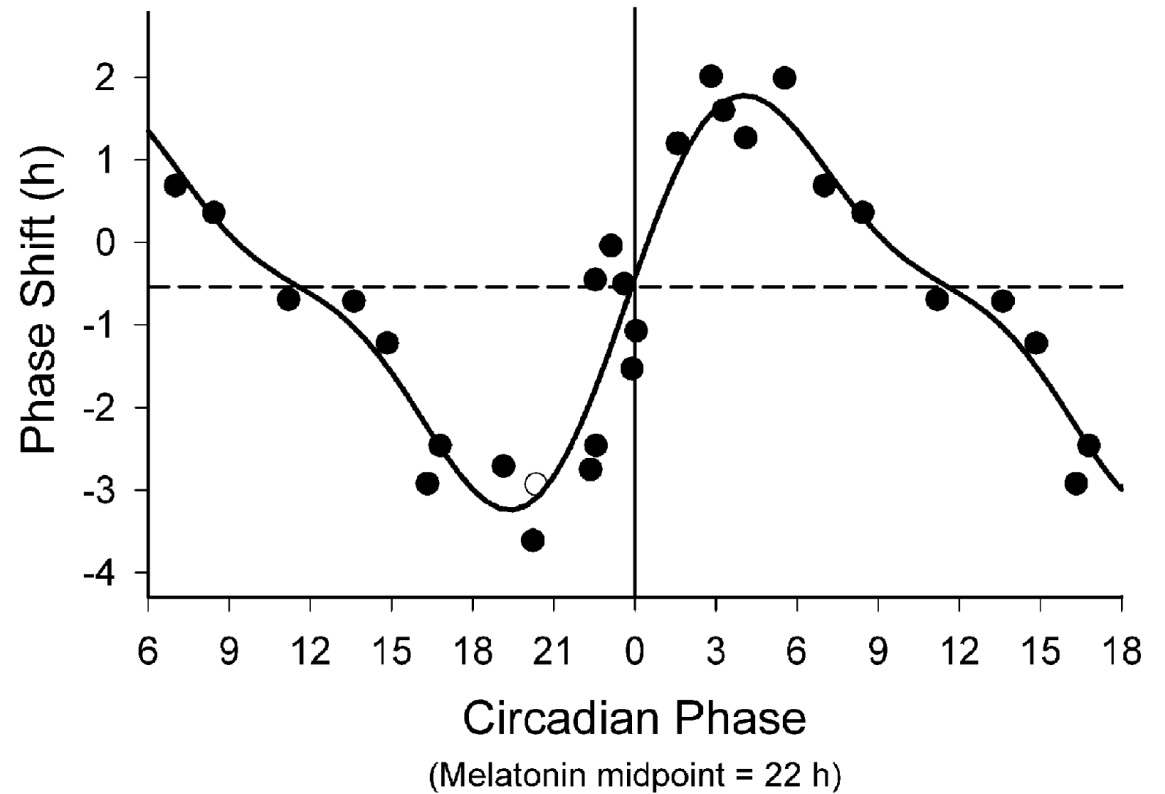
Sat Bir S. Khalsa, Megan E. Jewett, Christian Cajochen and Charles A. Czeisler

Division of Sleep Medicine, Department of Medicine, Brigham and Women's Hospital, Harvard Medical School, Boston, MA, USA

Light pulses shift human phase



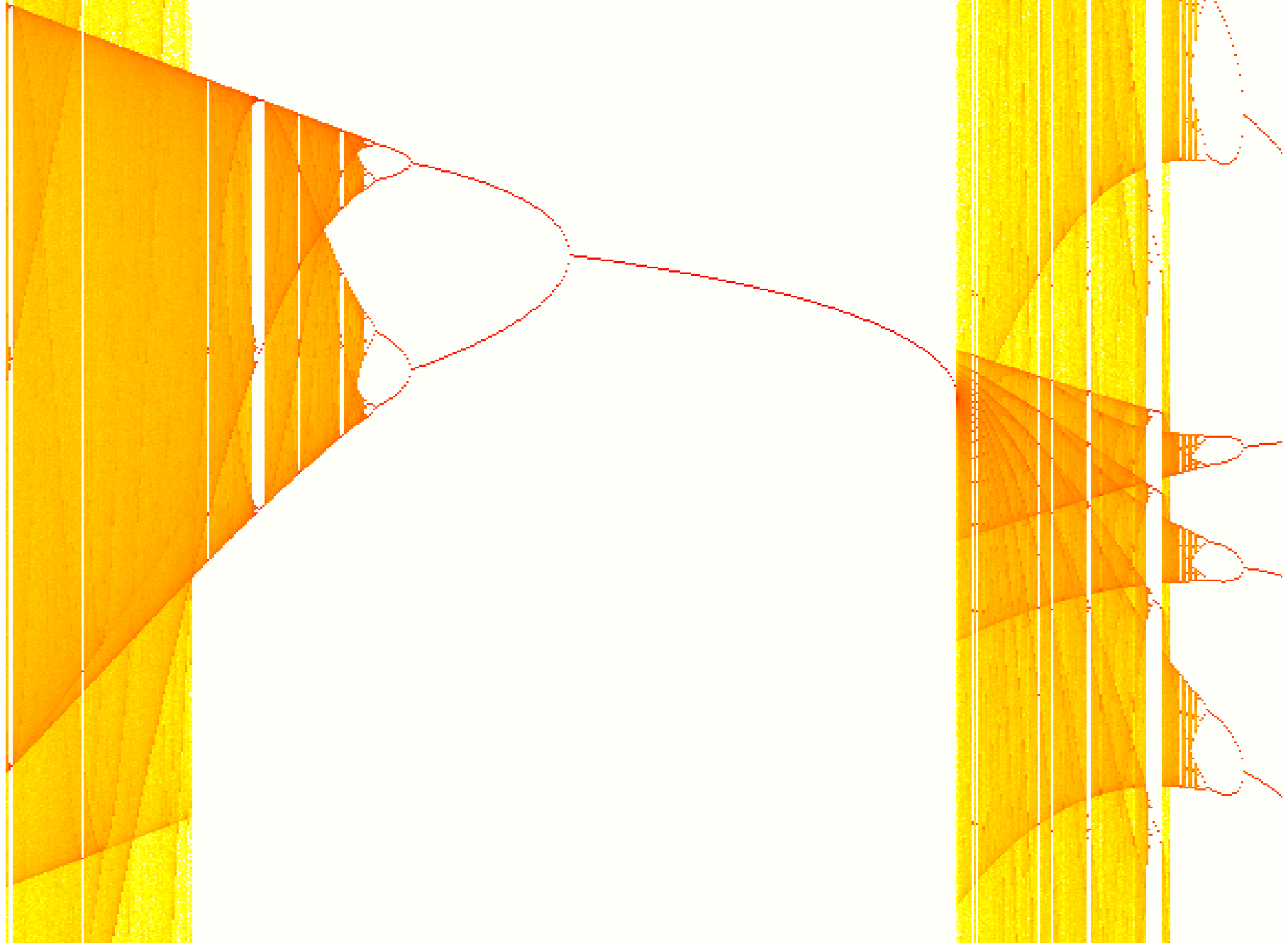
$g(\theta)$ measured in humans



Phase-dependent shift of free-running human circadian rhythms in response to a single bright light pulse

K. Honma, S. Honma and T. Wada* *Experientia* 43 (1987), Birkhäuser Verlag, CH-4010 Basel/Switzerland

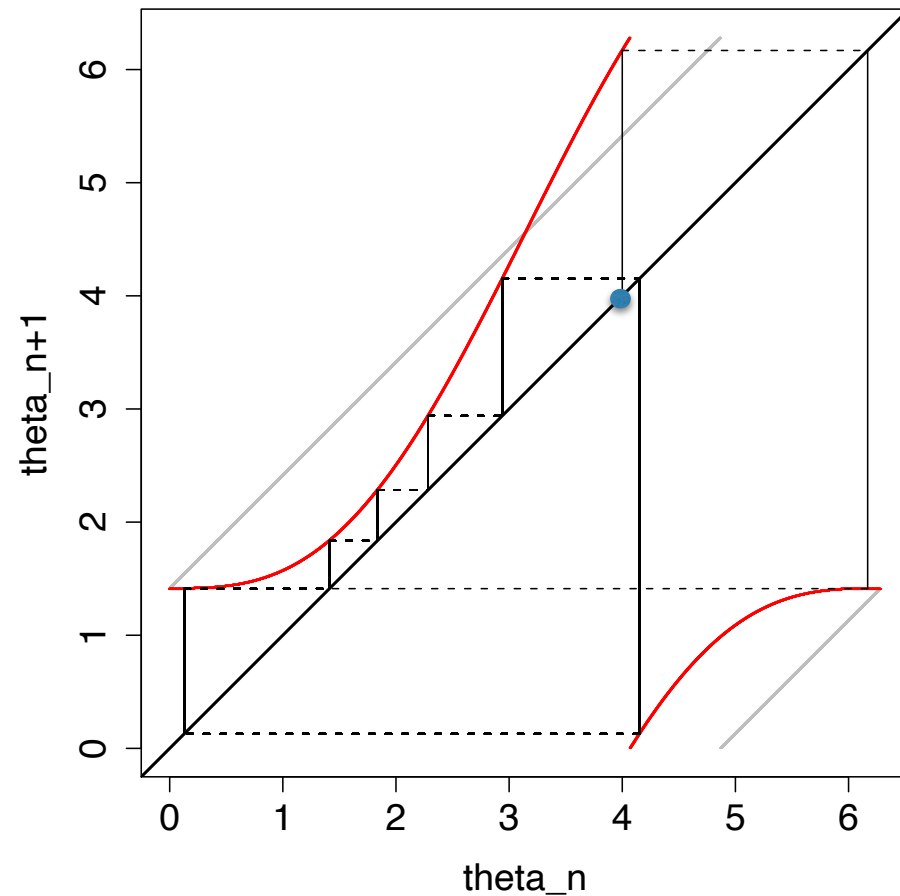
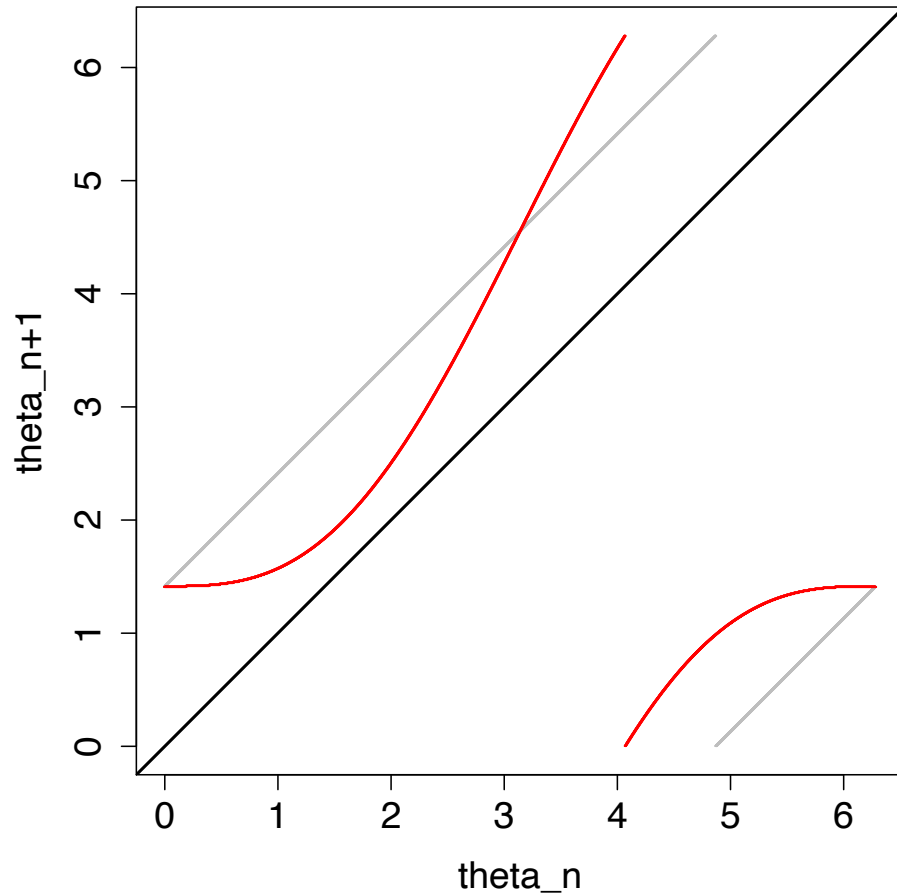
Model II (6.2)



Stability (no fixed point)

Here a period 6 orbit

$$|e| < T\omega \sim 1.4$$

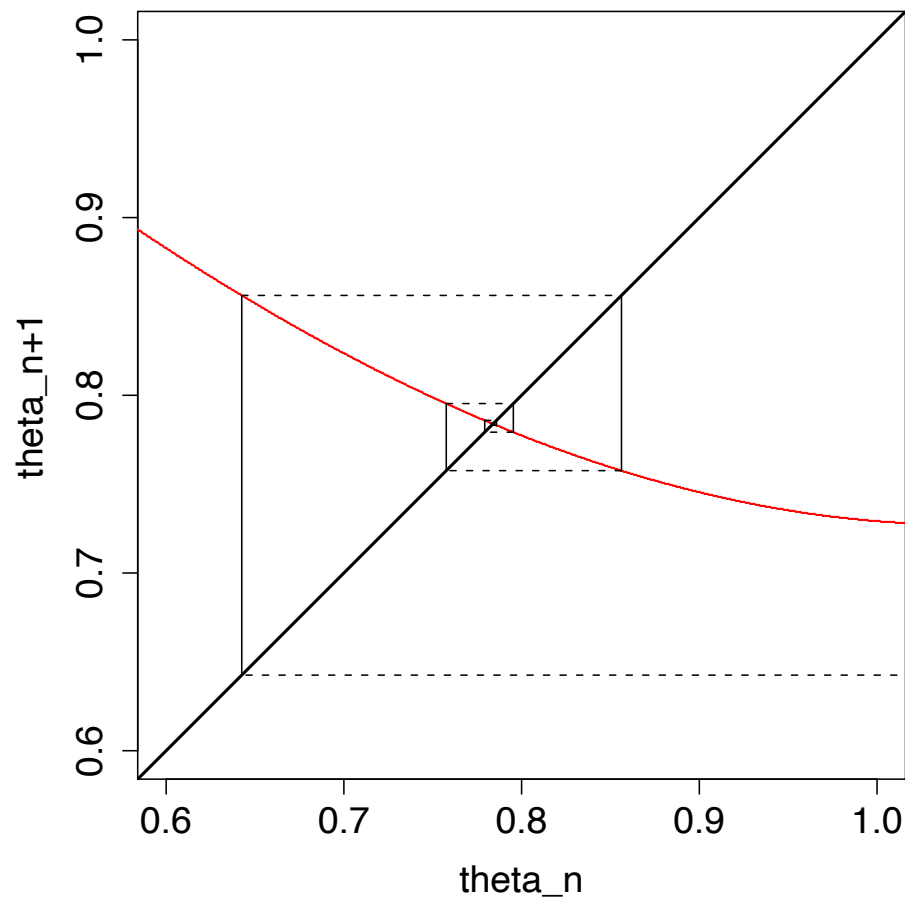
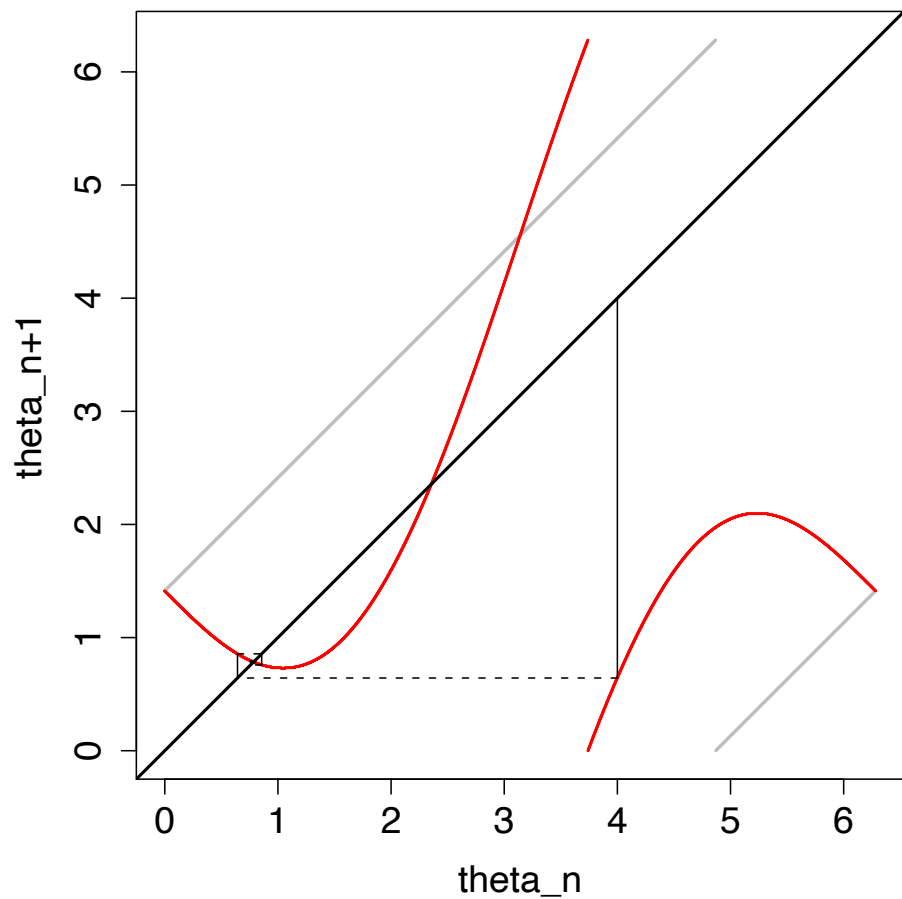


$$e = -1$$

• : θ_0

Stability (period 1 orbits)

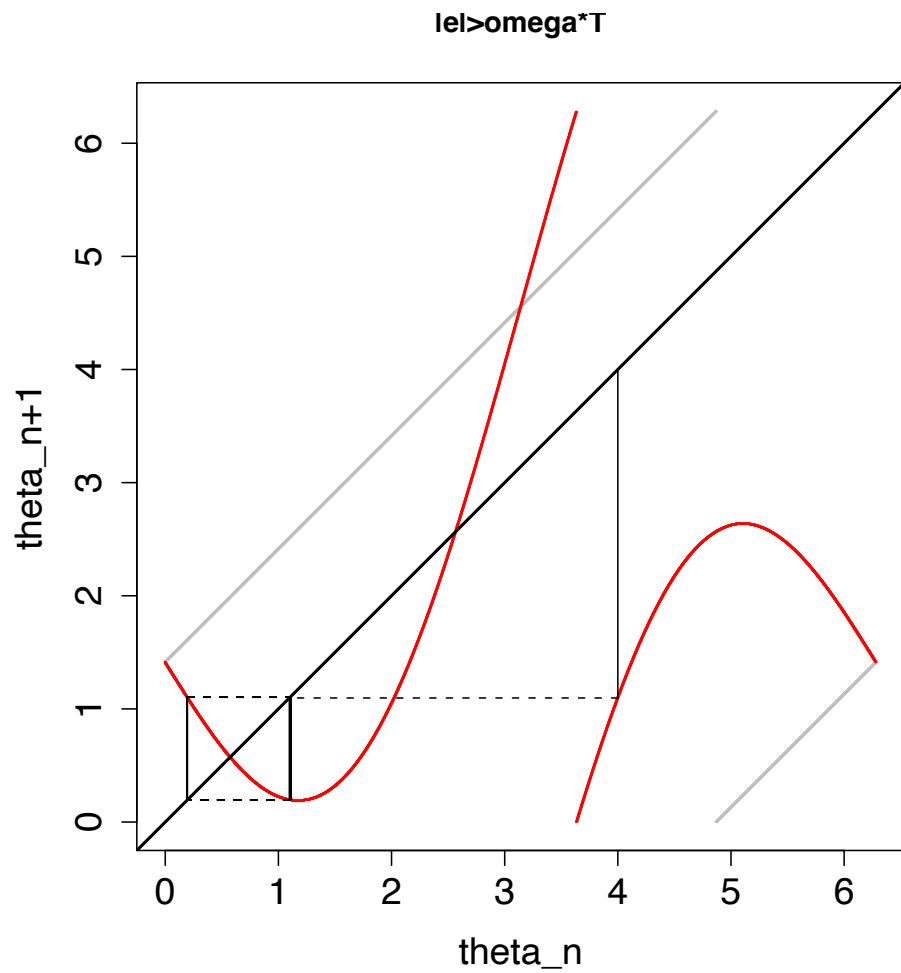
$$|e| > T\omega$$



$$e = -2$$

Stability (period 2 orbits)

$$|e| > T\omega$$

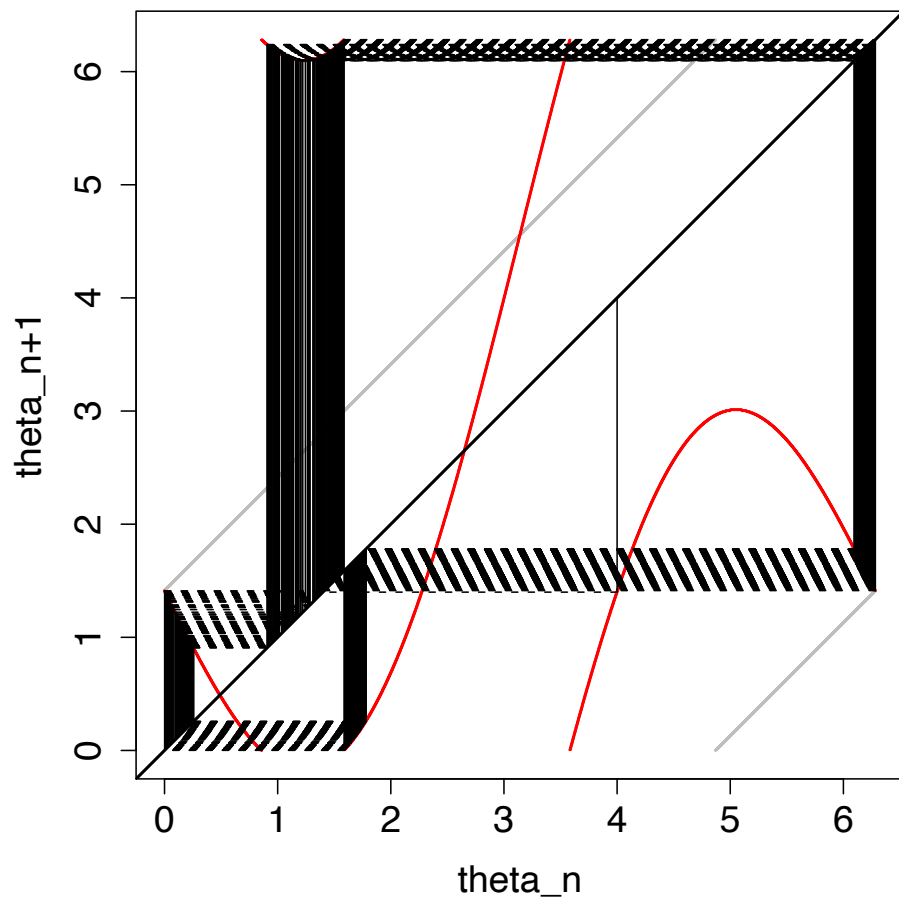


$$e = -2.6$$

Chaotic orbits

The trajectory fills a band (continuum)

$$|e| > T\omega$$



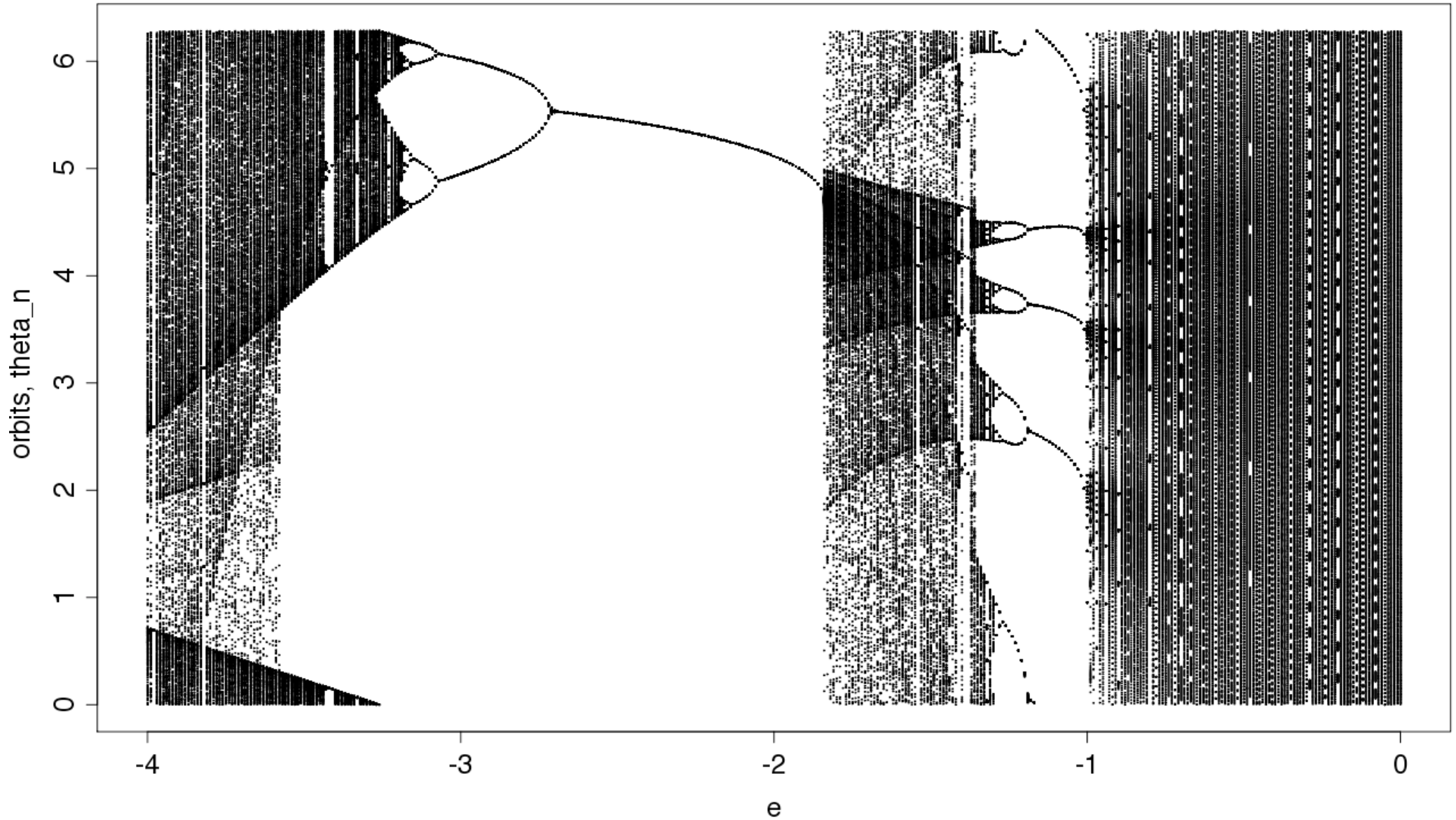
$$e = -3$$

$$\dot{\theta} = \omega + \sum_n \delta(t - nT)g(\theta(t))$$

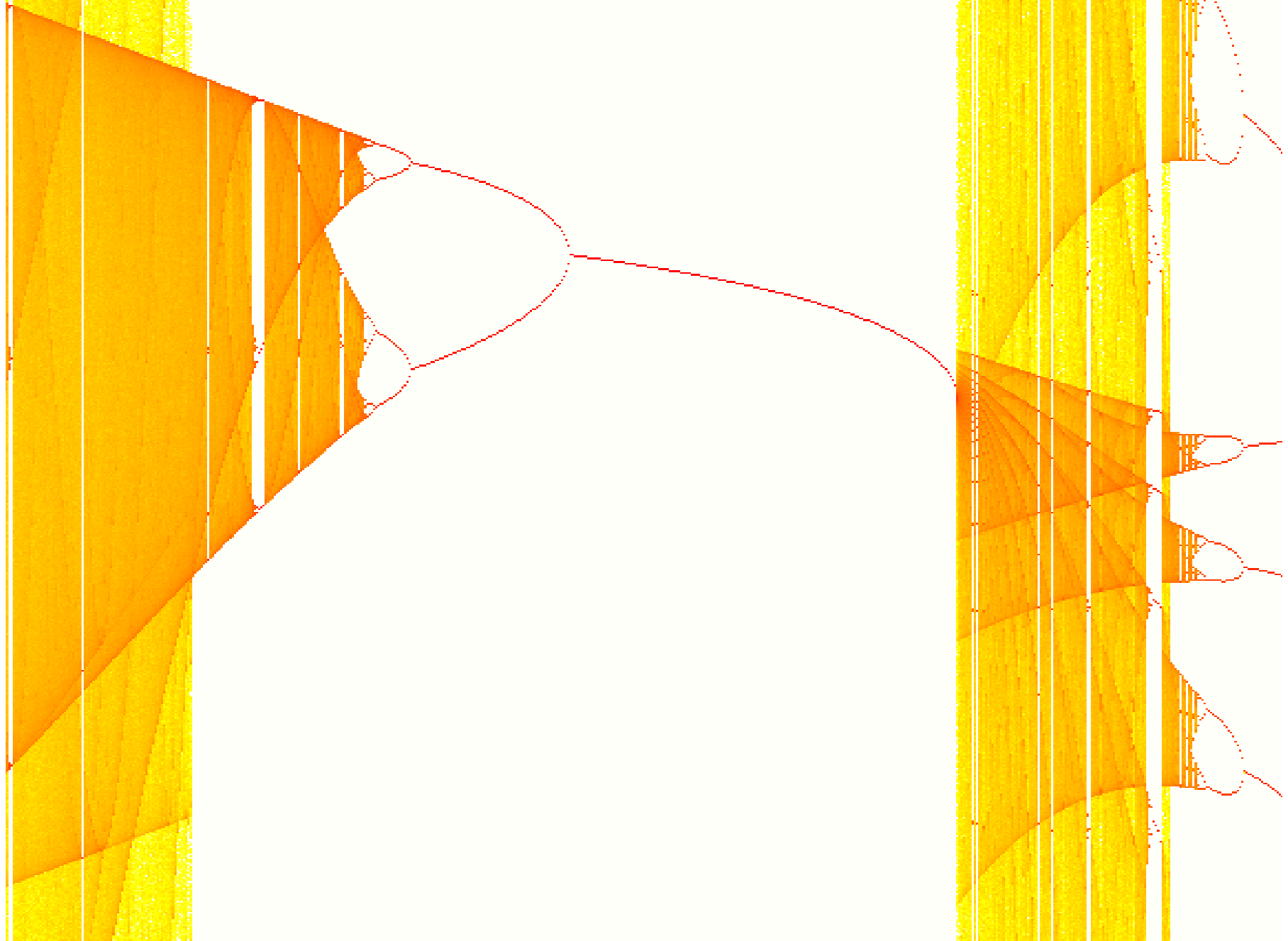
$$\theta_n = \theta_{n-1} + \omega T + g(\theta_{n-1})$$

Orbits

$$g(\theta) = e \sin(\theta)$$



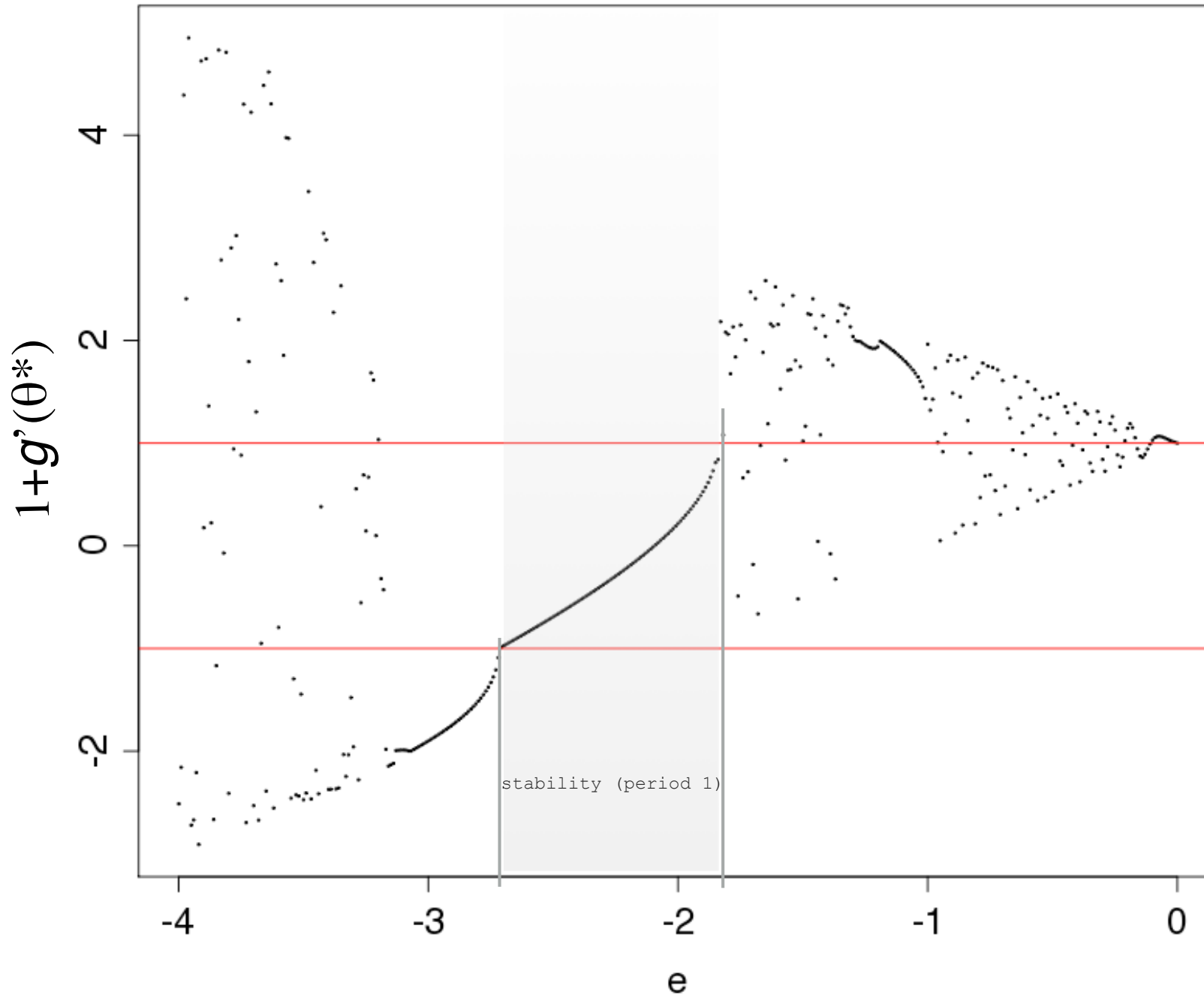
Densities



Stability region $|F'(\theta^*)| < 1$

(period 1 fixed point)

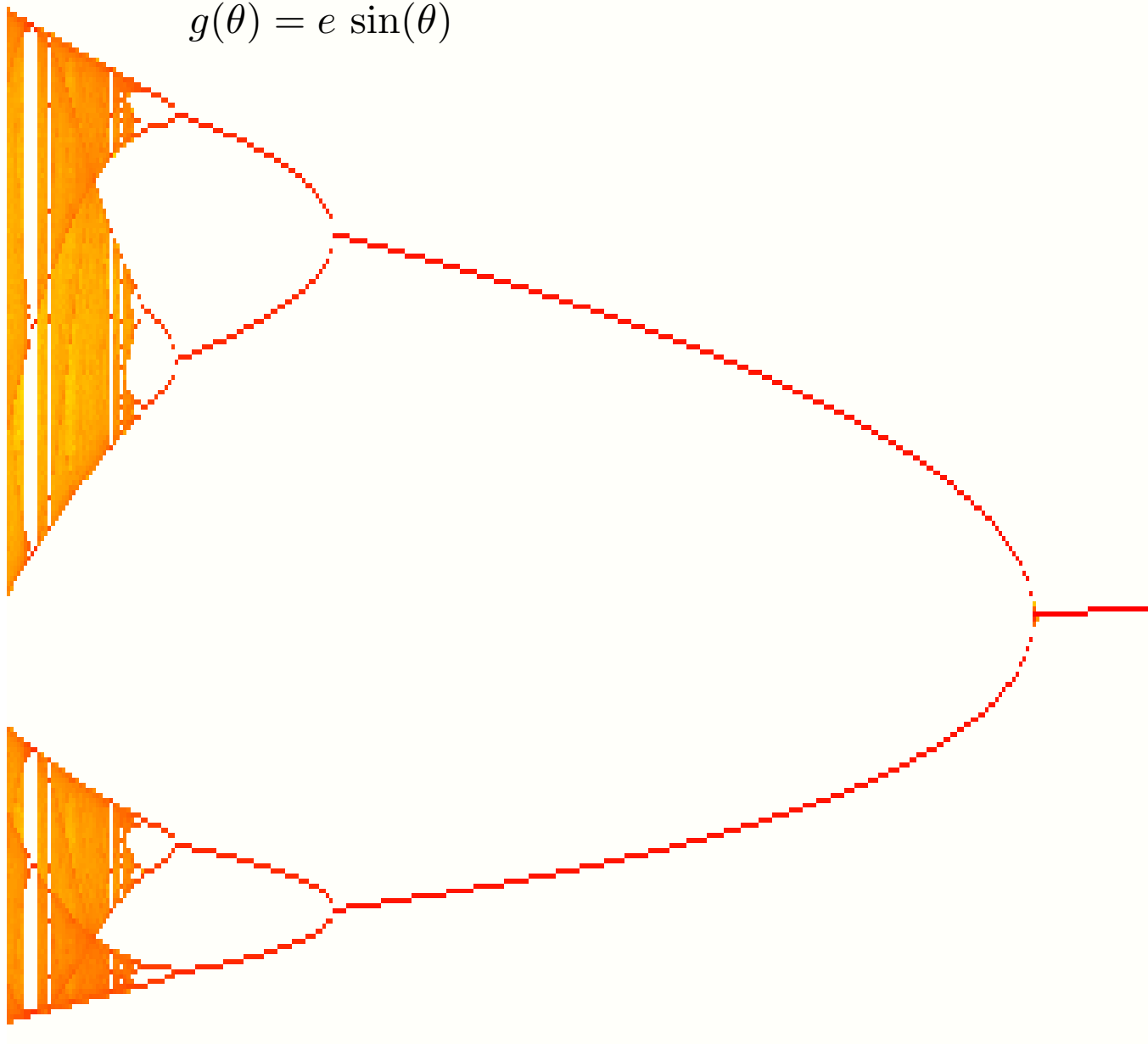
$$F'(\theta^*) = 1 + g'(\theta^*)$$



$$\dot{\theta} = \omega + \sum_n \delta(t - nT)g(\theta(t))$$

$$\theta_n = \theta_{n-1} + \omega T + g(\theta_{n-1})$$

$$g(\theta) = e \sin(\theta)$$



Remarks

- no chaos in 2D ODE system (but chaos in 3D)
- chaos in 1D maps (discrete dynamical systems)
- complex behavior in the kicked phase oscillator