#	Date	Content	Course notes ref.
1	Sep 9	Dynamical systems in 1D, fixed points; linear stability analysis; **Graphs 1 and 2, Recipe in 1D**	Chs. 1.1 - 1.3
2	Sep 16	Population models; logistic equation; 2 alleles problem	Ch. 1.4
3	Sep 23	Autocat. 1D genetic switch; hysteresis; Newton’s method for finding zeroes of a function	Ch. 2
4	Sep 30	Linear 2D systems; phase portraits; classification of fixed points; **Recipe for linear systems in 2D**	Ch. 3
5	Oct 7	Non-linear 2D systems; Jacobian at a fixed point; Euler and Runge-Kutta numerical integration schemes	Ch. 4.1
6	Oct 14	Non-linear 2D systems; predator-prey models, spirals; **Recipe for non-linear systems in 2D**	Ch. 4.2 contains a different model than in class, but still a spiral
	Oct 21	Semester break - no lecture / exercise period	
7	Oct 28	Limit cycles in 2D; Poincaré-Bendixson theorem; Biological oscillators (Selkow model)	Ch. 5
8	Nov 4	Bifurcations in 1D and 2D; saddlenode, transcritical, pitchfork; simple 2D model	Ch. 7.1
9	Nov 11	Genetic control system in 2D	Ch. 7.2
10	Nov 18	Forced oscillators, entrainment (Felix Naef)	Chs. 6.1 - 6.2
11	Nov 25	Kuramoto model (Felix Naef)	Ch. 6.3
12	Dec 2	Hopf bifurcation in 2D; Super- and sub-critical Hopf bifurcation, hard/dangerous transitions	Ch. 7.3
13	Dec 9	Chaos; Discrete logistic/sine map; period doubling	Ch. 8
14	Dec 16	Recap of course	

Comments on Graded Ex 1

Q1. It is a stable node not a stable fixed point. Fixed point could mean node, star, or spiral.

Q2 Label the eigenvectors of a saddlepoint as “stable manifold”, “unstable manifold” not slow / fast.

Q1, 2 Make sure trajectories NEVER intersect; do cross the nullclines at the correct angles; and there are no other places where a trajectory has a turning point (i.e. $dx/dt = 0$ or $dy/dt = 0$). Be accurate.

Q2 Don’t draw eigenvectors as long straight lines in a non-linear system’s phase portrait: they’re only like that in a linear system.

Q2 If a question asks for the general solution, don’t just give eigenvalues and eigenvectors on separate lines: it means an equation like:

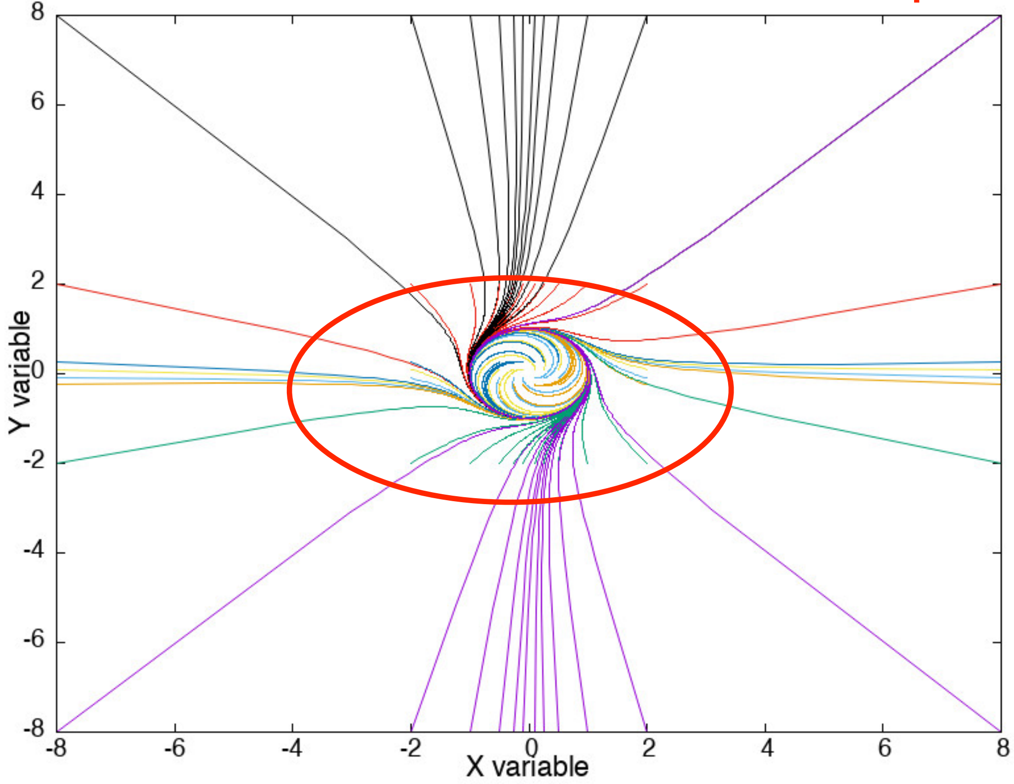
$$\underline{x}(t) = c_1 e^{3t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + c_2 e^{4t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Simple limit cycle

$$\frac{dx}{dt} = -y + x (1 - x^2 - y^2)$$
$$\frac{dy}{dt} = x + y (1 - x^2 - y^2)$$

$$dr/dt = r (1 - r^2)$$
$$d\varphi/dt = 1$$

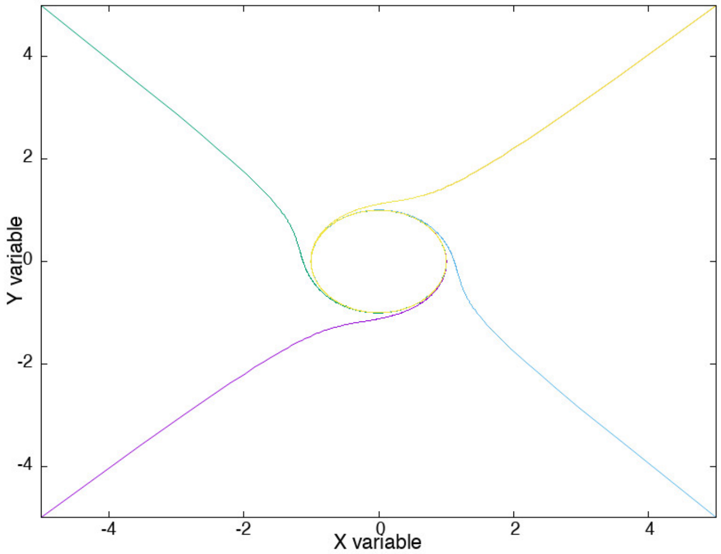
in plane polar coordinates



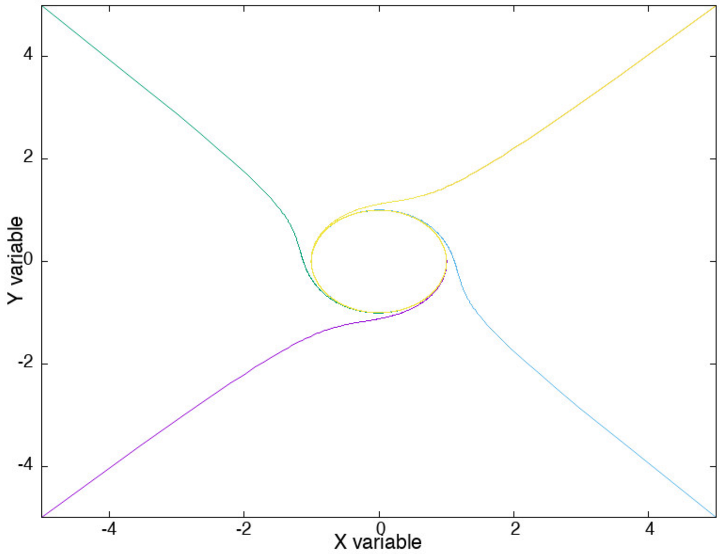
Trapping region for this case is any closed curve around the limit cycle

Why are trajectories at large x, y straight lines? Shouldn't they be rotating?

Close up



RK with 1000 points, dt = 0.01



10,000 points, dt = 0.001

$dr/dt = r (1 - r^2)$

$d\varphi/dt = 1$

For r >> 1, $dr \sim -r^3\, dt$ and is much greater than $d\varphi \sim dt$. Only when their magnitudes are comparable, do we see the curvature.

Note. Phase portraits only show the direction of trajectories not the speed along them.

Lecture 8

We have now seen all possible long-time behaviours of a 2D dynamical system: trajectories must:

- approach a fixed point (directly or spiral around it. NB. You never actually get there, only at time infinity or minus infinity.)
- or go to infinity
- or approach a limit cycle

- Limit cycles represent systems that can *oscillate* without an external driving force, e.g., heart contraction, chemical reactions, neurons
- The amplitude, frequency, and shape of a limit cycle are set by the equations not by the initial conditions; if perturbed, a system returns to the limit cycle (not so equilibrium reactions)
- How are fixed points/limit cycles created/destroyed? Bifurcations (back to 1D for a while …)

Background quiz: go.epfl.ch/turningpoint

Session Id: julian23

All input is anonymous; data are stored outside CH

Break

Tricky Points

In the tau-delta plot, if we are in the large, open areas, changing a parameter a little won't change a dynamical system's behaviour.

But if we are near a border … it can have catastrophic effects. Oscillatory behaviour can detach from a stable state, and blow up.

You must examine the full non-linear equations, NOT use the tau-delta plot.

Bifurcations

A bifurcation is a qualitative change in a system's behaviour as a parameter is changed by a small amount.

Fixed points can appear / disappear / change their type/stability.

There are only certain ways that FPs change; they occur in all dimensions, but 1D is easiest to visualise.