Recap of Lecture 1
Recipe for 1D, autonomous non-linear ODE

1) Given the 1D equation: $dx/dt = f(x)$, draw the graph of $f(x)$ against x (**Graph 1**)

2) Find the points x^* where $f(x^*) = 0$ (i.e., crosses the X axis), these are the fixed points $\{x^*\}$ of the system

3) Mark the direction of flow of the vector field dx/dt with arrows on the axes in all relevant regions

4) For each fixed point, x^*, evaluate $df/dx(x^*)$, and classify the stability according to the **sign** of $df(x^*)/dx$. Or use the graphical method of 3).

5) Draw some typical trajectories starting from various values of x_0 (**Graph 2**)

6) Integrate the equation explicitly if possible

Difficulties? Silly mistakes? Tricky points?

These are NOT the only important points in the lecture, but experience has shown that some people forget them.

- Be able to relate Graph 1 to Graph 2, i.e., go from the vector field to trajectories
- In 1D trajectories are monotonic to a fixed point or infinity (i.e., cannot pass through a fixed point)
- Trajectories never cross (but do meet at fixed points)
- Adjacent fixed points have opposite stability
- What about adjacent *semi-stable* fixed points?

Background quiz

Background quiz: go.epfl.ch/turningpoint

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All input is anonymous; data are stored outside CH

Break

Presentation

Tricky points

- Population models live in the first quadrant only
- For autonomous ODEs like $dx/dt = F(x)$, the vector field dx/dt depends only on the value of x NOT the time when the system arrives at x nor the initial condition x_0
- To model a dynamical system, we need to find an equation with the same *qualitative* behaviour as the real system. This is called being in the same "universality class."

  e.g., There is no *qualitative* change in the population dynamics equation if we change the Logistic equation to: $dN/dt = r N^2(1 - N / K)$? Only the *detailed* time-dependence will be different. (*this is not really tricky, just a reminder*)

Non-dimensionalising an equation with several parameters.

If we can eliminate a parameter it, it tells us 3 things:

1) the parameter is not really important; it doesn't determine the *shape* of the solution, only specific details, like a time scale, or the absolute scale of a variable for specific cases.

2) Removing a parameter makes it easier to understand the system, because we can explore how the behaviour depends on the remaining parameters. This is much harder when there are 3, 4, or 5 parameters.

3) Fewer parameters = fewer mistakes in algebra.