Final exam

When?   Friday, 16th January, 15.15 - 18.15

Where?   SG 1138, SG 0211

(seating plan will be distributed by email and at the door, come ~ 15 minutes early to find seat, special arrangements will be communicated in advance)

Bring coloured pencils/pens, ruler, eraser, etc.

5 pages (10 sides) of A4 paper with handwritten/drawn/printed notes are allowed

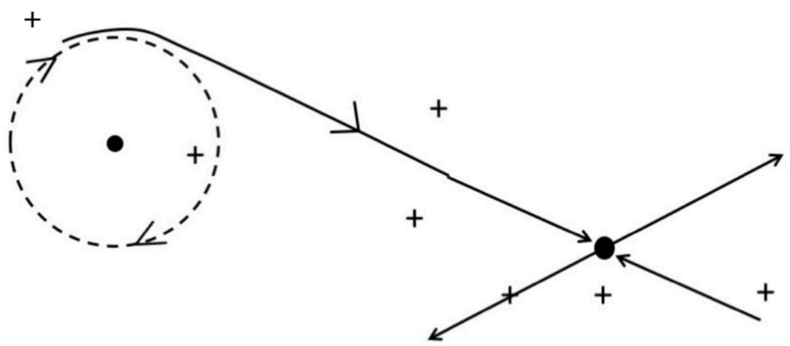
No calculators, phones, ipads, laptops, smart watches, etc.

Create your own exam question*

Question 4 (3 points)
Phase portrait with an unstable limit cycle

1. The dashed circle in the figure is an unstable limit cycle (with rotation in a clockwise direction). A saddlepoint with the indicated stable and unstable manifolds is shown nearby. The trajectory issuing from the limit cycle which forms the stable manifold of the saddlepoint is drawn.

   Draw the trajectories that start at the seven points marked with a + sign.



Draw 2, 3 or 4 fixed points (including spirals or limit cycles) in a phase portrait as for last year.

Mark 6 - 8 initial points by +

(Check that you can actually work out where trajectories from those points go)

If I get at least 20 possible questions, one of them will be on the exam (assuming it's not too easy, and that I can solve it.)

* I checked it is allowed by Lex rules.

What does chaos have to do with biology?

Some systems are inherently noisy or chaotic:

- turbulent rivers
- your heart rate (constant is bad, lack of flexibility)
- number of fish in a lake from year to year, etc.

If we assume that natural phenomena are governed by continuous differential equations, e.g., Navier-Stokes equations for fluid flow, how do they produce random fluctuations?

Period doubling provides a mechanism by which a deterministic system can become chaotic. The system jumps between FPs that are so close they are influenced by noise in the environment, and this makes the motion chaotic.

It appears that even highly complex natural systems, modelled by sets of ODEs, go along the same route to chaos as the simple 1D maps.

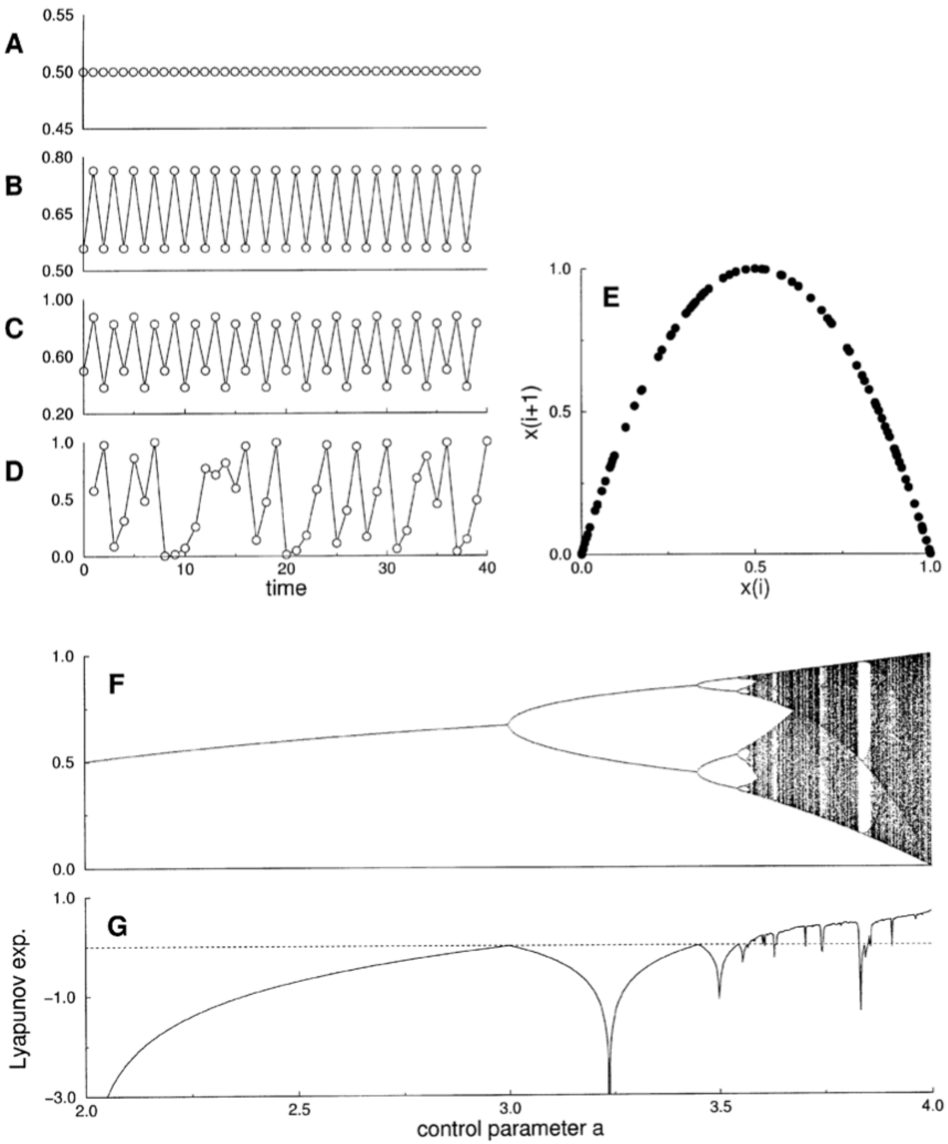


Fig. 1. The logistic equation $x_{i+1}=ax_i(1-x_i)$ with different values of bifurcation parameter a. Subsequent iterations x_i for increasing a give rise to the period-doubling route to chaos. (A) $a=2.0$ (period-1 orbit), (B) $a=3.1$ (period-2), (C) $a=3.5$ (period-4), and (D) $a=4.0$ (chaotic trajectory). When a is increased from 2 to 4, the system undergoes period-doubling bifurcations, i.e., the system switches to higher periodicities at certain parameter values. The right side (E) shows a plot

Cardiovascular Research 40 (1998) 257–264

Review

Chaos in the cardiovascular system: an update

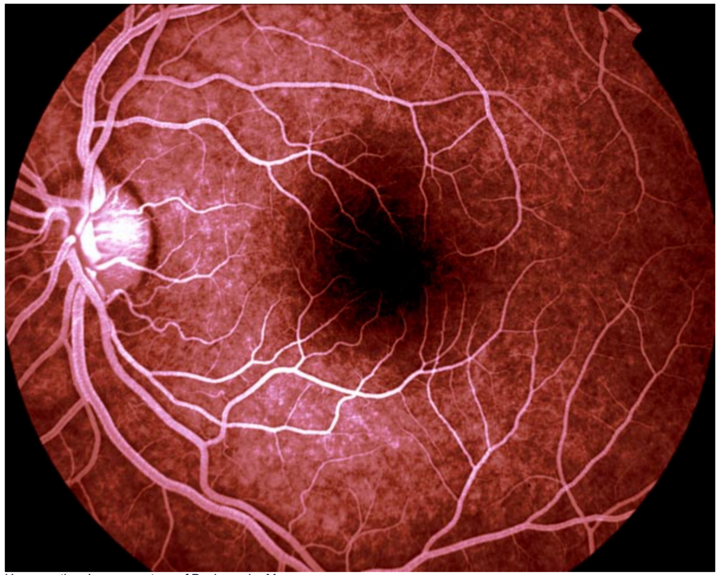
Claus D. Wagner, Pontus B. Persson*

Clinical Trial > Neuroimage. 2003 Nov;20(3):1765-74. doi: 10.1016/s1053-8119(03)00380-x.

Is the brain cortex a fractal?

Valerij G Kiselev [1], Klaus R Hahn, Dorothee P Auer

Fractal Blood Vessels

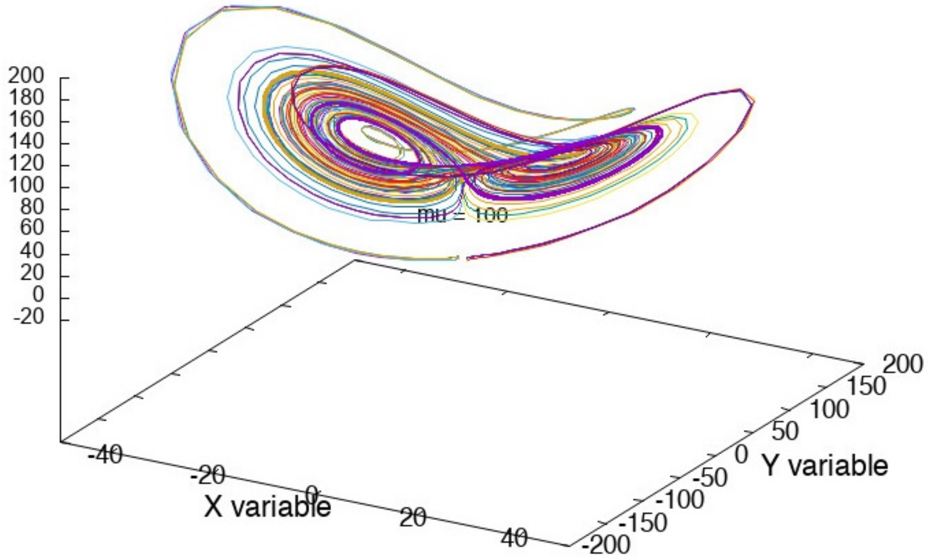


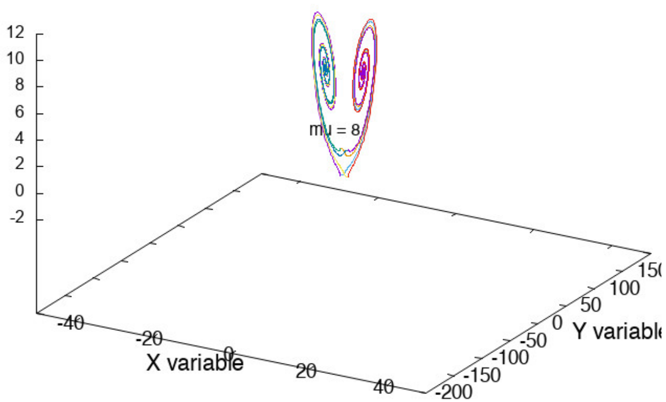
Human retina. Image courtesy of Paul van der Meer.

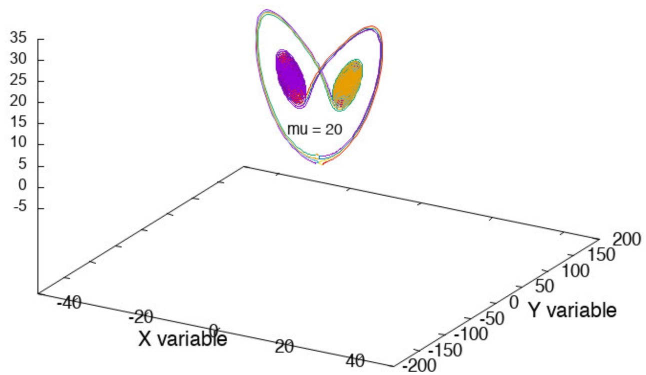
https://fractalfoundation.org

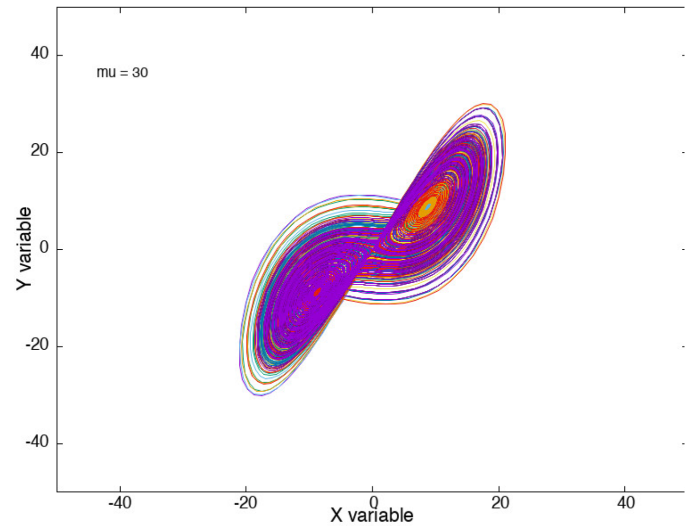
3D Runge-Kutta integration of the Lorentz attractor (on moodle for 9th December)

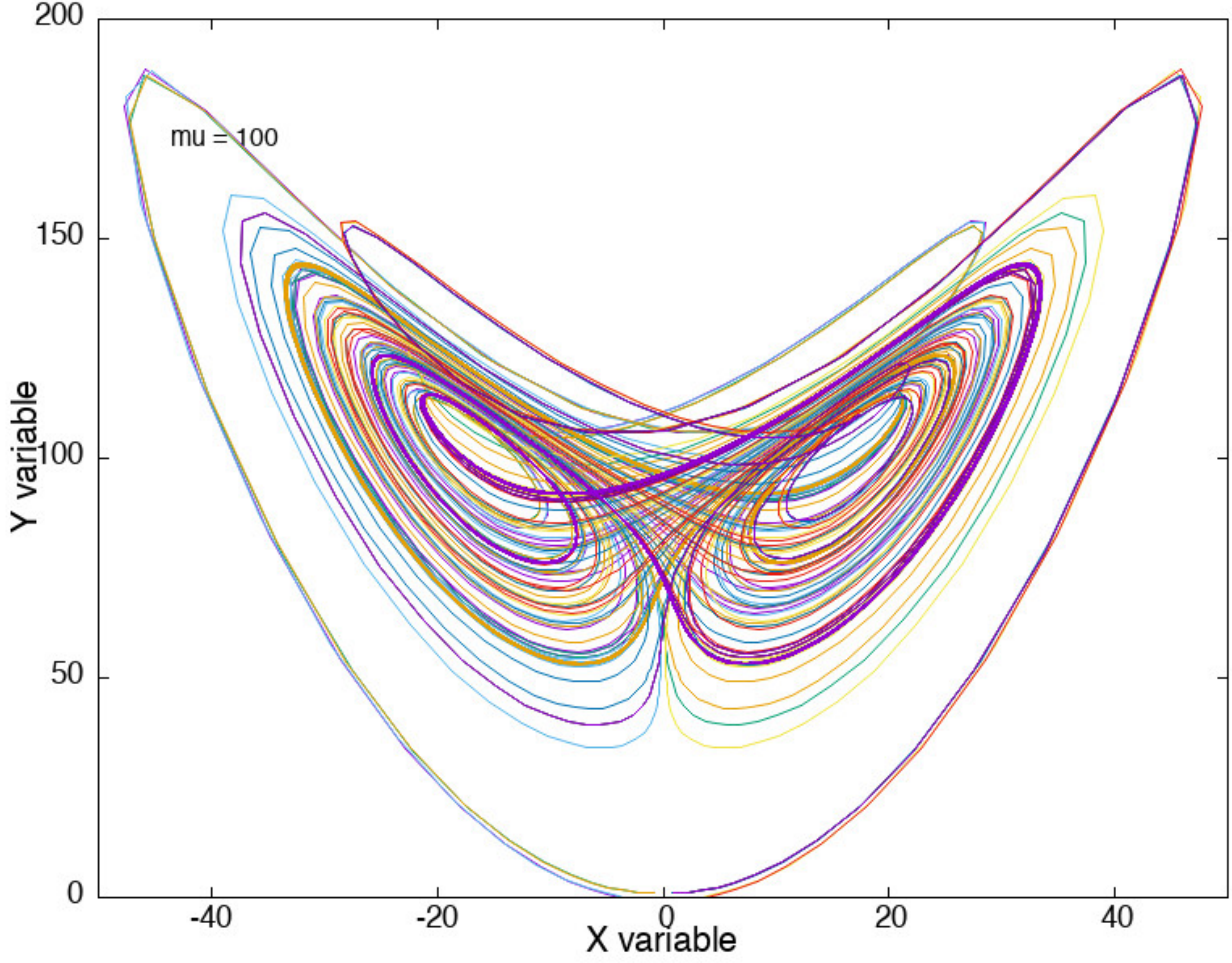
```
// ********************************************************************
// The trio of functions to return the time derivatives at a given time
//
// These are the Lorenz equations from page 319 of Strogatz.

double f1(const double a[6], double x, double y, double z)
{
    return 10.0*(y - x);
}

double f2(const double a[6], double x, double y, double z)
{
    return a[0]*x - y - x*z;
}

double f3(const double a[6], double x, double y, double z)
{
    return x*y - 2.6667*z;
```











mu = 100
Y variable
X variable
200
150
100
50
0
-40
-20
0
20
40

Background quiz: go.epfl.ch/turningpoint

Session Id: julian23

All input is anonymous; data are stored outside CH

Break

Comme vous le savez, tous les cours font l'objet d'une évaluation approfondie chaque semestre. L'enquête d'évaluation approfondie pour votre cours BIO-341_SA25/26 vient d'être ouverte aux étudiant·es et restera disponible jusqu'au 11.01.2026 23:59:00.

Le rapport sera disponible une fois la période de soumission de notes terminée, le 10 février 2026.

Les commentaires des étudiant·es vous seront plus utiles si le taux de réponse est élevé et nous vous recommandons donc, si possible, de consacrer 5 minutes au début ou à la fin d'un cours pour qu'ils et elles puissent répondre à l'enquête.
Les évaluations sont accessibles via la page d'accueil de moodle. Pour y accéder, les étudiant·es doivent :
Se connecter à moodle et rester sur la page d'accueil (tableau de bord, pas la page du cours).Cliquer sur la flèche en haut à droite de l'écran qui fera apparaître un bloc contenant la tuile intitulée "Évaluation approfondie" (veuillez noter que toutes les évaluations seront regroupées dans la tuile d'évaluation sur la page d'accueil de moodle, et non pas séparées dans chaque page moodle de cours).Les étudiant·es peuvent alors sélectionner votre cours et compléter le feedback.

Les étudiant·es peuvent également accéder aux évaluations de cours via l'application PocketCampus. Nous espérons que cela rendra les enquêtes plus accessibles et vous aidera ainsi à augmenter le taux de réponse.

Les enseignant·es pouvez accéder aux évaluations au même endroit sur la page d'accueil de moodle (Tableau de bord):
Accéder au taux de réponse pendant que l'évaluation est ouverte (Moodle - nouveau plugin). Accéder au rapport dès le 10 février 2026.

As you know, all courses receive an in-depth evaluation each semester. The in-depth evaluation survey for your course BIO-341_SA25/26 has just been opened to students and will remain available until 11.01.2026 23:59:00.

The report will be available to you after the deadline for entering the grades, on 10 February 2026.

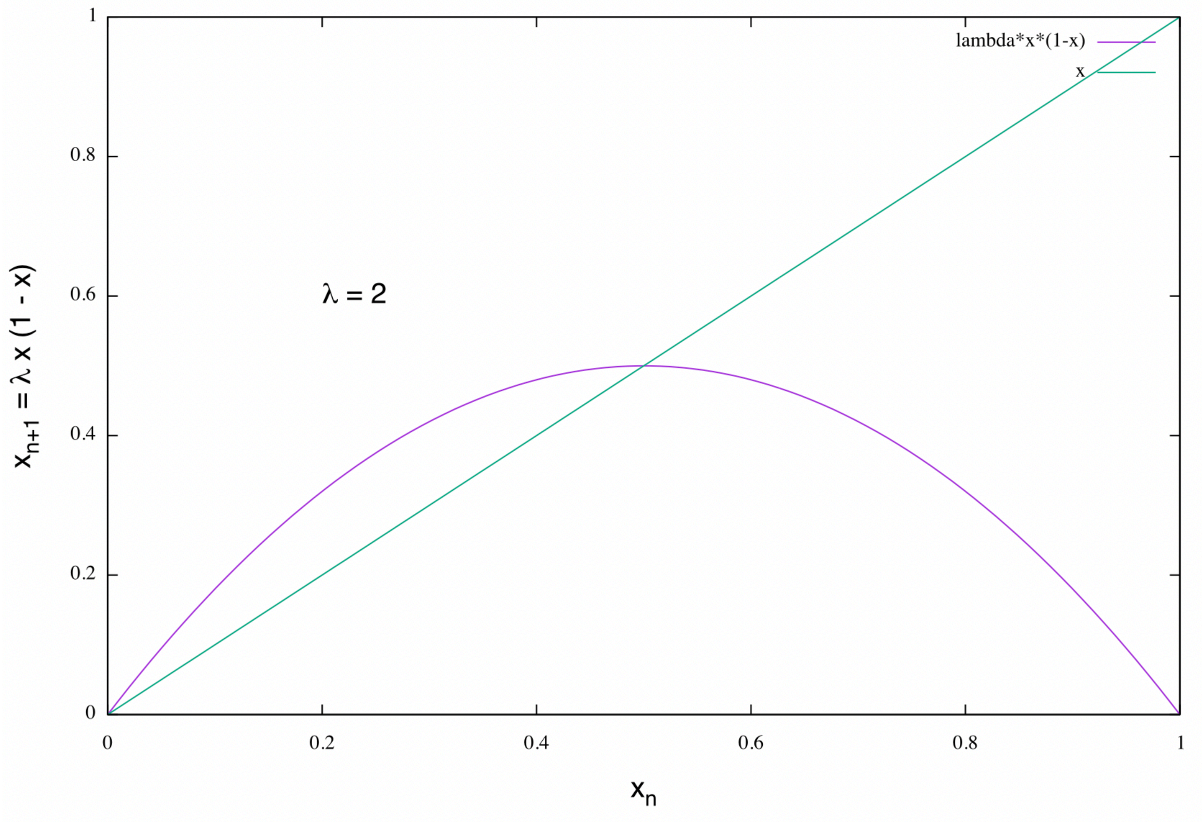
The student feedback will be more useful to you if the response rate is higher and so we recommend that, if possible, you dedicate 5 minutes at the beginning or end of a class for students to complete the survey.
The evaluations are accessible via the moodle. To access them, students have to:
Log onto moodle and stay on the moodle home page (dashboard, not the course page).Click on the arrow to the top right of the screen which will reveal a block that contains the entitled "In-depth evaluation" tile (please note: all evaluations will be together in the evaluation tile on the moodle home page, and not separate in each course moodle page). Students can then select your course and complete the feedback.

Students will also be able to access the course evaluations via the EPFL Campus App. We hope this will make the surveys more accessible and so help you to increase the response rate.
Teachers can access evaluations in the same location on the moodle home page. You will be able to:
Access the response rate while the evaluation is open (Moodle - nouveau plugin). Access the report from 10 February 2026.

If you share teaching responsibilities for this course, please inform your colleagues since, in order to avoid many teachers getting multiple emails, only one teacher per course has received this notification.

Iterating the discrete logistic map

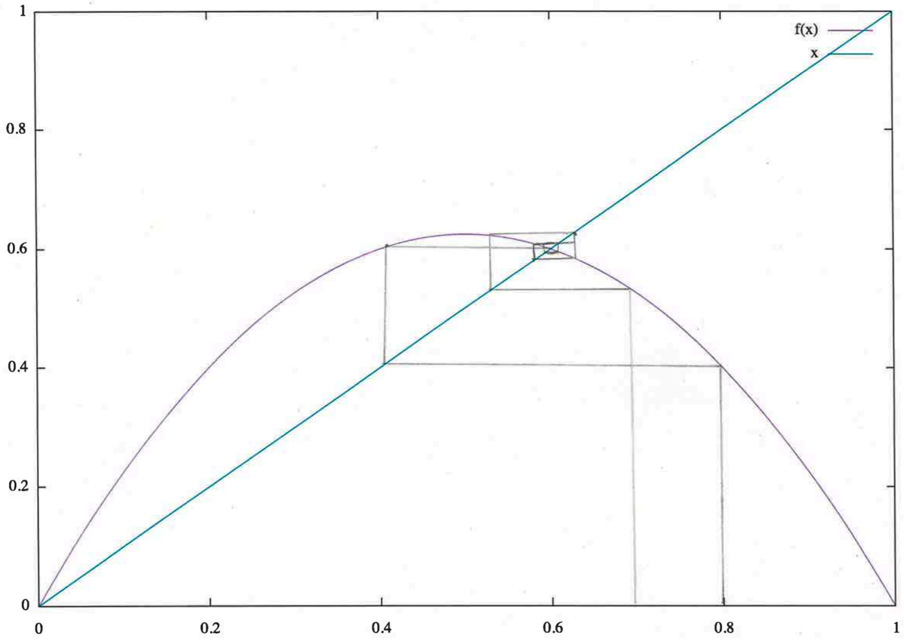
$$x_{n+1} = \lambda x_n (1 - x_n)$$

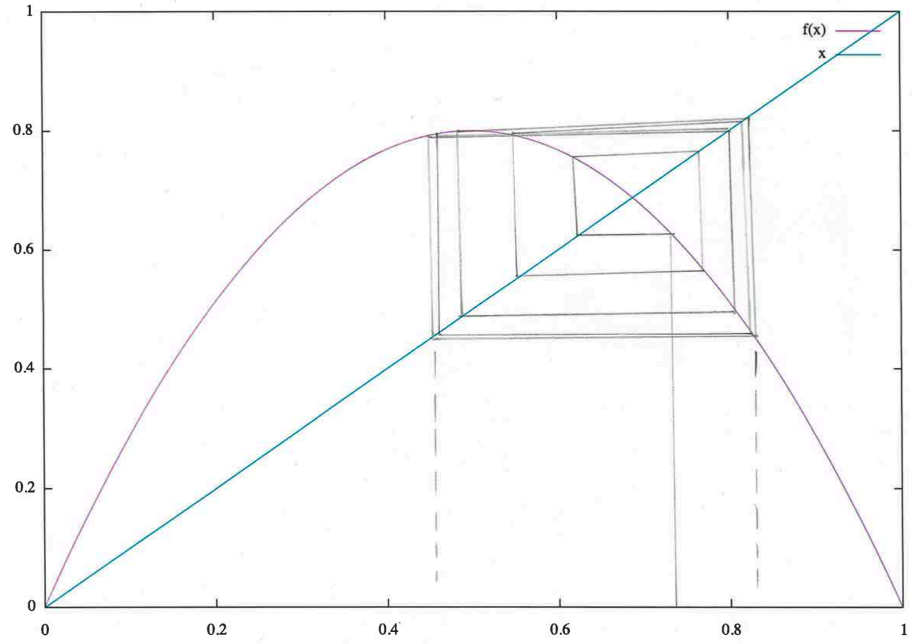


For a given value of λ in the range (0, 4), pick a (random) starting value x_0, in the range (0, 1), and apply the above map to get x_1. Repeat N times.

If there is a stable fixed point(s), the points will approach it

When is a fixed point in a discrete map stable?





r = 2.5

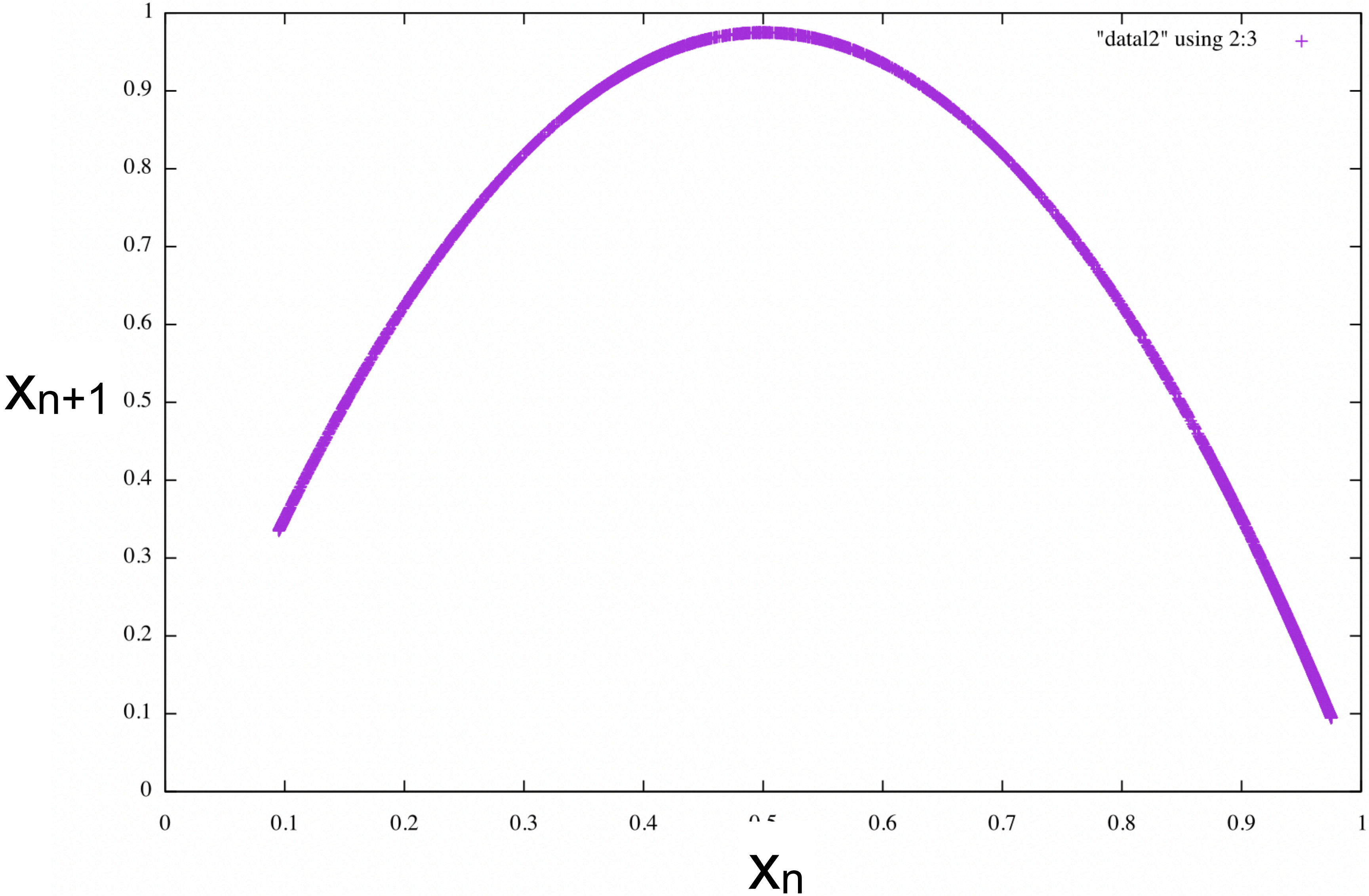
r = 3.2

Cobweb plot

Choose an x_n, go vertically to $x_{n+1} = f(x_n)$, then horizontally to x_{n+1} (diagonal), then vertically again to $f(x_{n+1})$, etc.

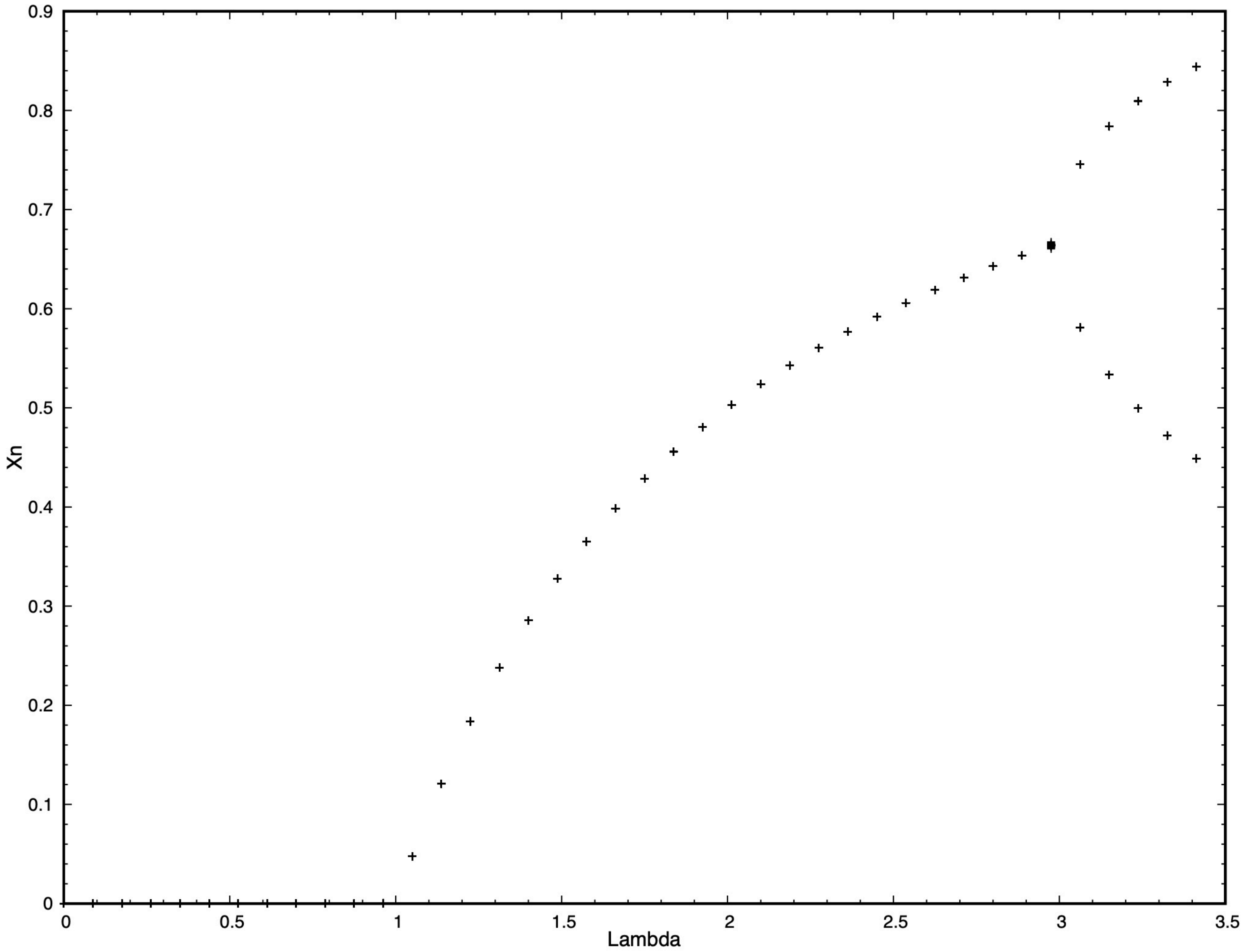
Either you spiral closer to the fixed point (stable), or you don't (unstable).

Return map for logistic map with r = 3.9, 5000 points ignoring first 1500

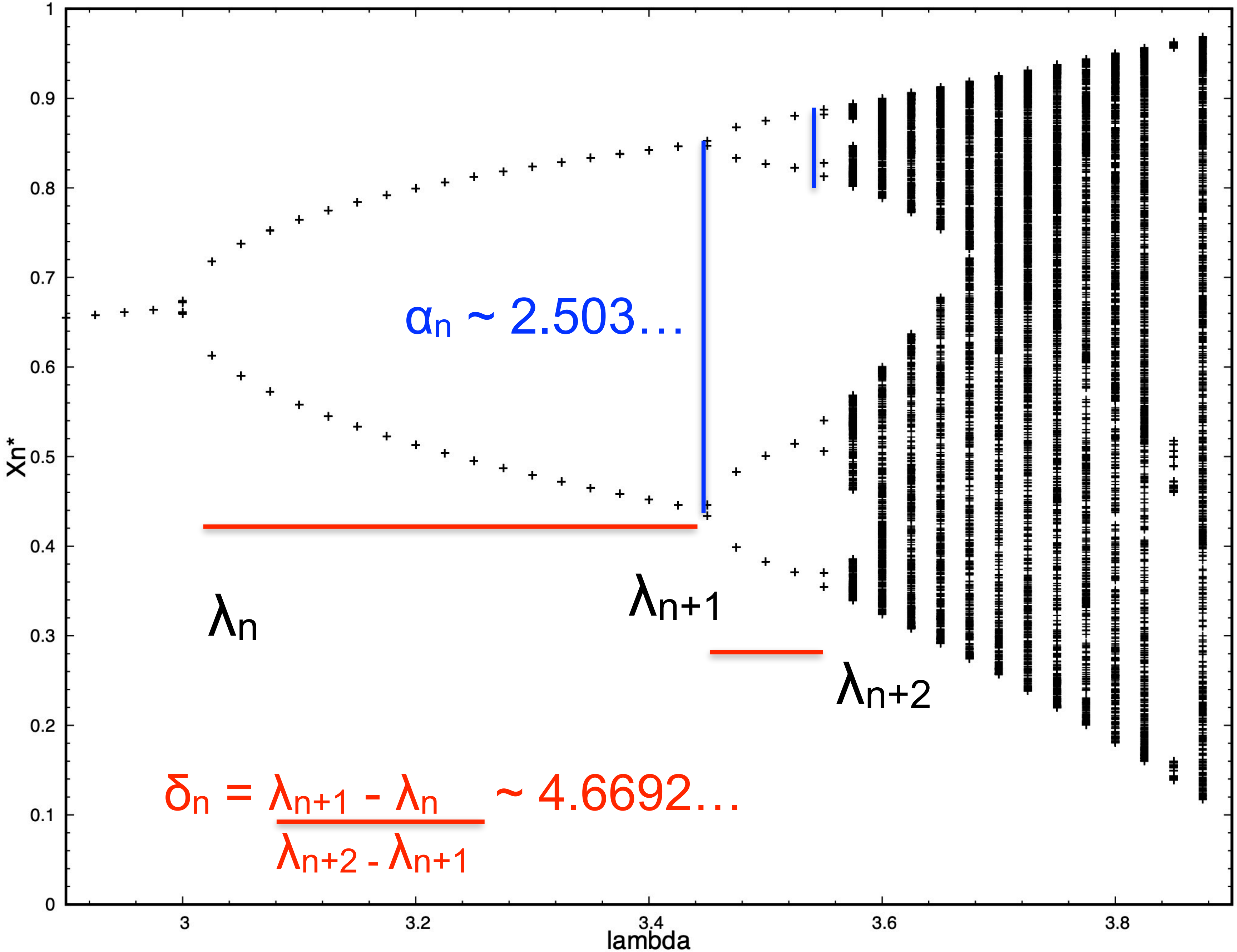


$x_{n+1} = \lambda x_n (1 - x_n)$

Iterate this for large n ~ 1000, and plot x_n against λ



the period doubling route to chaos



Feigenbaum numbers

$$\delta_n = \frac{\lambda_{n+1} - \lambda_n}{\lambda_{n+2} - \lambda_{n+1}} \sim 4.6692\ldots$$

$$\alpha_n \sim 2.503\ldots$$

δ tells you that to reach the next splitting you must increase the control parameter by about 1/5 of its previous increment.

Splittings get closer together!

α gives the ratio of the separation of the new fixed points.

This is called Period Doubling

Universality in Chaos, P Cvitanovic, Adam Hilger 1984

Exercise 13 explores the sine map

$$x_{n+1} = r \sin(\pi x_n)$$

For a given value of r in the range (0, 1), pick a (random) starting value x_0, also in (0, 1), apply the above map to get x_1, then repeat N times.

If there is a fixed point(s) in the map, the points will approach it.

How are the fixed points in the chaotic regime distributed?

“Tricky” points

- 3D system of ODEs is much harder than 1D, 2D because trajectories can “escape” from nearby ones
- A Poincare map transforms a 3D continuous problem into a 1D discrete iterated map
- Different 3D systems can have the same return map, and so be modelled by the same 1D iterated map
- Some 3D systems go chaotic via a process of period doubling, where their trajectories just “miss” being periodic as a parameter changes