For the final exam, you can bring 5 x A4 sheets of paper, written / drawn / printed with whatever you want (also by computer)

I recommend including a checklist for drawing a phase portrait. Check the following (if the item is asked for):

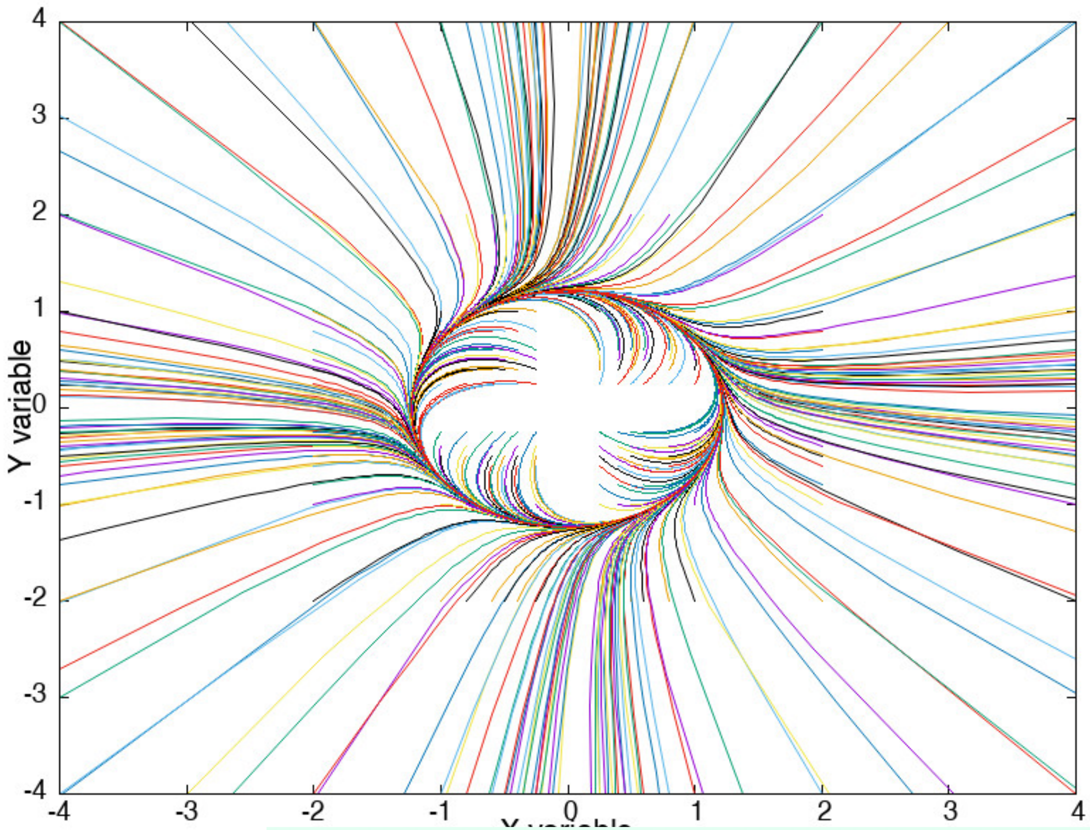
- trajectories don’t cross, and don’t end looking as if they will cross
- a trajectory is present in all distinct regions of the phase plane
- fixed point(s) are labelled with type/stability
- node eigenvectors are labelled fast/slow; saddlepoints with stable/unstable
- nullclines are labelled, and have arrows showing direction of other component
- trajectories only have a zero component on a nullcline
- check if the axes are nullclines
- if the problem is only defined in the first quadrant (popn, conc, …), don’t draw trajectories outside (extending nullclines and/or eigenvectors a little way outside is okay if it improves clarity, but NO trajectories)

Challenge: Can you find 2 limit cycles?

… starting from …

Edible prize

$$\frac{dx}{dt} = -y + x\,(1 - x^2 - y^2) + \ldots$$
$$\frac{dy}{dt} = x + y\,(1 - x^2 - y^2) + \ldots$$



First 3 people/groups who send me the phase portrait and the equations by 11th November.

Genetic control system

In Lecture 3 we had the gene expression model

$$dg/dt = s - r*g + g^2 / (1 + g^2)$$

Why another one?

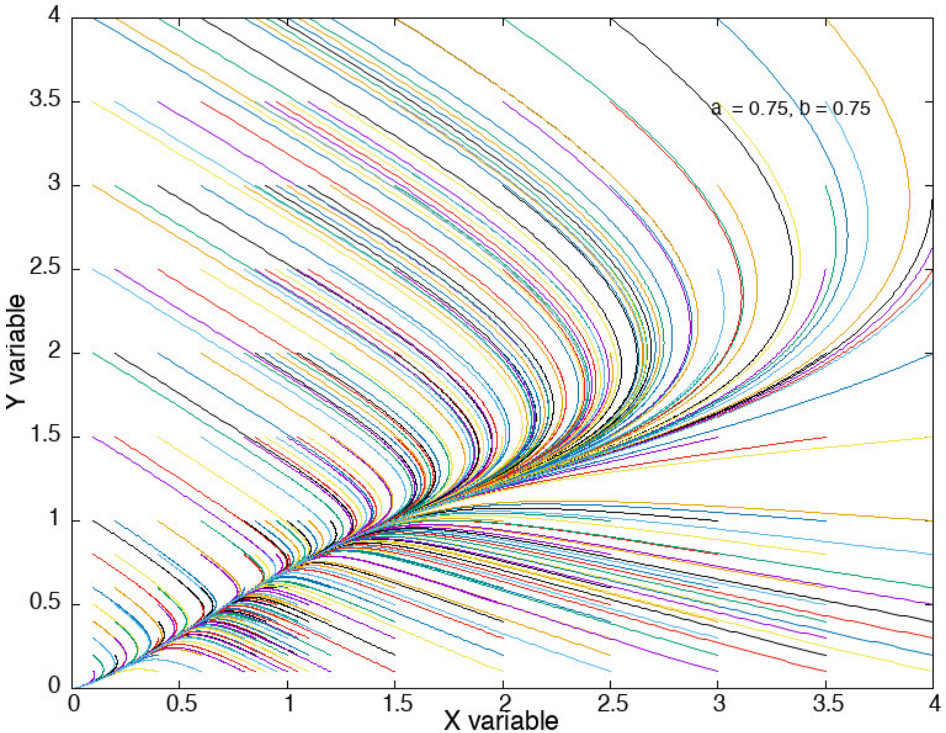
- production of protein depends on [mRNA], not a constant “s”
- transcription is “bursty” (see moodle today: Raj et al, 2006; Elowitz et al, 2002), high mRNA can keep translation high even if [protein] falls to low values
- Extensible: mRNA is degraded based on use not age, we could make b dependent on [protein], but then the model is 3D.

Background quiz: go.epfl.ch/turningpoint

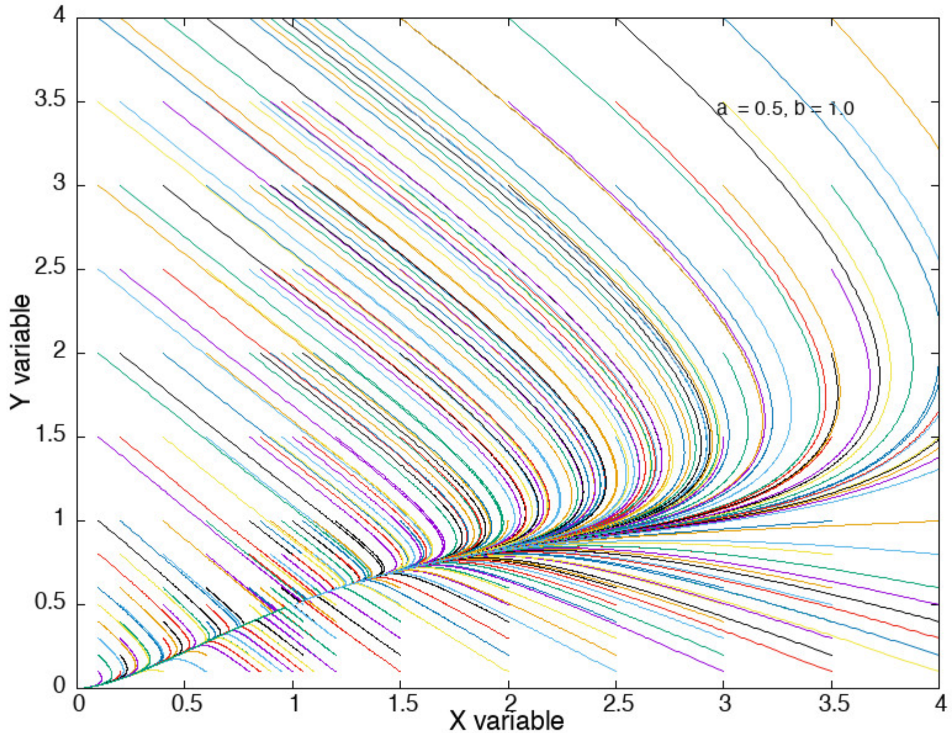
Session Id: julian23

All input is anonymous; data are stored outside CH

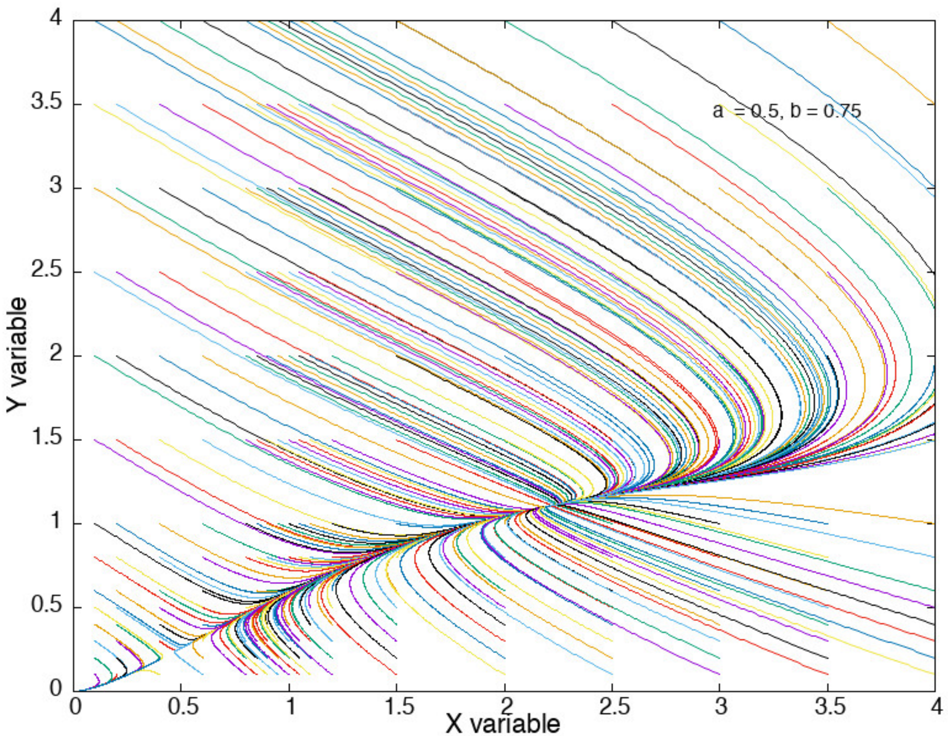
Break



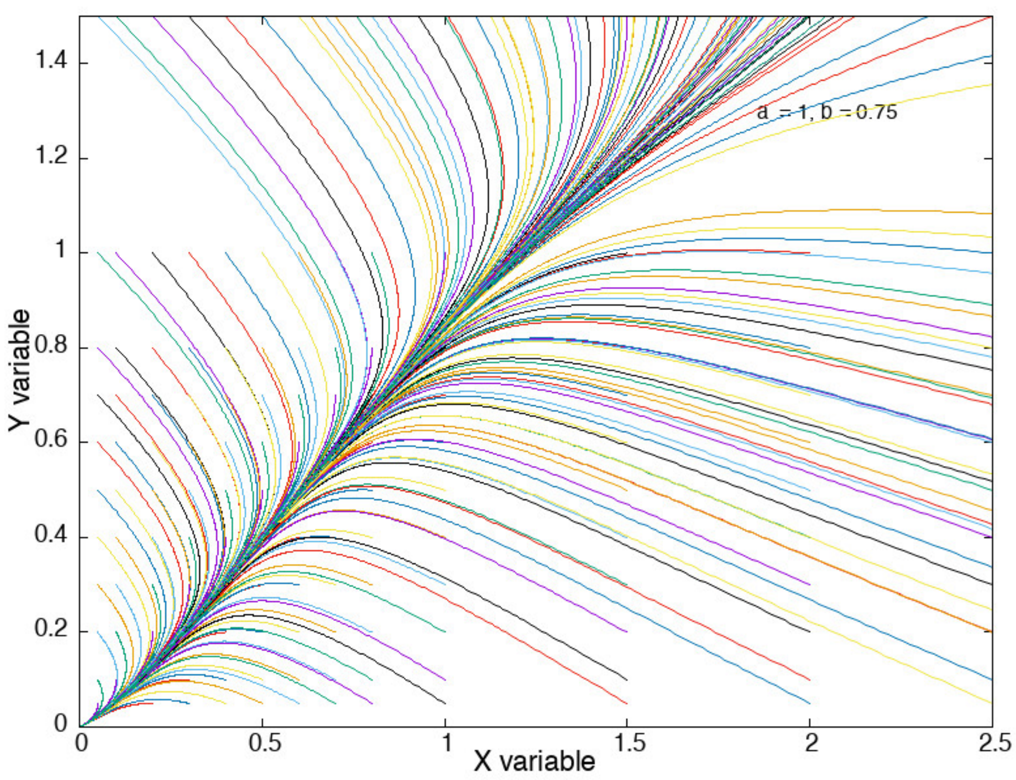
2ab > 1
(0,0) only

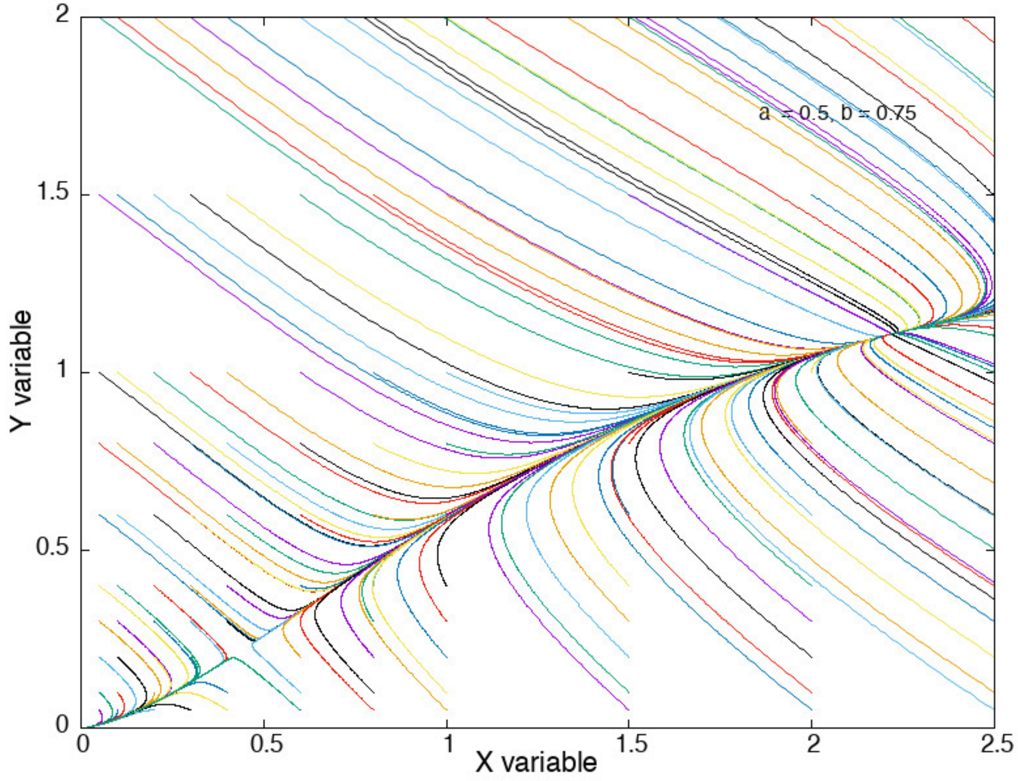


2ab = 1
(0,0), (1,0.5)



2ab < 1
(0,0),
(0.45,0.225),
(2.2, 1.1)





a = 1, b = 3/4

a = 1/2, b = 3/4

Only FP is a stable node at (0,0)

(0,0) and (~2.2, ~1.1) are stable nodes, separated by a saddle at (~0.45, ~0.22)

The stable manifold of the saddle is a separatrix.

“Tricky” points
(not the only important ones)

- The GCS creates a switch out of continuous ODEs as a parameter changes.
- A bifurcation is where the dynamics of a system (= the flow of the trajectories) changes topologically, i.e., fixed points appear or disappear or change stability
- The separatrix of the saddlepoint between the two stable nodes is the stable manifold, and divides the phase plane into disjoint parts; initial points slightly away from it on each side go to very different endpoints. This is not true for the unstable manifold.