Final Exam

Friday, 16th January 2026

15:15 - 18:15

Rooms: SG 0211, SG 1138

Instructions for the procedure, seating plan, etc will be distributed nearer the time.

Background quiz: go.epfl.ch/turningpoint

Session Id: julian23

All input is anonymous; data are stored outside CH

Break

Lecture 7 Limit Cycles

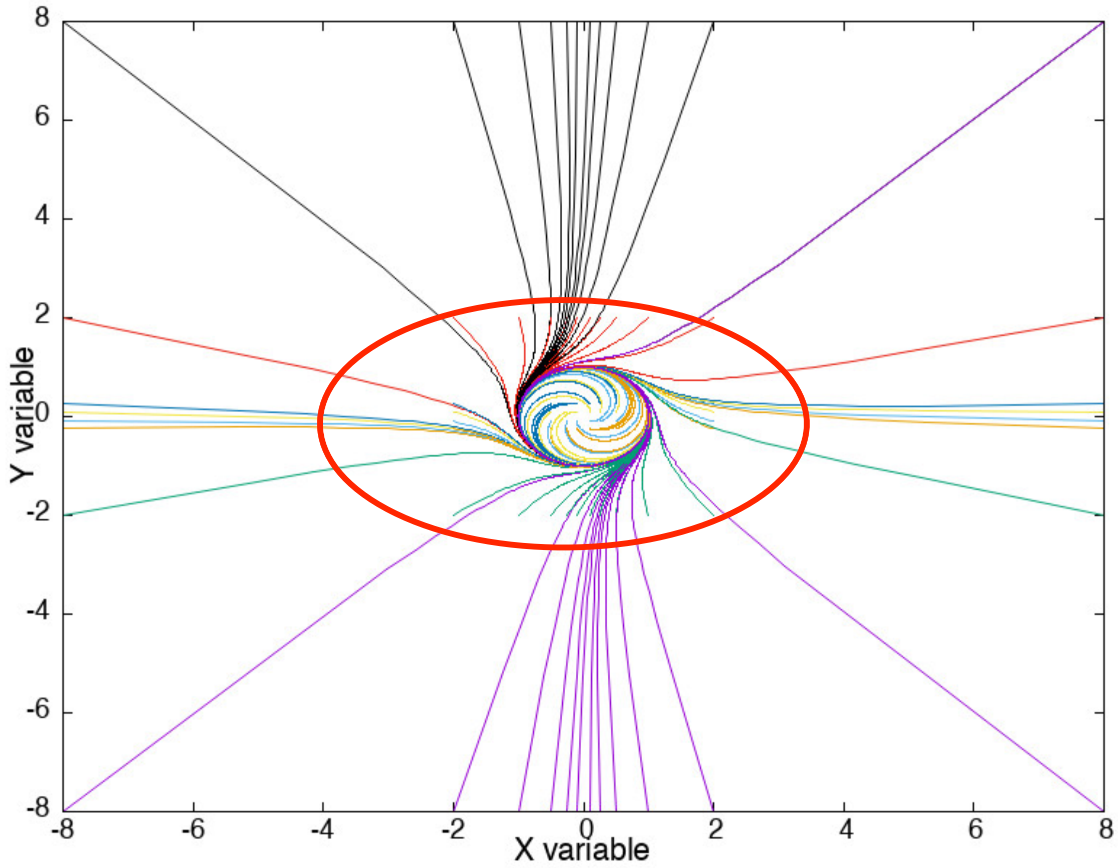
- Why do we care so much about fixed points? Because we want to understand how living organisms are so robust. FPs represent the long-time behaviour of a dynamical system.
  - Fixed points were the first attempt (but they're fixed!)
  - Hysteresis was next (stable state resists small perturbations)
- We have seen all types of fixed point in *linear* 2D systems (tau-delta plot), and these also occur in non-linear systems
- Limit cycles represent systems that can *oscillate* without an external driving force, e.g., heart contraction, neurons, electrical circuits …
- The amplitude, frequency, and shape of a limit cycle are set by the equations not by the initial conditions (cp centres); if perturbed, they return to the limit cycle

Simple limit cycle

$dx/dt = -y + x(1 - x^2 - y^2)$
$dy/dt = x + y(1 - x^2 - y^2)$

$dr/dt = r(1 - r^2)$
$d\varphi/dt = 1$

in plane polar coordinates



Trapping region for this case is any closed curve around the limit cycle

Q. Where are the nullclines?

Q. Why are trajectories at large x, y straight lines? Shouldn't they be rotating? (Think about this for next week)

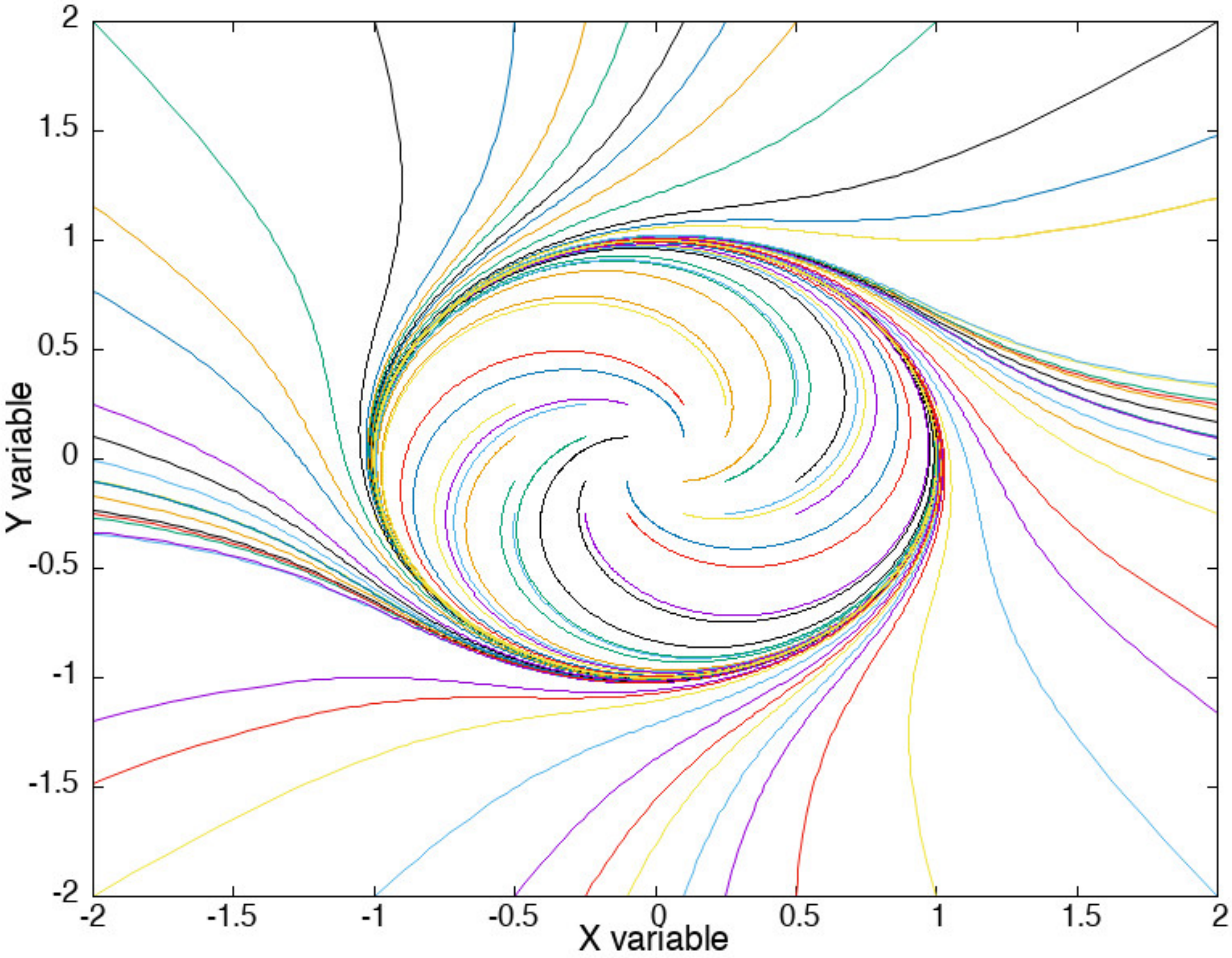
Where are the nullclines? They're clearly not the axes.

$dx/dt = -y + x (1 - x^2 - y^2) = 0$

$dy/dt = x + y (1 - x^2 - y^2) = 0$

They're cubic equations, e.g.

$y^3 - (1 - x^2) y - x = 0$ for $dy/dt = 0$

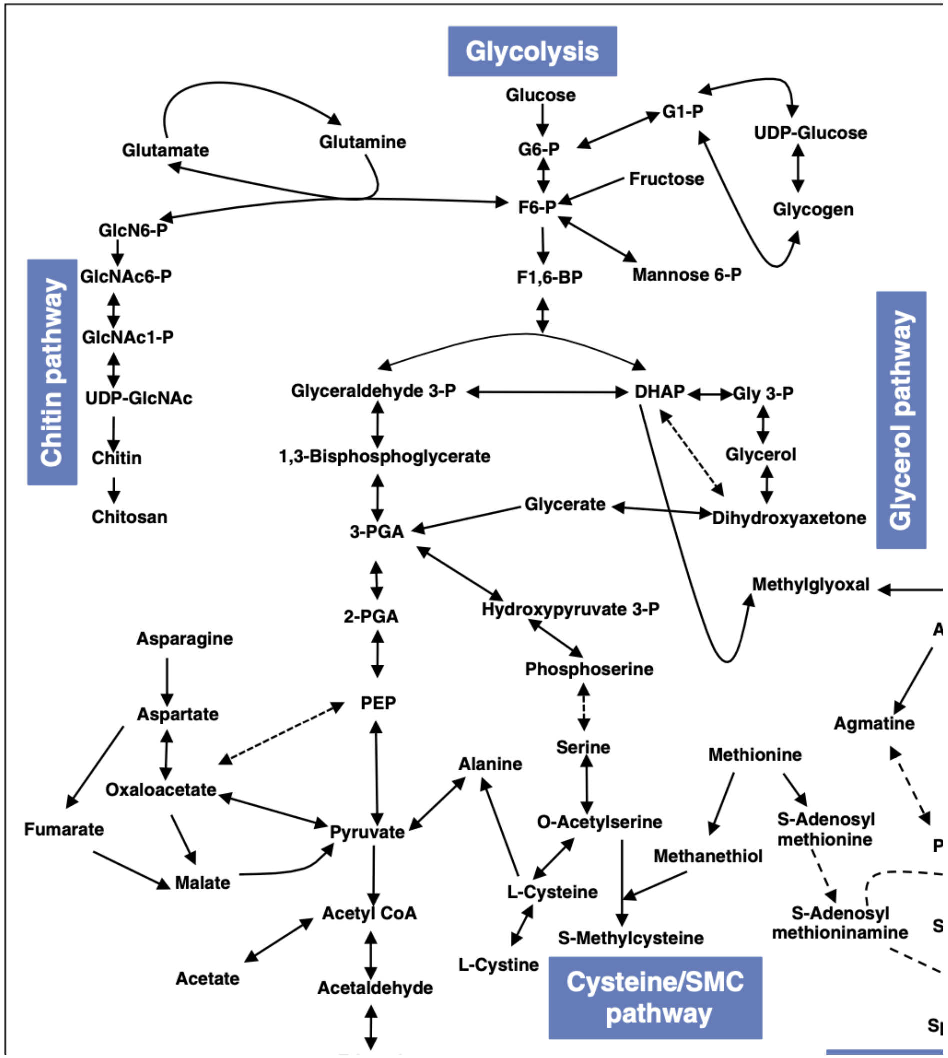


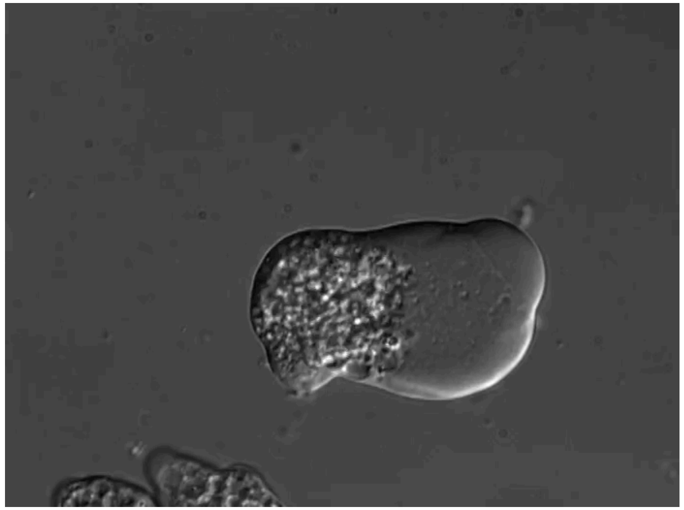
Go through:

(+/-1, 0) for x

(0, +/- 1) for y

Glycolysis (course notes Ch. 5.2)





Entamoeba histolytica

Anaerobic
No oxidative phosphorylation
No mitochondria

Produces ATP in cytoplasm

Metabolomic analysis of Entamoeba,
Jeelani and Nozaki, Current Op. Microbiol. 20:118 (2014)

Sel'kov model of glycolysis

Strogatz, Ex. 7.3.2, page 207, course notes Ch. 5, page 42.

Sel'Kov, Self oscillations in glycolysis. A simple kinetic model.
Euro. J. Biochem. 4:79 (1968)

ATP + F6P + E —> ADP + F1,6BP + E

Let x = [ADP] = product
y = [ATP] = reactant

The model is:

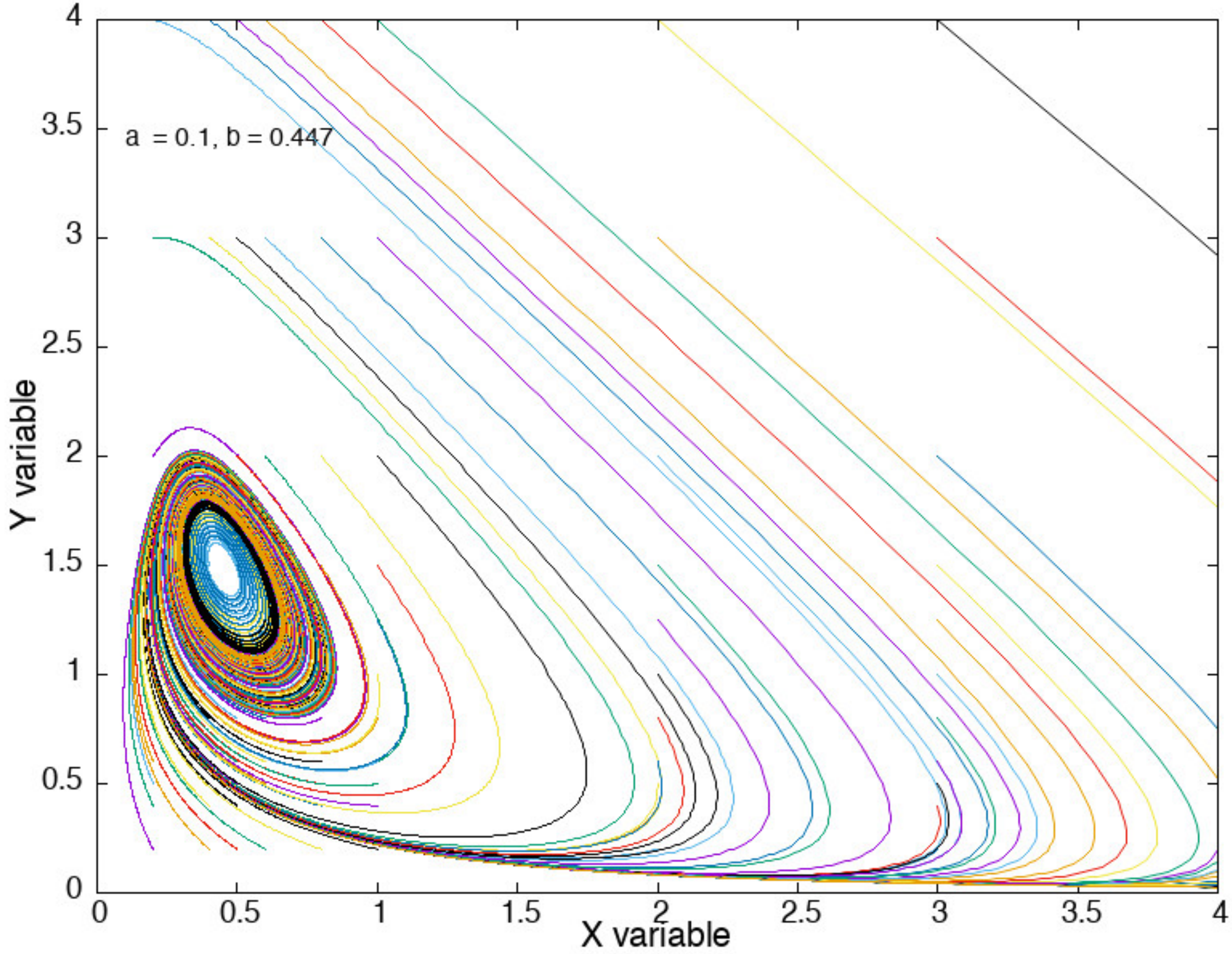
$$dx/dt = -x + a\,y + x^2 y$$

$$dy/dt = b - a\,y - x^2 y$$

with $a, b > 0$.

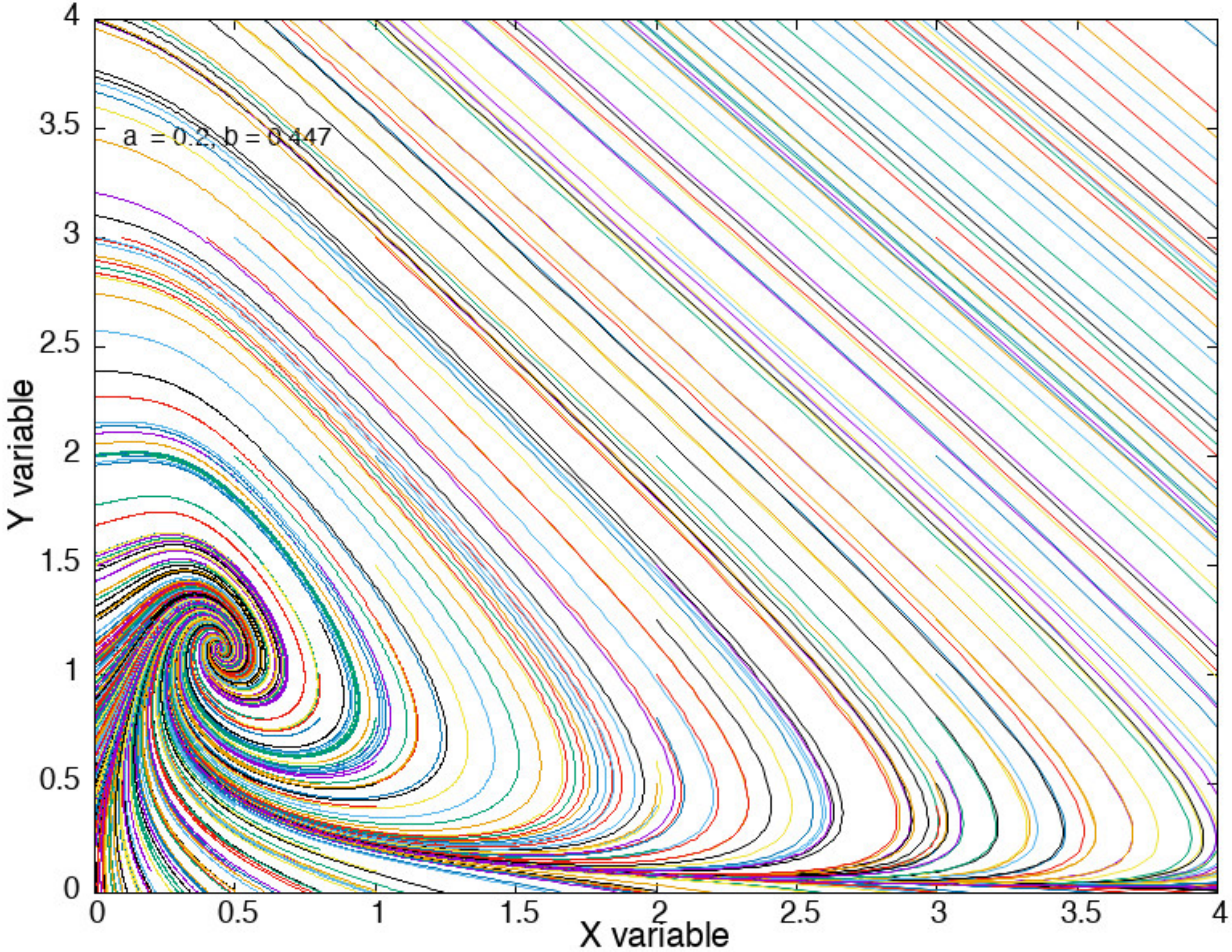
where do the oscillations come from?

Try a = 0.1, b = 0.447
Fixed point at (0.447, 1.49)



a = 0.2, b = 0.447

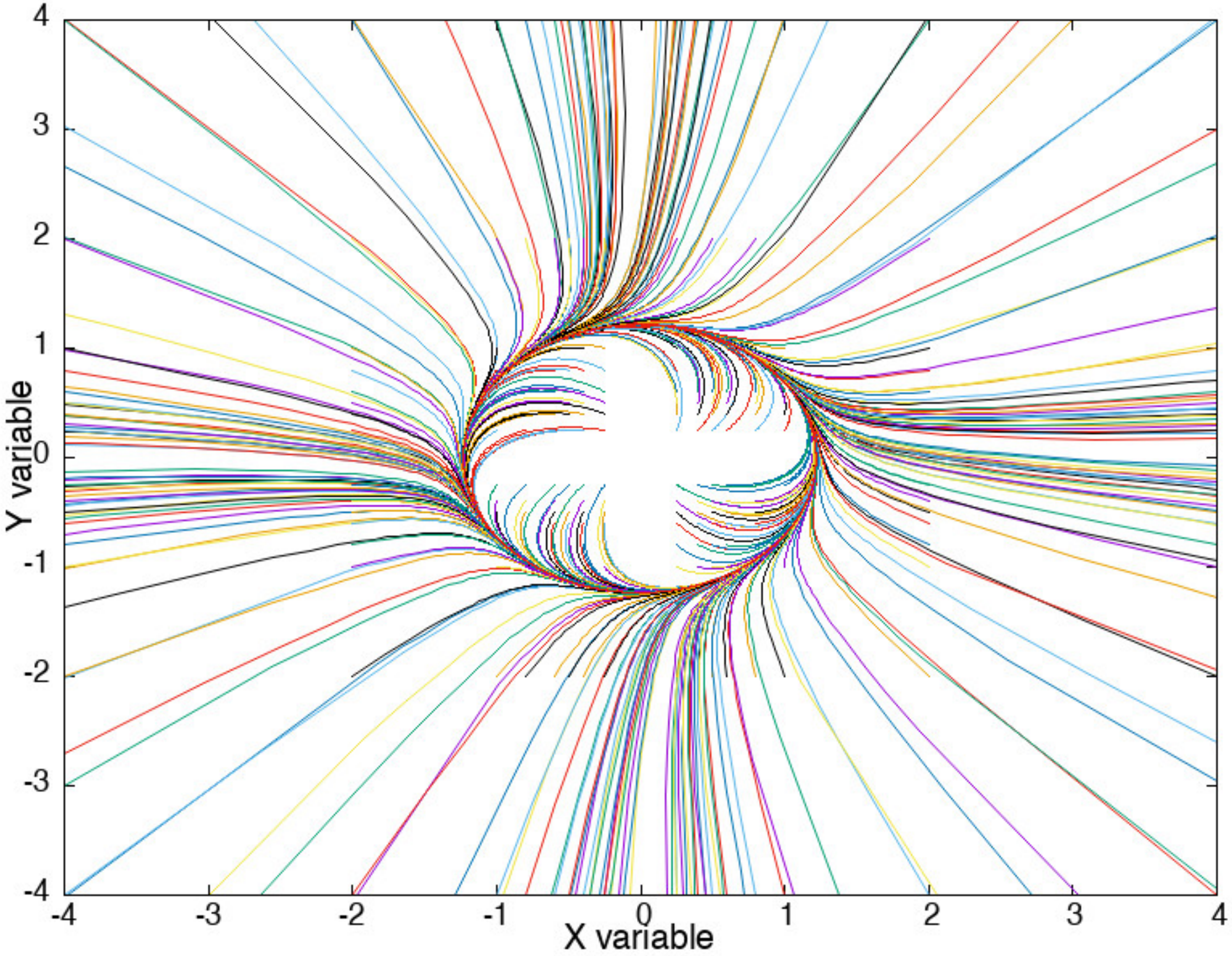
Fixed point at (0.447, 1.12) is now a stable spiral



I was playing with the equations, trying to find 2 limit cycles

and …

$$\frac{dx}{dt} = -y + x(1 - x^2 - y^2) + (x - 4)(1 - x^2 - y^2)$$
$$\frac{dy}{dt} = x + y(1 - x^2 - y^2) + (y - 4)(1 - x^2 - y^2)$$

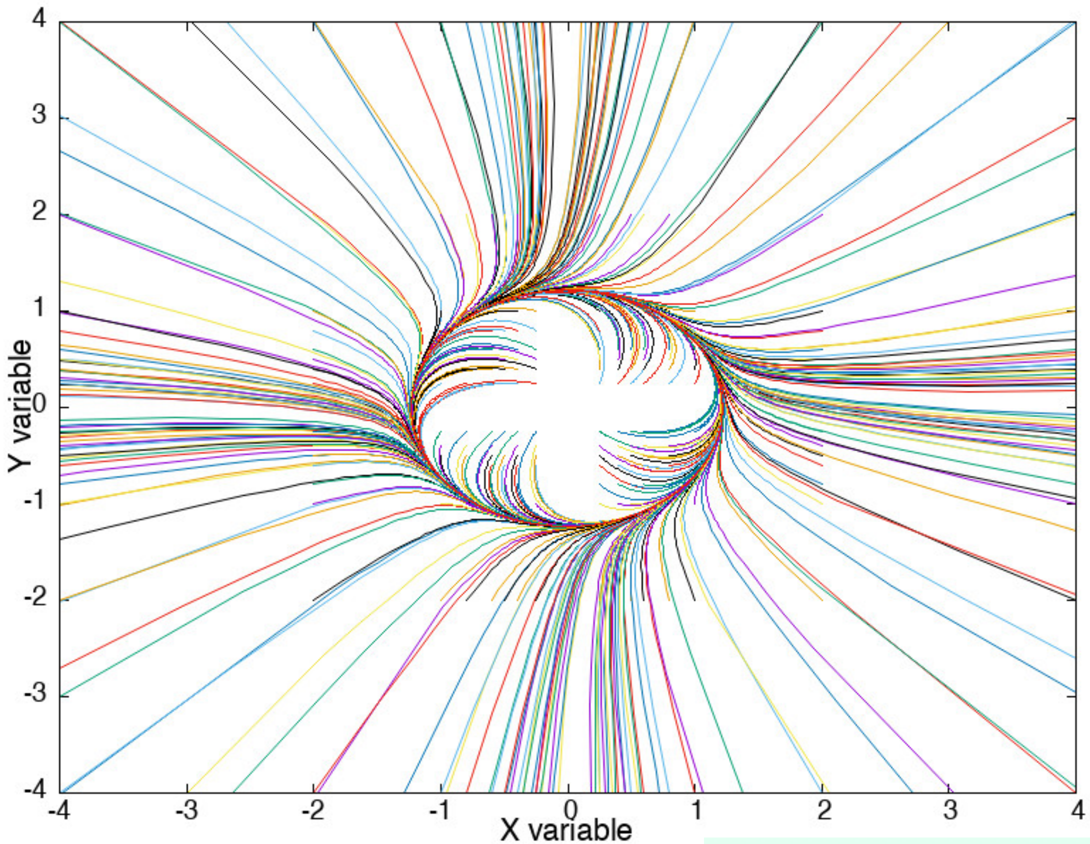


Challenge: Can you find 2 limit cycles?

… starting from …

Edible prize

$$\frac{dx}{dt} = -y + x\,(1 - x^2 - y^2) + \ldots$$
$$\frac{dy}{dt} = x + y\,(1 - x^2 - y^2) + \ldots$$



First 3 people/groups who send me the phase portrait and the equations

“Tricky” points

- Distinction between a limit cycle and a centre is that the limit cycle is isolated, whereas centres are closed cycles infinitesimally close together
- Poincare Bendixson theorem needs two things: a node/spiral inside a closed bounding curve / trapping region. Finding a trapping region is usually the harder part.
- Boundaries of a trapping region don’t have to be straight lines
- Check the field vectors on the corners of a trapping region: trajectories can squeeze through a tiny space (see slide 3 in the population model)
- Be able to find the direction of the trajectories for large X, Y coordinates far from any fixed point (e.g., curve 3 of the trapping region in the Selkow model)