

BIO-341 Dynamical systems in biology 2025

4 credits

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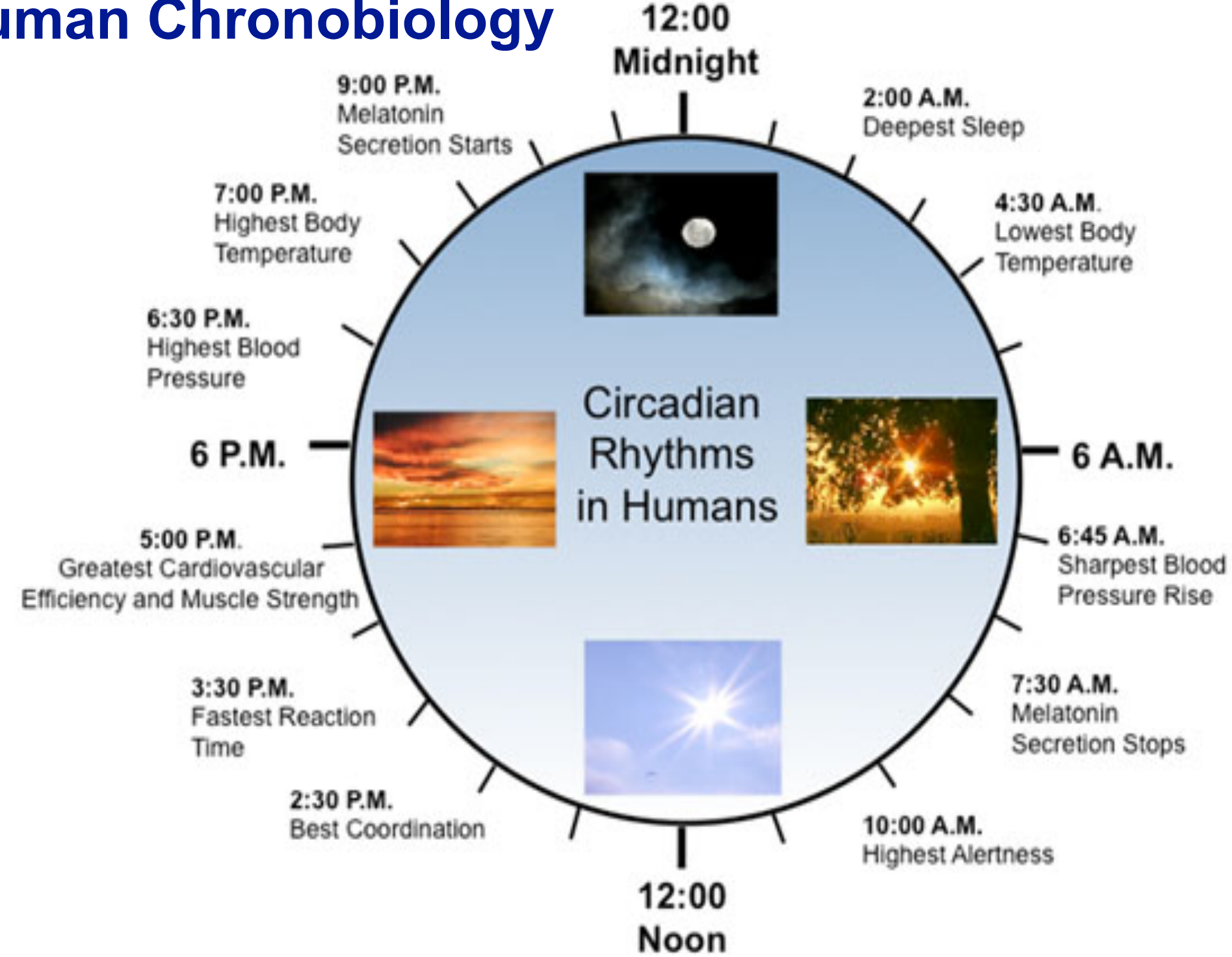
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Computational biology, modelling, bioinformatics,
gene regulation, biological oscillators, circadian rhythm, chronobiology

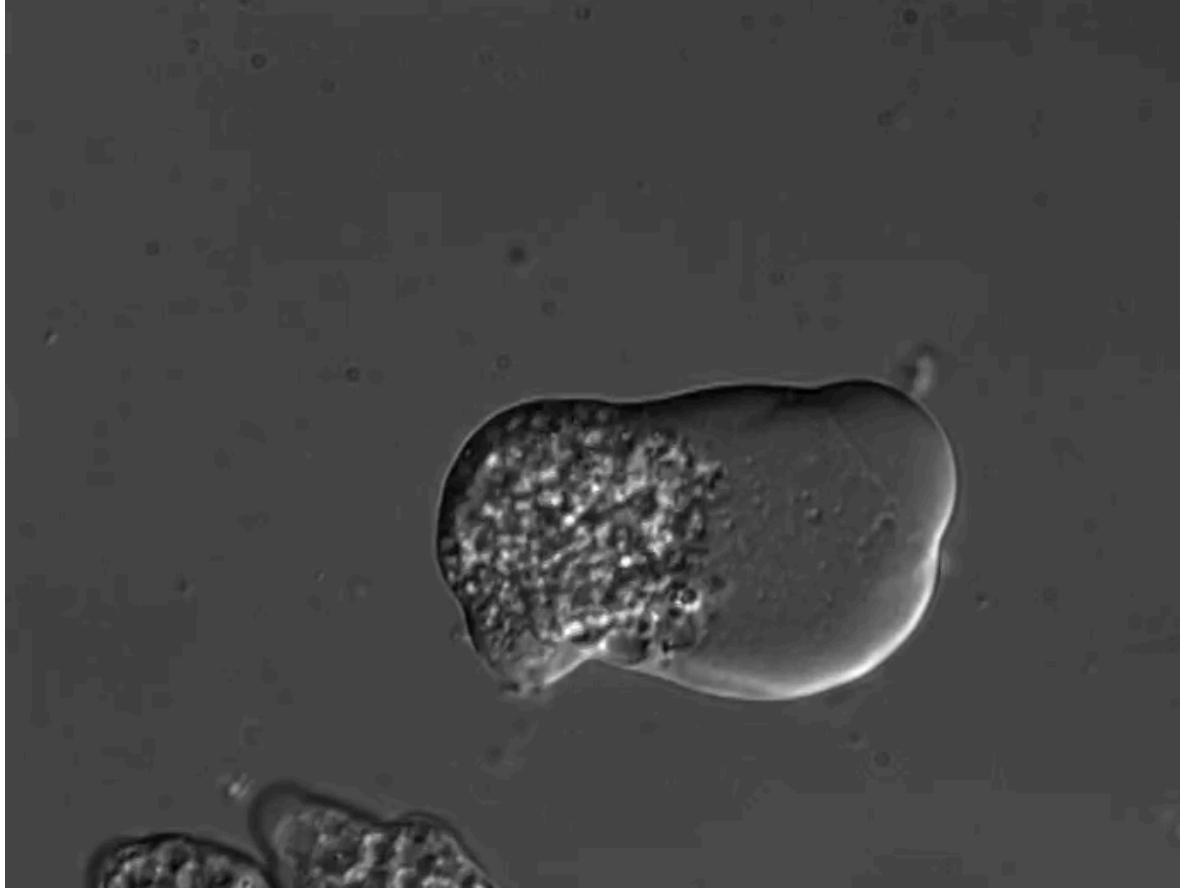
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no need to write this information, the slides will be on moodle

Human Chronobiology



This course is about life, although it might not look like it at first
— *there's a lot of equations*



Entamoeba histolytica

- anaerobic protozoan,
- no mitochondria
- produces ATP by glycolysis

hyaline - clear cytosol
vesicle-filled - granular cytosol
leading edge - lobopod

Differential interference
microscopy, 5x speedup

The Roberto Stock group at IBt-UNAM

We're not looking at biology in a microscope ...

... we're looking at it through equations ... why?

Example of a model system: Circadian oscillator

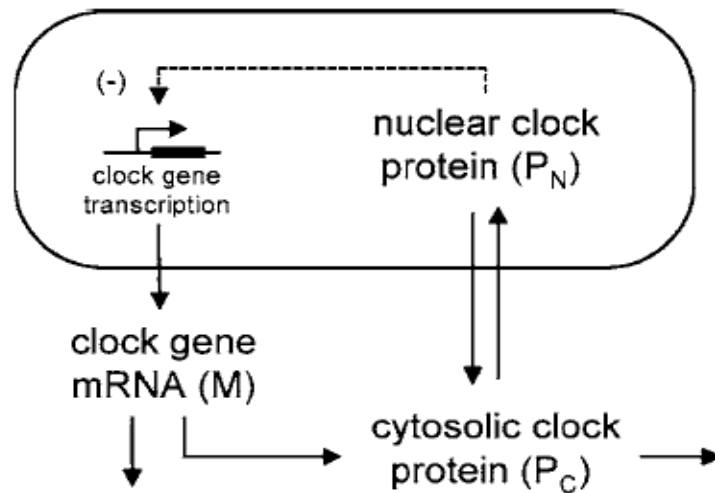
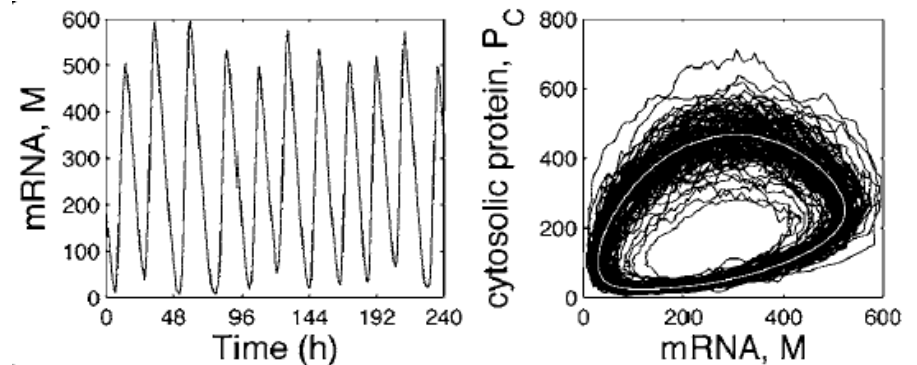


FIG. 1. Scheme of the core model for circadian oscillations. The model is based on the effect exerted by a clock protein (P_C) which can enter the nucleus (P_N) to repress the transcription of its gene into mRNA (M).

$$\left| \begin{aligned} \frac{dM}{dt} &= v_s \frac{K_I^n}{K_I^n + P_N^n} - v_m \frac{M}{K_m + M}, \\ \frac{dP_C}{dt} &= k_s M - v_d \frac{P_C}{K_d + P_C} - k_1 P_C + k_2 P_N, \\ \frac{dP_N}{dt} &= k_1 P_C - k_2 P_N. \end{aligned} \right.$$



What can this course be used for?

- How is gene expression switched on and off *reliably* given that levels of proteins are *continuous* functions? (**cell biology**)
- Will two or more populations competing for resources reach a stable equilibrium, or go extinct? (**ecology**)
- Why did the Tacoma narrows bridge collapse? (**mech eng**)
- Why is flying a plane too fast dangerous? (**aerodynamics**)
- Why do weakly-coupled oscillatory systems (fire flies, metronomes) synchronise? (**mathematical modelling**)
- Is there a limit to how accurately we can describe the world using equations? aka non-linear equations are very strange (**philosophy**)



TACOMA BRIDGE COLLAPSE

Organisation Autumn 2025

- **Lectures**

- Tuesday 15.15-17h, Room **CE 15**
- Course notes (PDF, etc) on Moodle

- **Exercises**

- Friday 10-12h **CM 3**
- **Ed** (cf. Moodle Forum)
 - Asynchronous discussion channel
- Exercises/Jupyter notebook/python tutorials on moodle.

Exercises

- 12 sets
 - Theory (pencil and paper)
 - Programming (jupyter-notebooks)
 - Appear on moodle each Tuesday after lecture, solutions posted after Friday's exercise
- The exercises are **crucial** in order to pass the written exam (which is pencil and paper only!)

Assistants (1 PhD student + 4 SA):

Responsible person: yves.paychere@epfl.ch

Evaluation

- Final written exam in January 2026 (80% of grade)
- Graded exercises
 - 2 graded exercises (20% of the grade)
 - weeks 5-6 and 12-13, to be confirmed

Prerequisites

- analysis, linear algebra
- biology, programming, physics

Methods and techniques developed

- 1st order ordinary differential equations (ODE)
- qualitative (graphical) analysis of ODEs
- numerical simulations (cf. Numerical Analysis)
- programming in python, jupyter-notebooks

PDF notes (moodle)

any exchange students who cannot access moodle, send me an email

<http://moodle.epfl.ch> (ModelBio, enrollment key: modelbio99, but you should already be added)

- exercises + solutions (PDFs, jupyter notebooks)
- graded exercise solutions to be uploaded to moodle
- course notes in PDF + supplementary material
- Ed discussion forum on moodle
- tricky concept(s) for each lecture (not the *only* important point, just the one(s) that seem difficult)

Books

- **S. Strogatz, Nonlinear Dynamics and Chaos (Perseus)**
(Copies are available in the section — use them! it has many good examples)
- A.W. Murray, Mathematical Biology (Springer)

Learning objectives

- Understand the basics of dynamical systems in 1D and 2D, in particular the *qualitative* analysis of ordinary differential equations (ODE): why? because different physical systems map to the same model
- Introduction to modelling in biology
⇒ systems biology:
How do you apply mathematical, physical, and computational tools in biology?
- **Interdisciplinarity**, the course combines elements of mathematics, physics, biology, and numerical analysis (in the exercises)
- **Learn how to construct dynamical models** of simple biological processes, identify their dynamics and dependence on parameters;
- This course lays the foundation for learning quantitative biology, and synthetic biology (BIOENG-320, project iGEM)

Background quiz

Background quiz: go.epfl.ch/turningpoint

Session Id: [julian23](#)



All input is anonymous; data are stored outside CH

Break

Difficulties? Silly mistakes? Tricky points?

These are NOT the only important points in the lecture, but experience has shown that some people forget them.

- In 1D trajectories are monotonic to a fixed point or infinity
- Be able to relate Graph 1 to Graph 2, i.e., go from the vector field to trajectories
- Trajectories never cross (but do meet at fixed points)
- Adjacent fixed points have opposite stability
- What about adjacent *semi-stable* fixed points?

#	Date	Content	Exercise
1	Sep 9	Dynamical systems in 1D, fixed points; linear stability analysis; Graphs 1 and 2, Recipe in 1D	Growth models in 1D, Gompertz model
2	Sep 16	Population models; logistic equation; 2 alleles problem	Allee effect, dimers
3	Sep 23	Autocat. 1D genetic switch; hysteresis; Newton's method for finding zeroes of a function	Insect outbreaks
4	Sep 30	Linear 2D systems; phase portraits; classification of fixed points; Recipe for linear systems in 2D	Linear 2D systems, gene networks
5	Oct 7	Non-linear 2D systems; Jacobian at a fixed point; Euler and Runge-Kutta numerical integration schemes	First graded exercise (1 week allowed)
6	Oct 14	Non-linear 2D systems; predator-prey models, spirals; Recipe for non-linear systems in 2D	Population dynamics with a limit cycle
	Oct 21	Semester break - no lecture / exercise period	
7	Oct 28	Limit cycles in 2D; Poincaré-Bendixson theorem; Biological oscillators (Selkow model)	Biochemical circuit with feedback
8	Nov 4	Bifurcations in 1D and 2D; saddlenode, transcritical, pitchfork; simple 2D model	Bifurcations
9	Nov 11	Genetic control system in 2D	van der Pol oscillator
10	Nov 18	Forced oscillators, entrainment (Felix Naef)	Felix question
11	Nov 25	Kuramoto model (Felix Naef)	Model exam question
12	Dec 2	Hopf bifurcation in 2D; Super- and sub-critical Hopf bifurcation, hard/dangerous transitions	Second graded exercise (1 week allowed)
13	Dec 9	Chaos; Discrete logistic/sine map; period doubling	Model exam question
14	Dec 16	Recap of course	