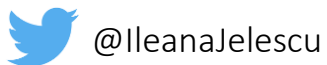


Principles and applications of diffusion-weighted imaging

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Microstructure Mapping Lab



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UNIL | Université de Lausanne

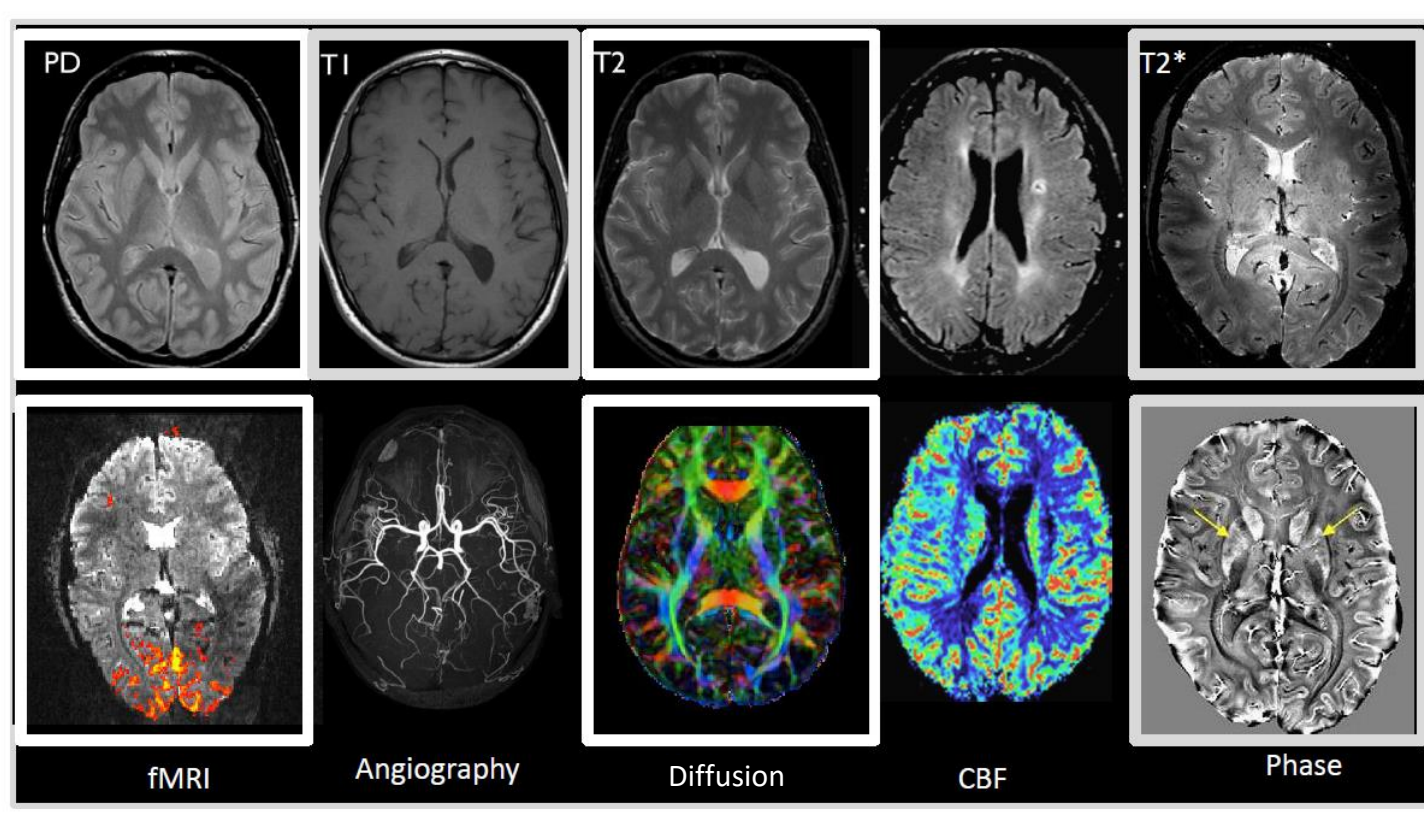
Faculty of Biology and Medicine



MRI – imaging water

Image contrast can be manipulated to be sensitized to a variety of features

- Depending on: chemical & magnetic properties of immediate environment, mobility of molecules...



Diffusion MRI: sensitizing the signal to the Brownian motion (random walk) of water molecules

Outline

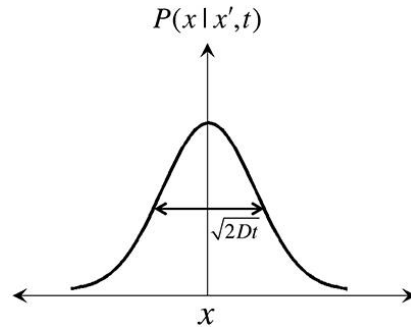
- I. Diffusion-weighting
- II. Gaussian approximation regime: ADC and DTI
- III. Beyond Gaussian:
 - i. Signal representations: DKI
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 - iv. Thinking outside the brain

Diffusion MRI gives access to the mesoscale

➤ Unrestricted homogeneous medium:



MRI voxel size:
1 – 2 mm

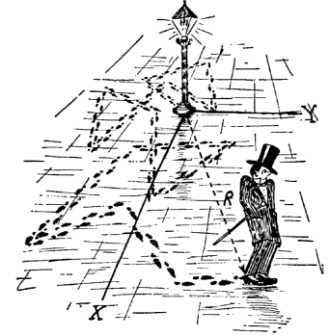
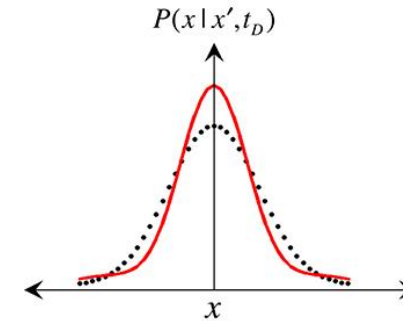


$$\langle x^2 \rangle = 2nDt$$

Typical length scale for
microstructure: 1 – 10 μm



➤ Biological tissue:

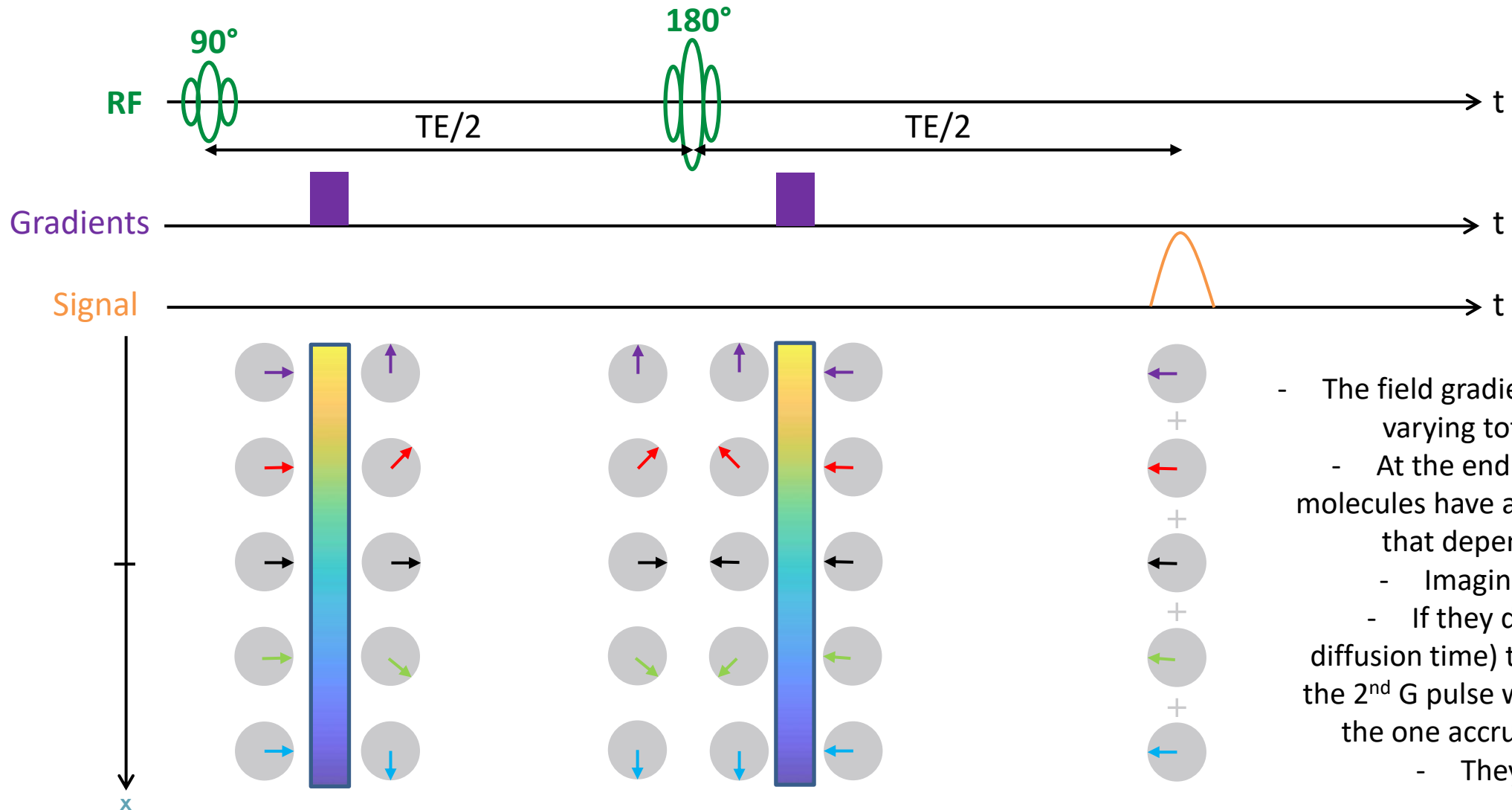


Mean displacement of water molecules :

- $t = 2 - 500 \text{ ms} \longrightarrow \sqrt{\langle x^2 \rangle} \approx 2 - 50 \mu\text{m}$

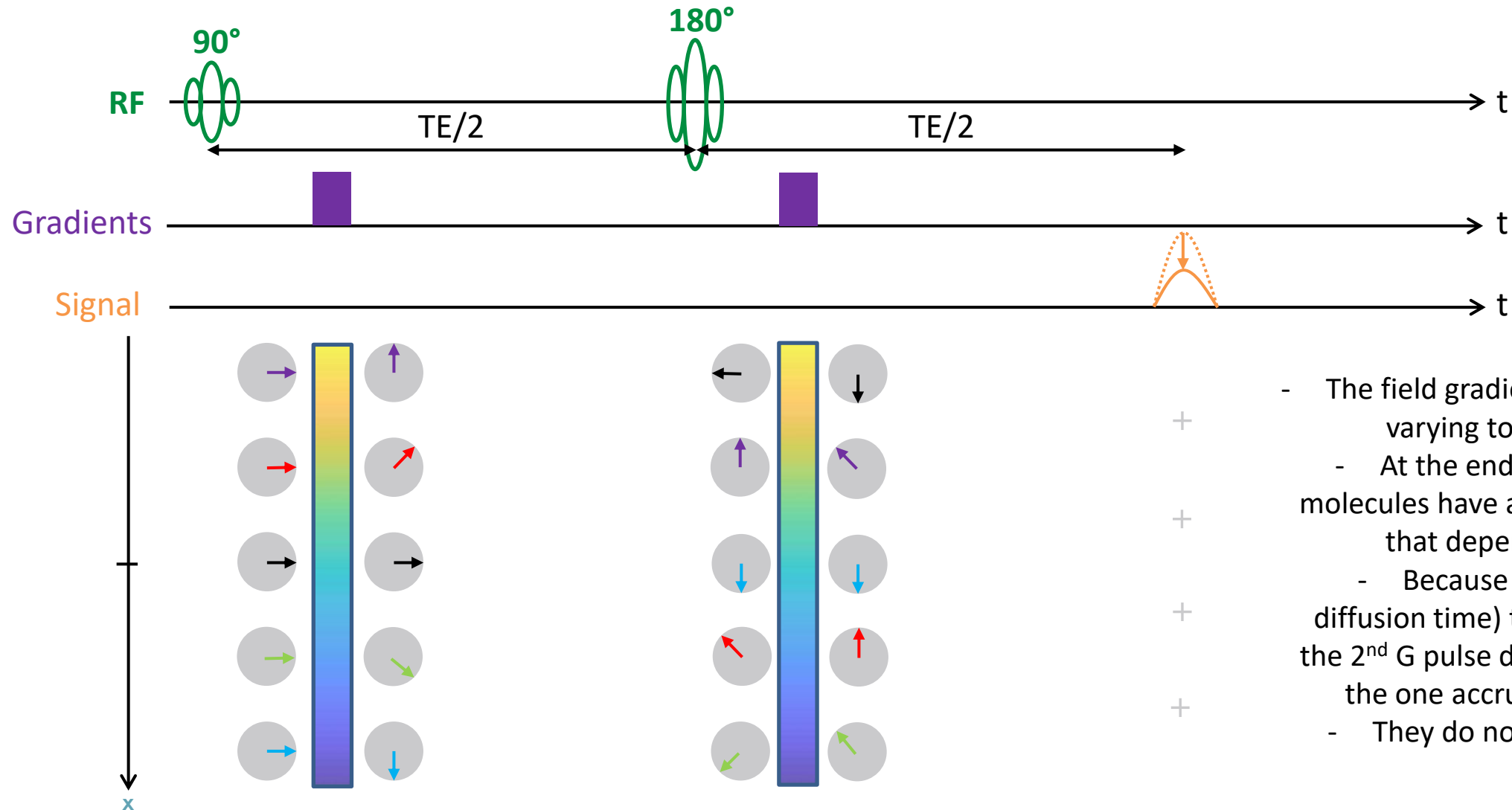
➤ Excellent probe of tissue microstructure

Diffusion weighting in MRI – pulsed gradient spin-echo (PGSE)



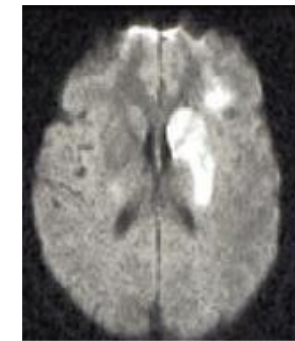
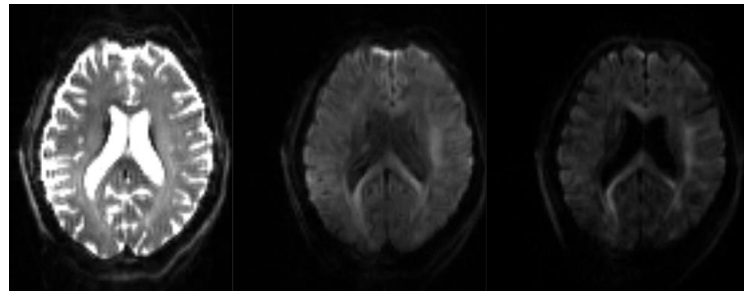
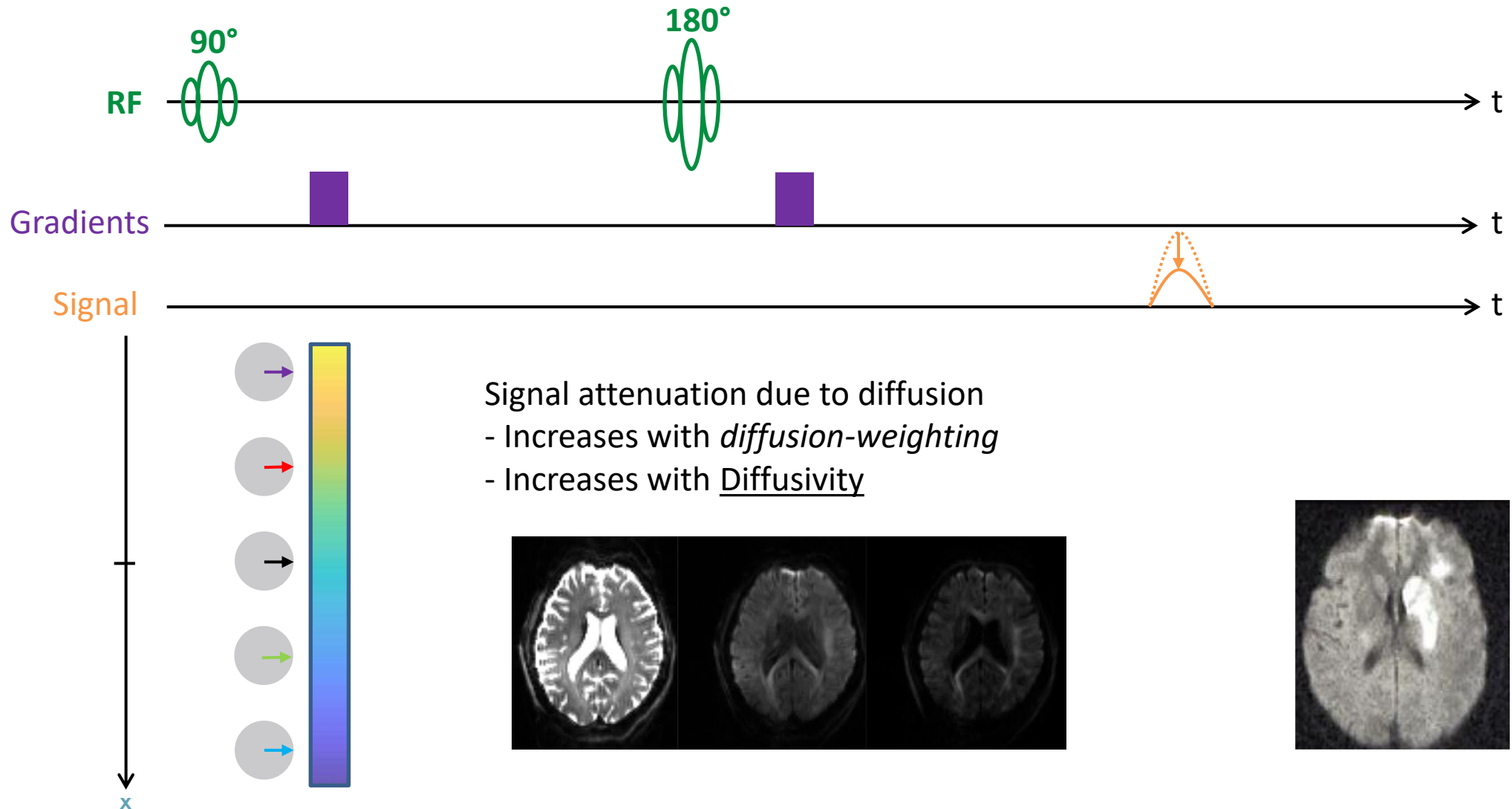
- The field gradient creates a linearly varying total field: $B(x) = B_0 + G \cdot x$
- At the end of the (short) pulse, molecules have accumulated a phase that depends on their position
 - Imagine: no diffusion ($D=0$)
 - If they don't mix (during the diffusion time) the phase accrued in the 2nd G pulse will fully compensate the one accrued in the 1st G pulse
 - They add up coherently!

Diffusion weighting in MRI – pulsed gradient spin-echo (PGSE)



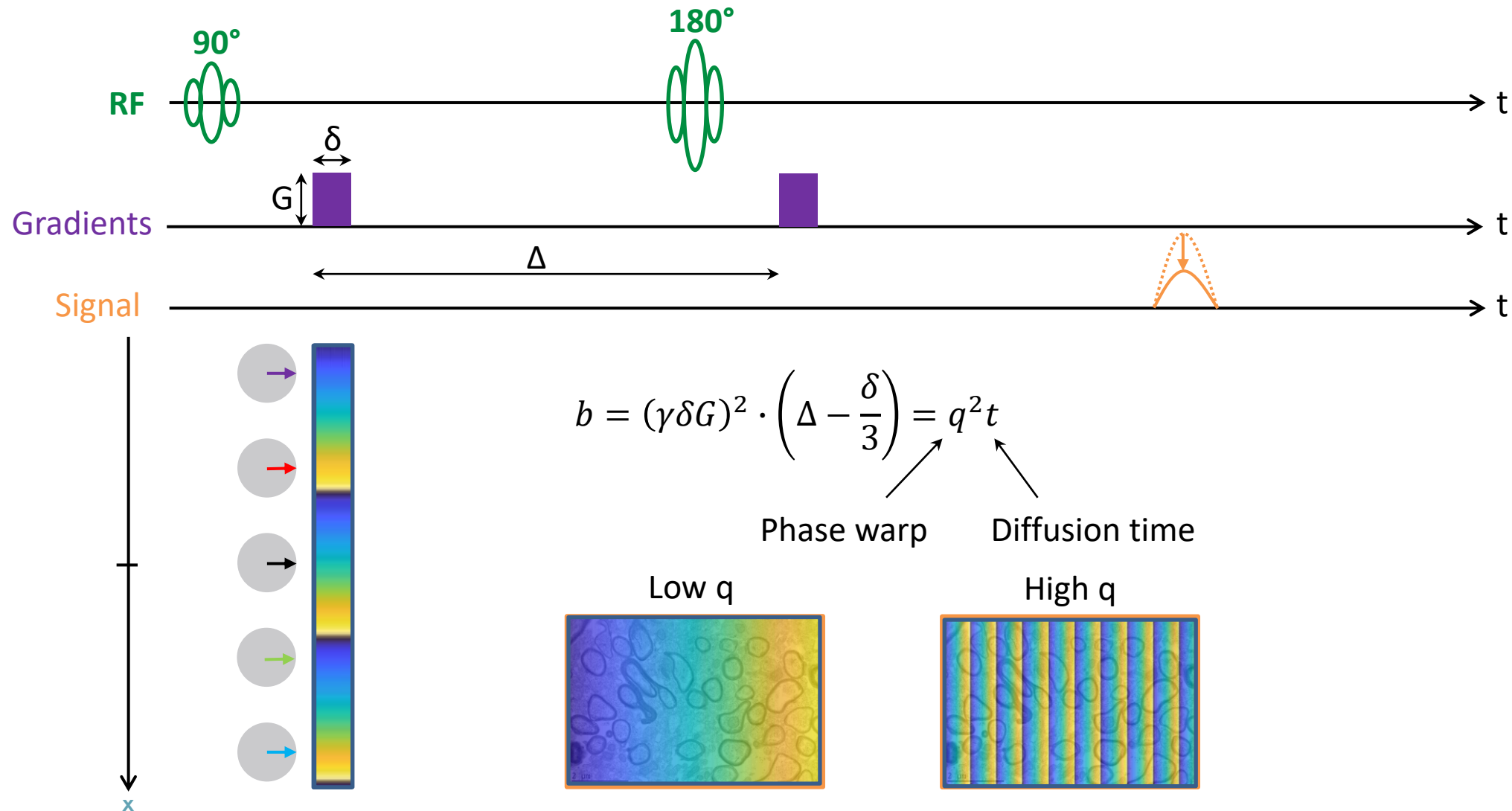
- The field gradient creates a linearly varying total field: $B(x) = B_0 + G \cdot x$
- At the end of the (short) pulse, molecules have accumulated a phase that depends on their position
- Because they mix (during the diffusion time) the phase accrued in the 2nd G pulse does not compensate the one accrued in the 1st G pulse
- They do not add up coherently!
Signal attenuation

Diffusion weighting: signal attenuation & contrast

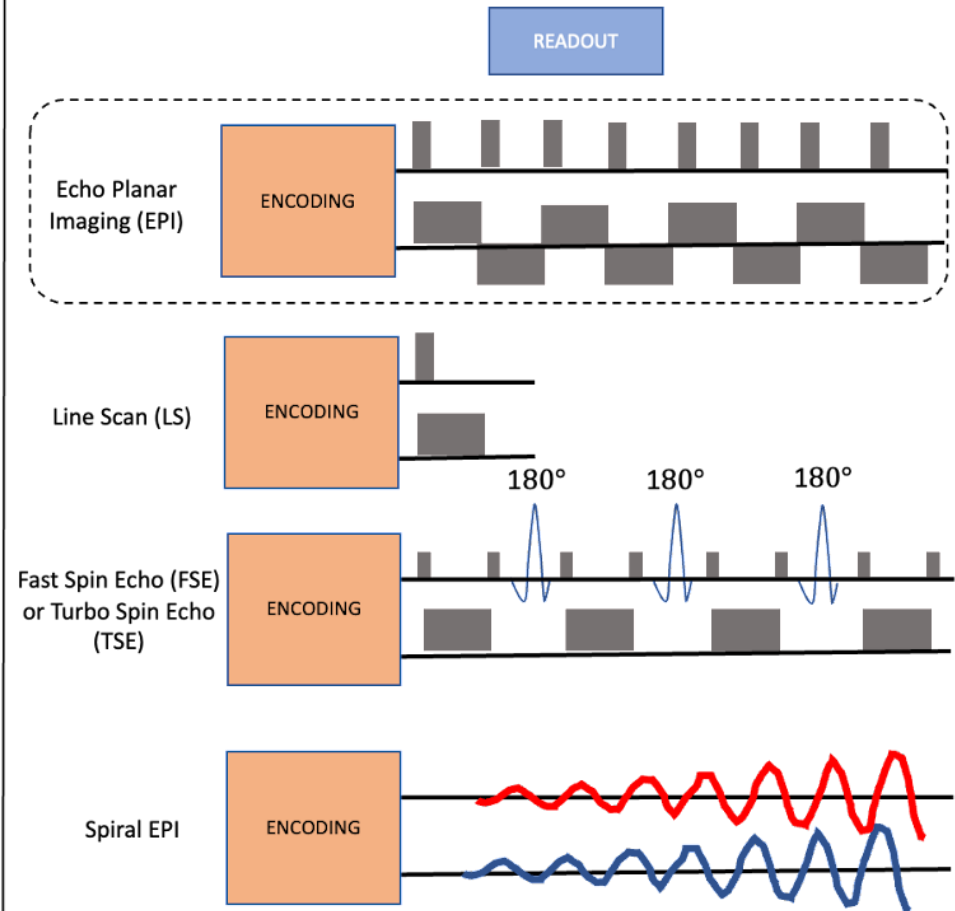
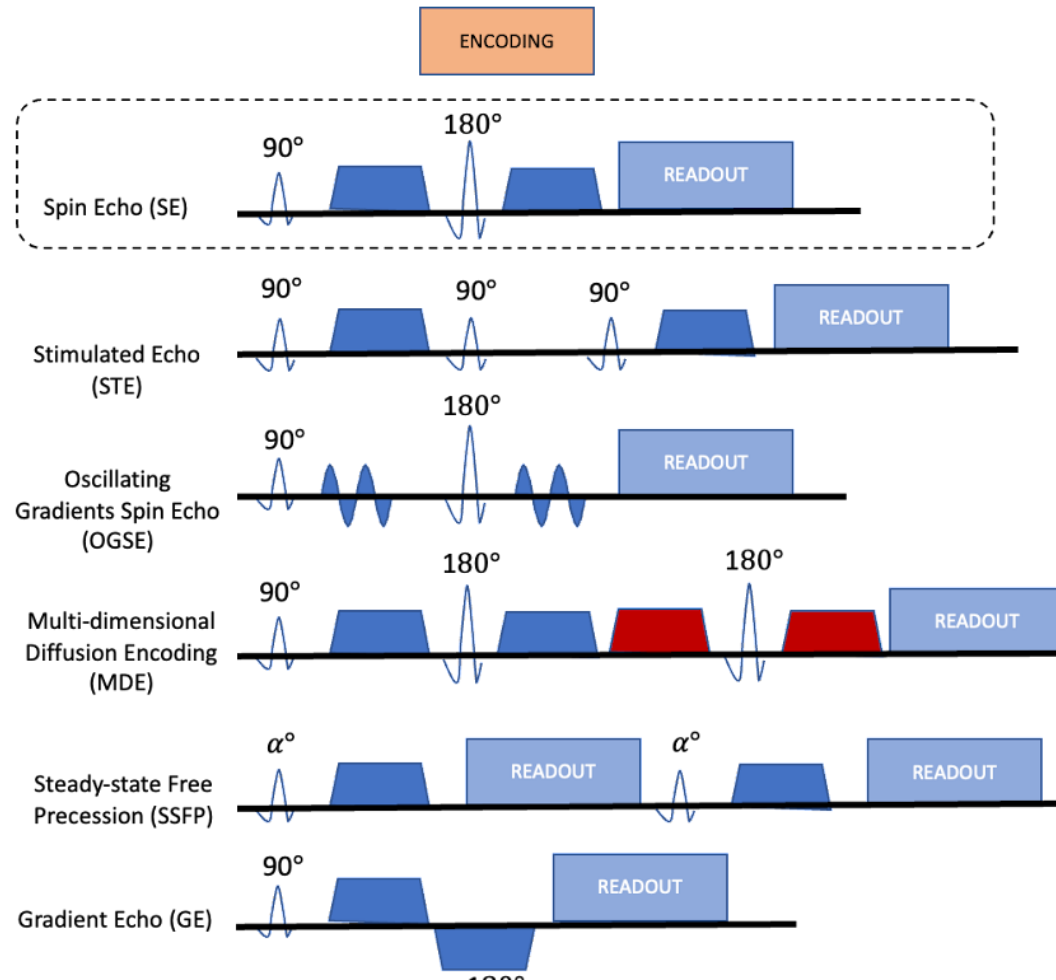


Stroke

Diffusion weighting in MRI – q, t and the b-value



Diffusion encoding schemes and image read-out schemes



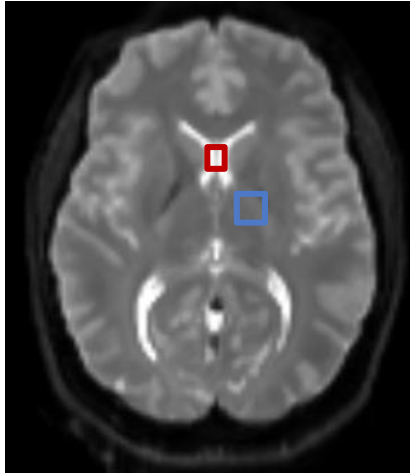
Summary so far

- Diffusion of water molecules in biological tissue (e.g. brain):
 - Is a stochastic process due to thermal agitation, it is always present
 - Is not free: it is affected by the cellular structures (restriction & hindrance)
 - Can provide information about tissue microstructure below the MR image spatial resolution
- In Diffusion MRI, we sensitize the signal amplitude to the diffusion of water molecules using strong magnetic field gradients G
- The MRI signal attenuation depends on:
 - The diffusion properties of the medium (that we want to measure)
 - The diffusion weighting imparted: $b=q^2 \cdot t$ (that we set as acquisition parameter)

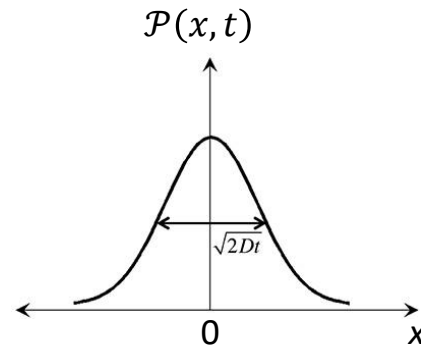
Outline

- I. Diffusion-weighting
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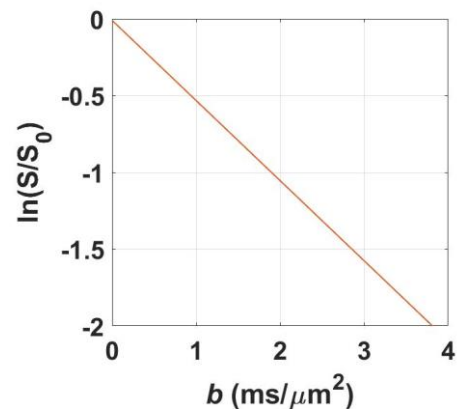
From a free medium to biological tissue (and from D to ADC)



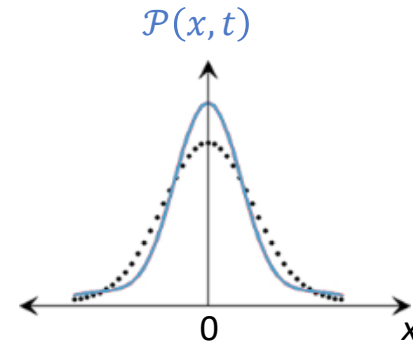
Free homogeneous medium
Gaussian diffusion



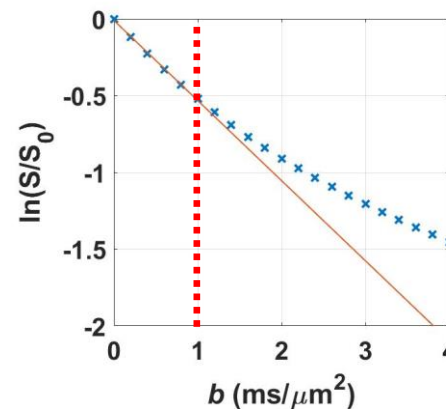
$$\ln\left(\frac{S}{S_0}\right) = -bD$$



Biological tissue
Non-Gaussian diffusion



$$\ln\left(\frac{S}{S_0}\right) = F(b)$$



Low b-value regime:
Gaussian approximation
“DTI” regime

$$\ln\left(\frac{S}{S_0}\right) \approx -b \cdot ADC + \mathcal{O}(b^2)$$

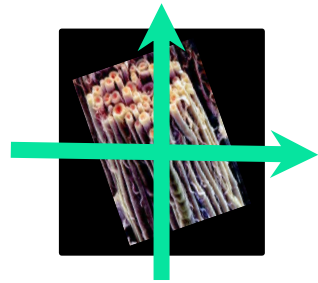


Apparent Diffusion Coefficient

From the ADC to the diffusion tensor (DTI)

Apparent Diffusion Coefficient

...
along a direction $\vec{g}$,
given by $\vec{G} = G \cdot \vec{g}$



$$\ln \frac{S(b, \vec{g})}{S_0} = -b \cdot ADC_{\vec{g}}$$

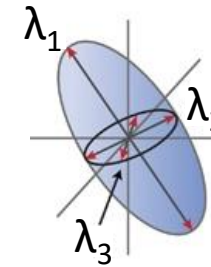
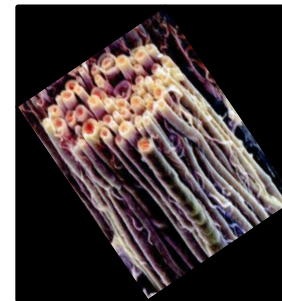
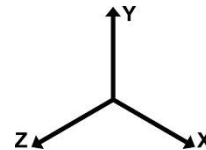
$$\vec{g} = [g_1 \ g_2 \ g_3]$$

$$\ln \frac{S(b, \vec{g})}{S_0} = -b \sum_{i,j=1}^3 g_i g_j D_{ij} = -b \vec{g}^T \hat{D} \vec{g}$$

$$\vec{g} = [1 \ 0 \ 0]$$

$$\vec{g} = [0 \ 1 \ 0]$$

$$\hat{D}_{\{\vec{x}, \vec{y}, \vec{z}\}} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix}$$



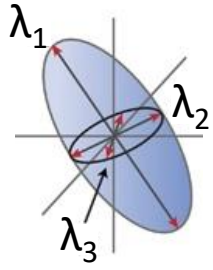
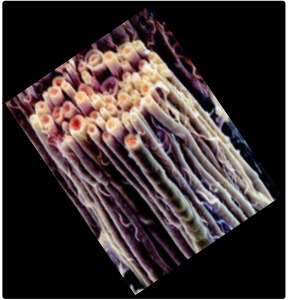
$$\hat{D}_{\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

- Symmetric $\rightarrow$ 6 coefficients to estimate
- Invertible with all eigenvalues real and positive

Minimal data:

- 1 $b = 0$
- 6 non collinear directions on 1 shell (e.g. $b = 1 \text{ ms}/\mu\text{m}^2$)

DTI scalars

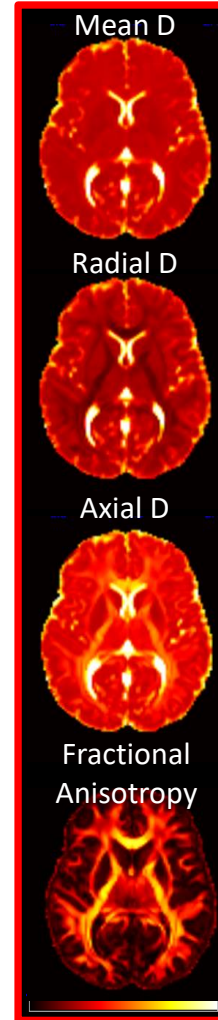


$$\hat{D}_{\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

In white matter in particular:

$\vec{e}_1$: direction of faster diffusion in the voxel
(along the axon bundle)

→ Useful in early days of tractography for tracking fiber orientations from voxel to voxel



Mean diffusivity:

$$MD = (\lambda_1 + \lambda_2 + \lambda_3)/3$$

$$MD = \text{Tr}(\hat{D})/3$$

$MD \sim 1 \frac{\mu m^2}{ms}$ in brain, little GM/WM contrast

Radial diffusivity:

$$RD = (\lambda_2 + \lambda_3)/2$$

Good GM/WM contrast, lowest in WM

Axial diffusivity:

$$AD = \lambda_1$$

Some GM/WM contrast, highest in WM

Fractional anisotropy:

$$FA = \frac{\sqrt{\frac{1}{2}[(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_3 - \lambda_1)^2]}}{\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}$$

FA = 0 : isotropic diffusion, FA = 1 : fully anisotropic (1D)

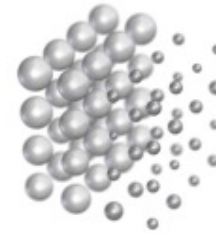
Excellent GM/WM contrast, highest in WM

Some important jargon & nuances

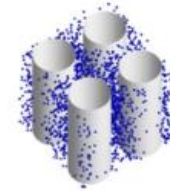
- “Diffusion MRI” $\neq$ “DTI”
- (Free) Diffusivity $\neq$ ADC
- In anisotropic tissue: MD $\neq$ ADC
- Units!

“widespread”		“natural choice”	
Diffusion coefficient	b-value	Diffusion coefficient	b-value
mm^2/s	s/mm^2	$\mu\text{m}^2/\text{ms}$	$\text{ms}/\mu\text{m}^2$
$3 \cdot 10^{-3}$	1000	3	1
$4 \cdot 10^{-4}$	10'000	0.4	10

- Anisotropy $\neq$ non-Gaussian diffusion



Isotropic diffusion
Non-Gaussian (restricted compartments)

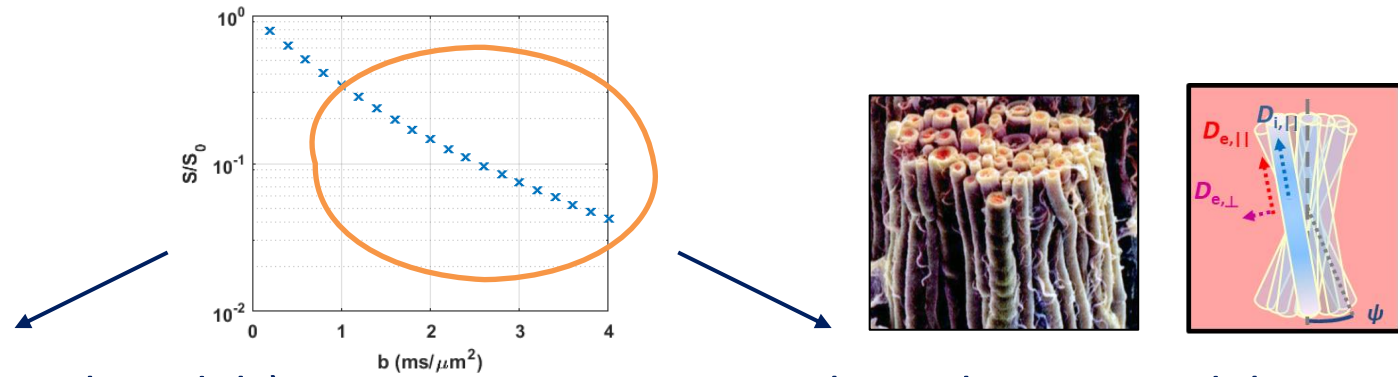


Anisotropic diffusion
yet Gaussian in each direction

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Signal representations vs biophysical models



Signal representations (statistical models)

- “Formulas” to describe signal decay
- ☺ Applicable to any type of tissue
- ☺ Provide sensitive biomarkers
- ☹ Lack specificity: difficult to make inferences about microstructural changes

Biophysical tissue models

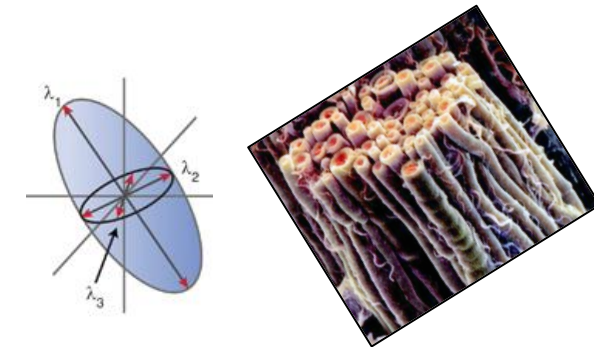
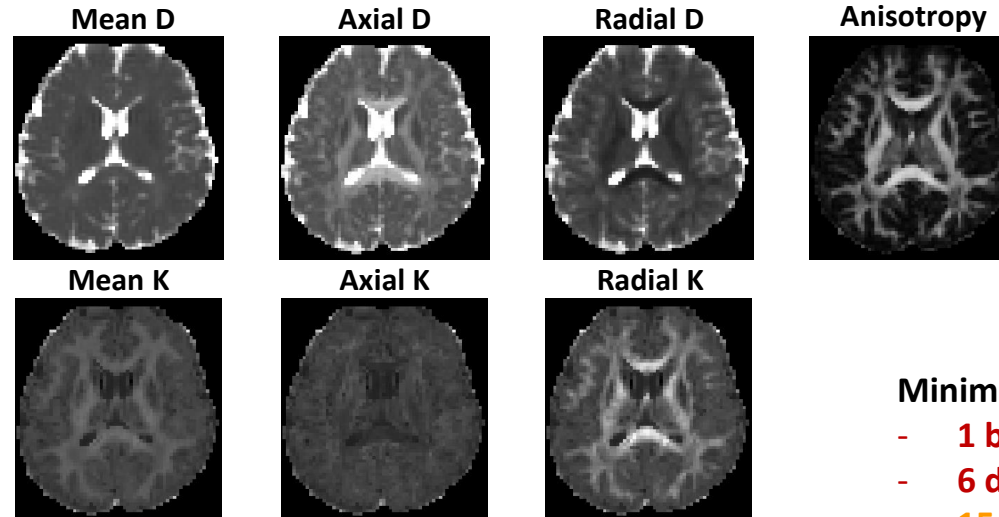
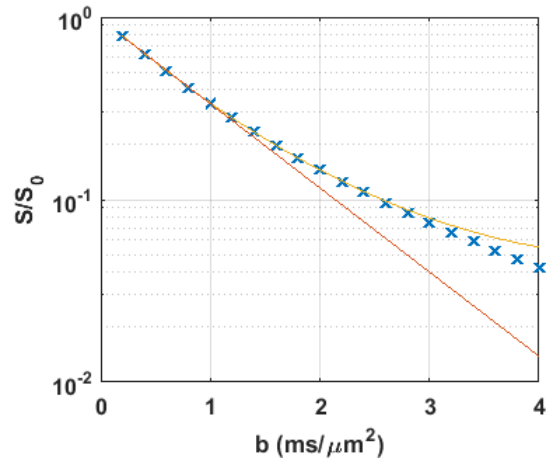
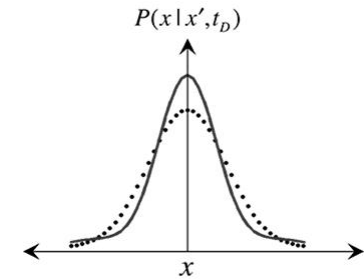
- Simplified “pictures” of medium in which water molecules are diffusing
- ☹ Tissue-specific
- ☺ Specific biomarkers that characterize microstructure
- ☹ Assumption validity?
- ☹ Fit stability?

Representations: Diffusion Tensor Imaging (DTI) and Kurtosis (DKI)

$$\ln \frac{S}{S_0} = -bD + \frac{1}{6} b^2 D^2 K + \dots$$

$$\ln \frac{S}{S_0} = -b \sum_{i,j=1}^3 g_i g_j D_{ij} + \frac{1}{6} b^2 \bar{D}^2 \sum_{i,j,k,l=1}^3 g_i g_j g_k g_l W_{ijkl} + \dots$$

diffusion tensor (DTI) & kurtosis tensor (DKI)



Minimal data:

- **1 $b = 0$**
- **6 directions on 1 shell (e.g. $b = 1 \text{ ms}/\mu\text{m}^2$)**
- **15 directions on a 2nd shell (e.g. $b = 2/2.5$)**

DKI: Valuable

PubMed.gov DTI [Advanced](#) [Create alert](#) [Create RSS](#) [User Guide](#)

Sorted by: Best match

MY NCBI FILTERS

RESULTS BY YEAR

1994 2022

TEXT AVAILABILITY

☐ Abstract

☐ Free full text

12,530 results

1 ☐ [Diffusion tensor imaging \(DTI\)-based white matter mapping in brain research: a review.](#)

Cite Assaf Y, Pasternak O.
J Mol Neurosci. 2008;34(1):51-61. doi: 10.1007/s12031-007-0029-0.
PMID: 18157658 Review.

Share Diffusion tensor imaging (DTI) has become one of the most popular MRI techniques in brain research, as well as in clinical practice. The number of brain studies with DTI is growing steadily and, over the last decade, has produced more than 700 publications. ...

2 ☐ [Mild traumatic brain injury: Is DTI ready for the courtroom?](#)

Cite Shenton ME, Price BH, Levin L, Edersheim JG.
Int J Law Psychiatry. 2018 Nov-Dec;61:50-63. doi: 10.1016/j.ijlp.2018.09.002. Epub 2018 Nov 1.

PubMed.gov DKI [Advanced](#) [Create alert](#) [Create RSS](#) [User Guide](#)

Sorted by: Best match

MY NCBI FILTERS

RESULTS BY YEAR

2005 2022

851 results

1 ☐ [Physics, Techniques and Review of Neuroradiological Applications of Diffusion Kurtosis Imaging \(DKI\).](#)

Cite Marrale M, Collura G, Brai M, Toschi N, Midiri F, La Tona G, Lo Casto A, Gagliardo C.
Clin Neuroradiol. 2016 Dec;26(4):391-403. doi: 10.1007/s00062-015-0469-9. Epub 2015 Nov 20.
PMID: 26589207 Review.

Share These limitations can be partially overcome by means of diffusion kurtosis imaging (DKI). The aim of this paper is the description of the theory of DKI, a new topic of growing interest in radiology. DKI is a higher order diffusion model that is a straightforward ...

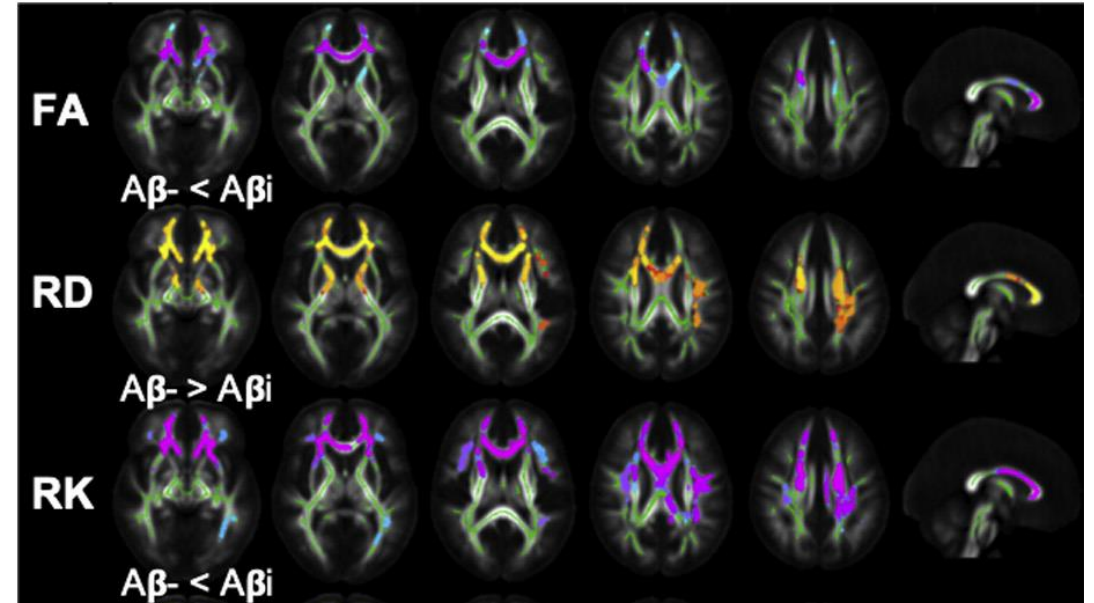
Practicality

- “Standard” multi-shell acquisition:
 - $1\ b = 0, 30\ b_1, 60\ b_2$
 - ~7-8 min using acceleration (GRAPPA, multiband...)

DKI: Valuable

- Sensitive to tissue complexity
- Complementary information to DTI:
 - Brain development
 - Dementia
 - Stroke
 - TBI
 - Glioma
 - Prostate cancer
 - Other body cancers

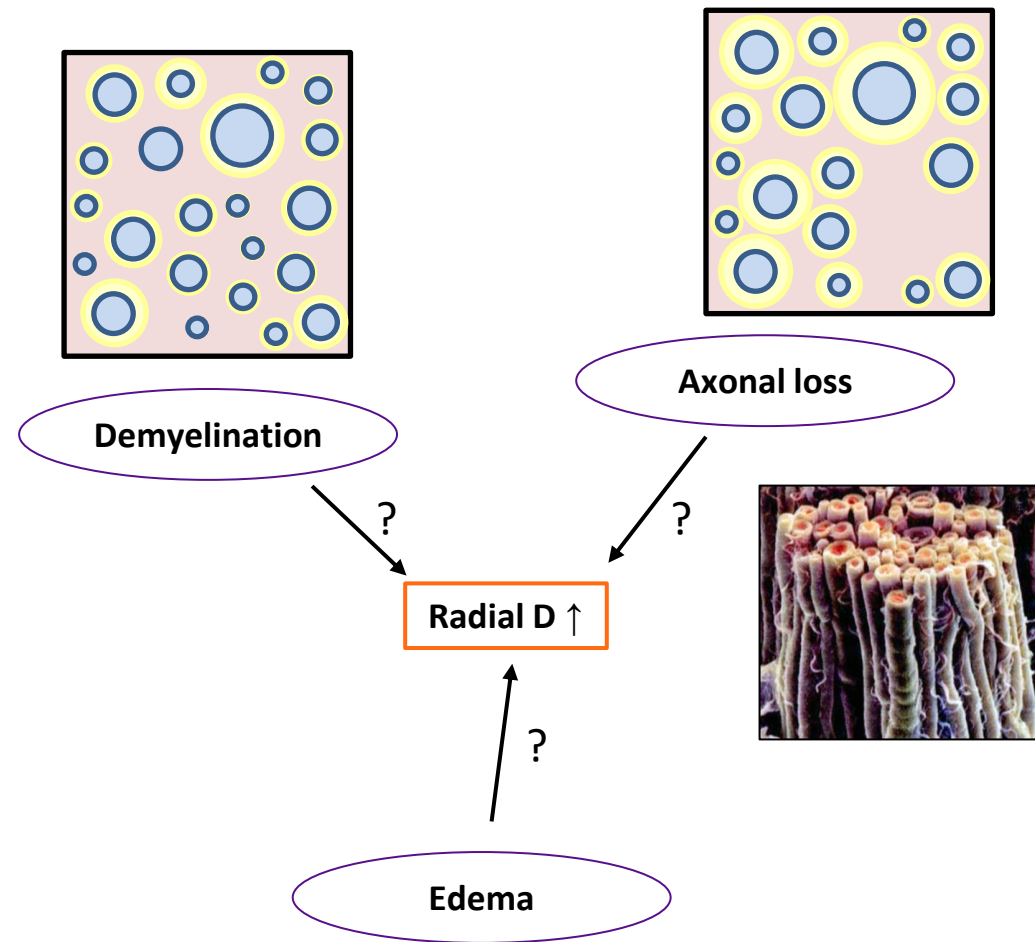
Jensen and Helpert, NMR Biomed 2010
Hui, Stroke 2012
Paydar, AJNR 2014
Rosenkrantz, JMIR 2015



Relationship between altered WM microstructure & amyloid load in MCI patients

Dong, Neurobiol Aging 2020

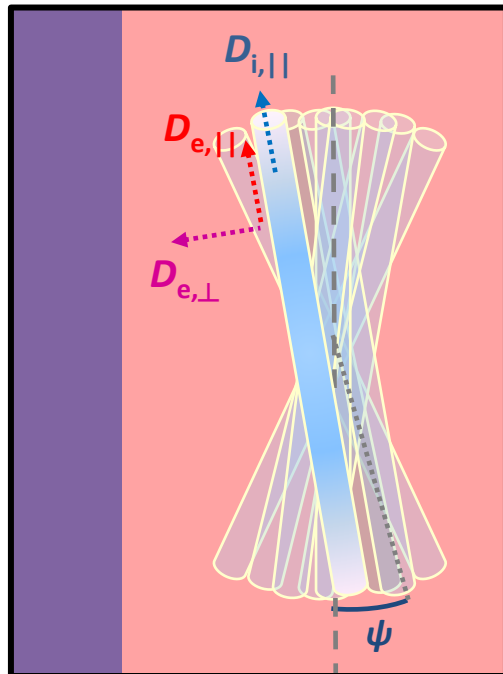
Representations: Valuable... but still lack specificity



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What is a relevant model for white matter?



$$\frac{\text{blue box}}{\text{blue box} + \text{red box}} = f$$

■ Intra-axonal space

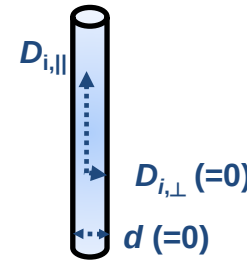
■ Extra-axonal space

■ CSF

Intra-axonal compartment:
Collection of cylinders (or “sticks”)

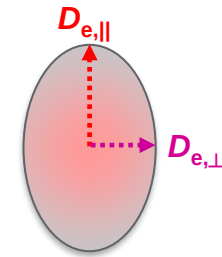
Extra-axonal compartment:
(Gaussian) anisotropic connected space

CSF:
Gaussian isotropic

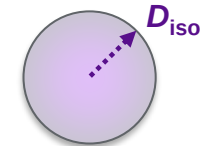


With some
ODF $P(\mathbf{n})$

f



$1-f-f_{iso}$



f_{iso}

Markers of:

f : axonal density / loss

ODF: alignment / pruning

d : axon caliber

$D_{i,||}$: axonal injury

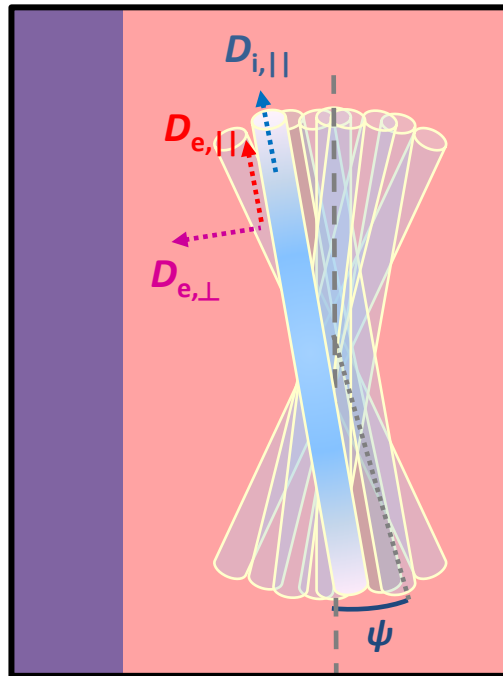
$D_{e,\perp}$: demyelination

$D_{e,||}$: inflammation

f_{iso} : edema, atrophy

Most cases: “Standard model”

Long-time limit & moderate q



$$\frac{\text{blue box}}{\text{blue box} + \text{red box}} = f$$

■ Intra-axonal space

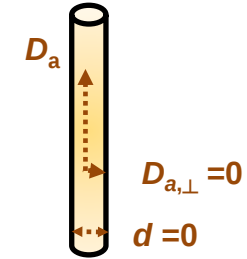
■ Extra-axonal space

■ CSF

Intra-axonal compartment:
Collection of **sticks**

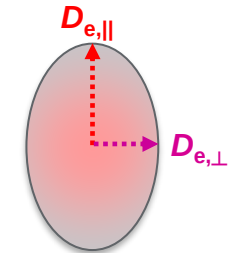
Extra-axonal compartment:
Gaussian anisotropic connected space

CSF:
Gaussian isotropic

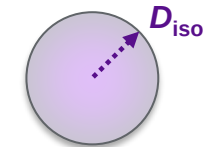


With some ODF $P(\mathbf{n})$

f



$1-f-f_{iso}$



f_{iso}

Moving away from spurious assumptions

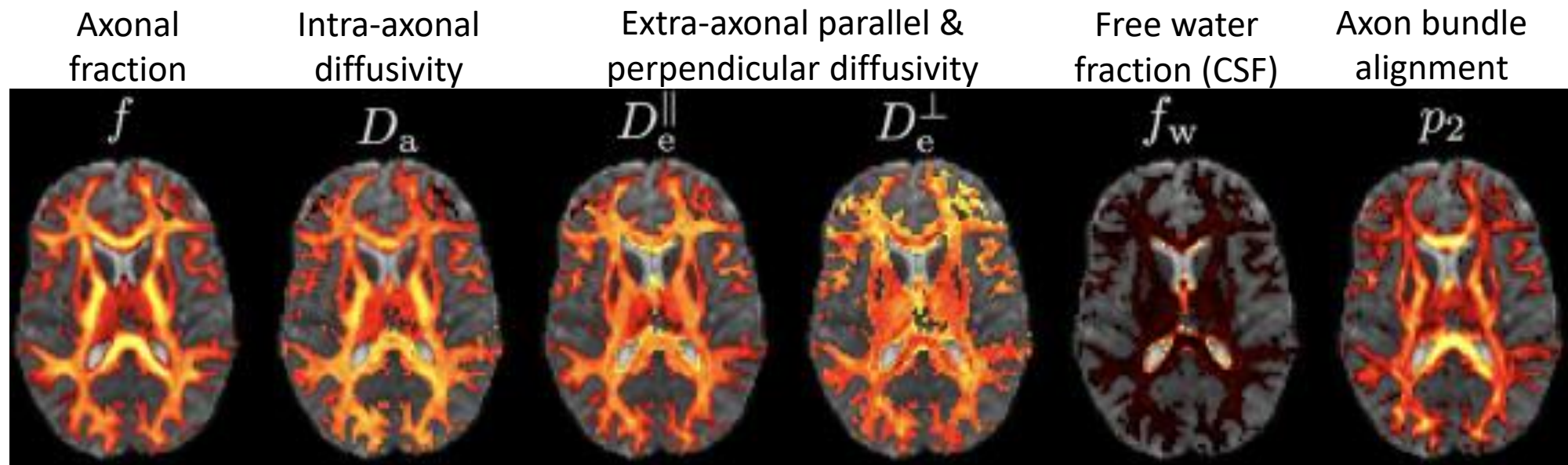
➤ Favor Standard Model frameworks that introduce minimum assumptions

- WMTI-Watson (from DKI)
- SMI (from Rotational Invariants)
- Diffusion-relaxometry with tensor encoding

https://github.com/Mic-map/WMTI-Watson_DL

<https://github.com/NYU-DiffusionMRI/SMI>

Jespersen, NeuroImage 2018; Novikov, NeuroImage 2018; Lampinen, MRM 2020

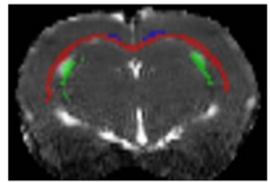


Coelho, NeuroImage 2022

Example applications

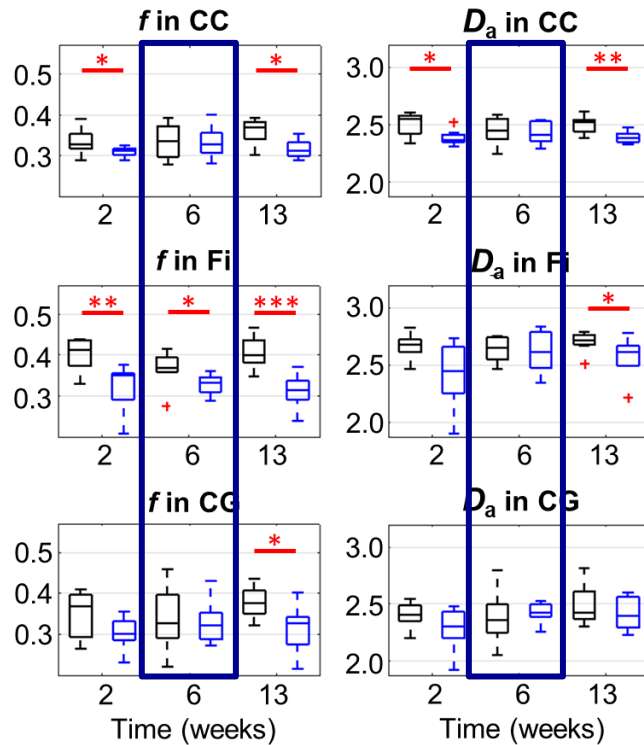
Acute & chronic damage in rat model of AD (“brain diabetes”)

WM microstructure

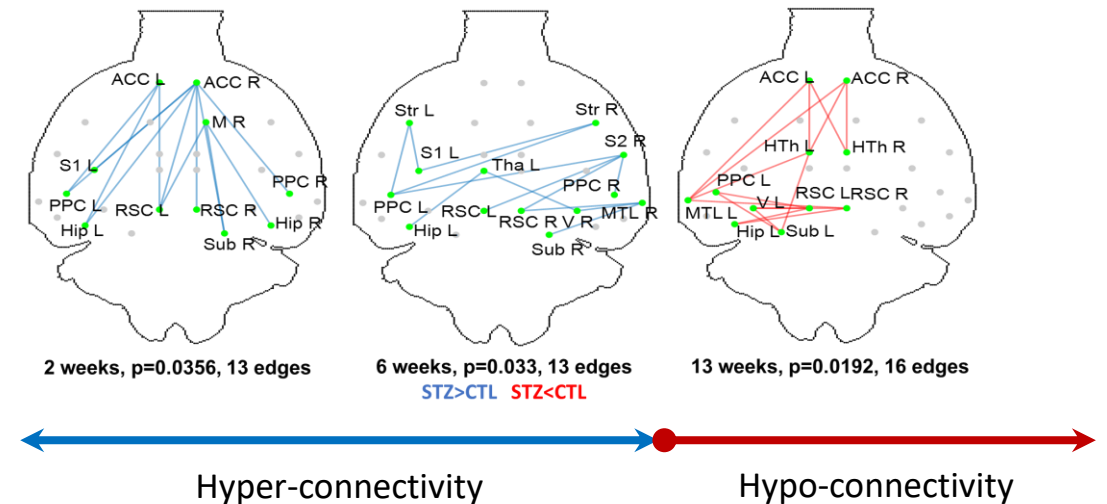


CC: corpus callosum
Fi: fimbria of hippocampus
CG: cingulum

- STZ
- CTL



Functional connectivity



Pereira, Neurolmage 2021; Diao, Front Neurosci 2021

Competition between inflammation & neurodegeneration

WM demyelination and remyelination in cuprizone-fed mice (model for Multiple Sclerosis)

	3W CPZ				6W CPZ			
	Histological analyses	DKI	DTI	WMTI	Histological analyses	DKI	DTI	WMTI
Body	↓ MBP*** ↑ Iba1*** ↑ GFAP*	↑ MK*** ↑ RK** ↑ AK**	↓ MD*** ↓ RD***	↓ D_a *** ↓ $D_{e,\perp}$ ** ↓ $D_{e,\parallel}$ **	↓ MBP* ↑ Iba1*** ↑ GFAP***	↓ AK**	↑ MD*** ↑ RD***	↓ AWF*** ?
Splenium	↓ MBP*** ↑ Iba1**	↑ MK*** ↑ RK*** ↑ AK***	↓ MD*** ↓ RD** ↓ AD**	↓ D_a ***	↓ MBP*** ↑ Iba1* ↑ GFAP**	↓ MK* ↓ RK** ↓ AK**	↑ MD*** ↑ RD**	↓ AWF**

$$AWF \equiv f$$

Competing mechanisms?

- Cellular crowding
- Demyelination

Table 5. Spearman's correlation between diffusion metrics and histological quantitative measures for the cuprizone group (CPZ); correlation coefficient significant (bold) at * $p \leq 0.05$, ** $p \leq 0.01$, and *** $p \leq 0.001$ (two tailed)

		FA	MD	$D_{\parallel}$	$D_{\perp}$	MK	$K_{ }$	$K_{\perp}$	AWF	D_a	$D_{e,\parallel}$	$D_{e,\perp}$	α
Myelin	Solochrome	-0.637***	0.837***	0.518**	0.672***	-0.510**	-0.495**	-0.498**	-0.365*	0.431*	0.271	0.623***	-0.276
	Amino	0.035	-0.080	-0.085	0.033	-0.003	-0.258	0.279	-0.239	-0.347	-0.072	0.043	-0.157
Microglia	GFAP	0.042	-0.161	-0.441**	0.184	0.019	-0.111	0.225	0.175	-0.207	-0.321	0.072	0.018
	Iba1	-0.340	0.175	-0.376*	0.649***	-0.423*	-0.483**	-0.024	-0.326	-0.316	-0.435*	0.434*	-0.365*

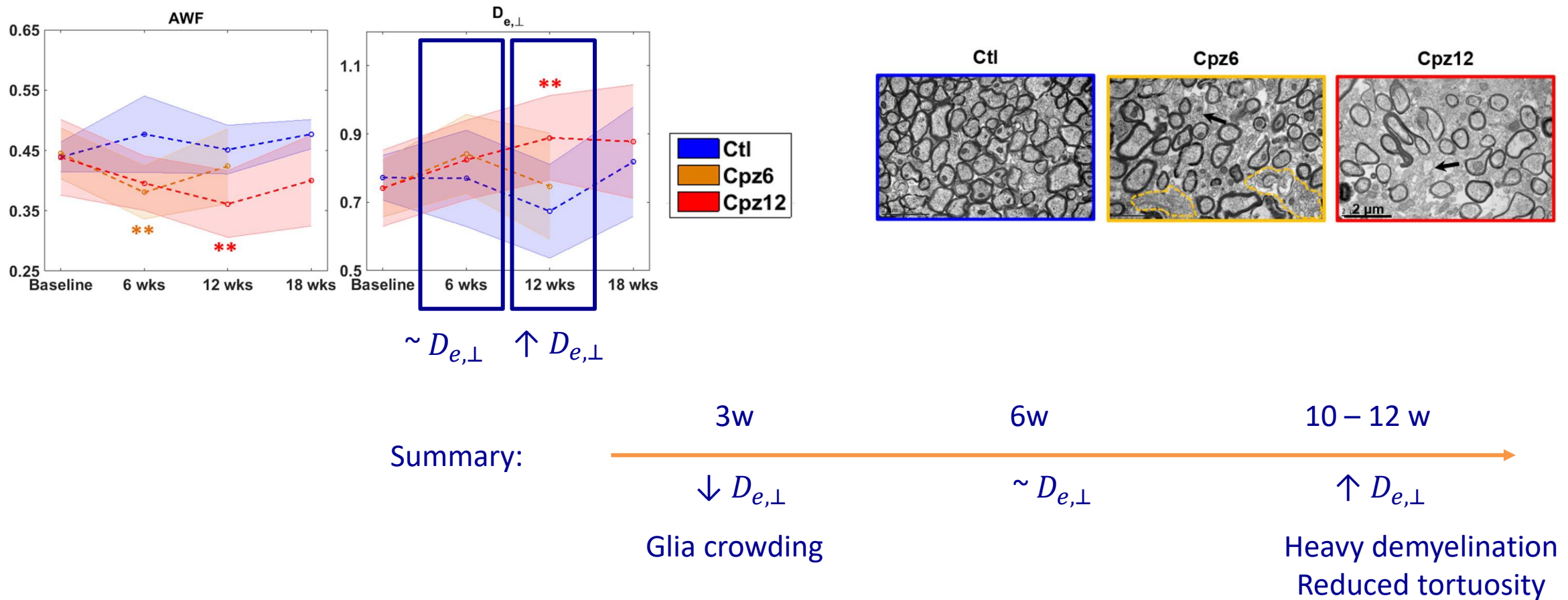
10-week CPZ diet

↑ Microglia
↓ $D_{e,\parallel}$ / ↑ $D_{e,\perp}$

Falangola et al., NMR in Biomed 2014; Guglielmetti et al., NeuroImage 2016

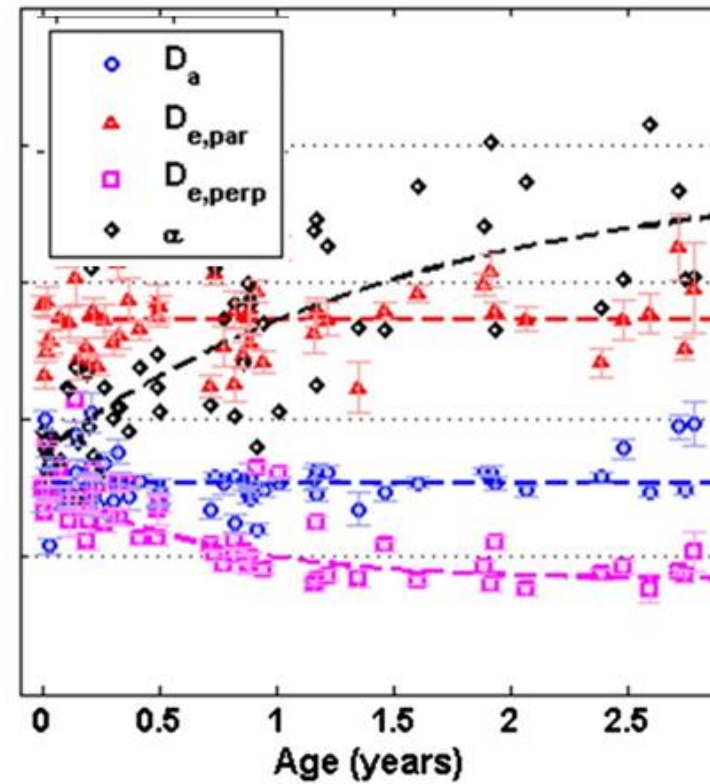
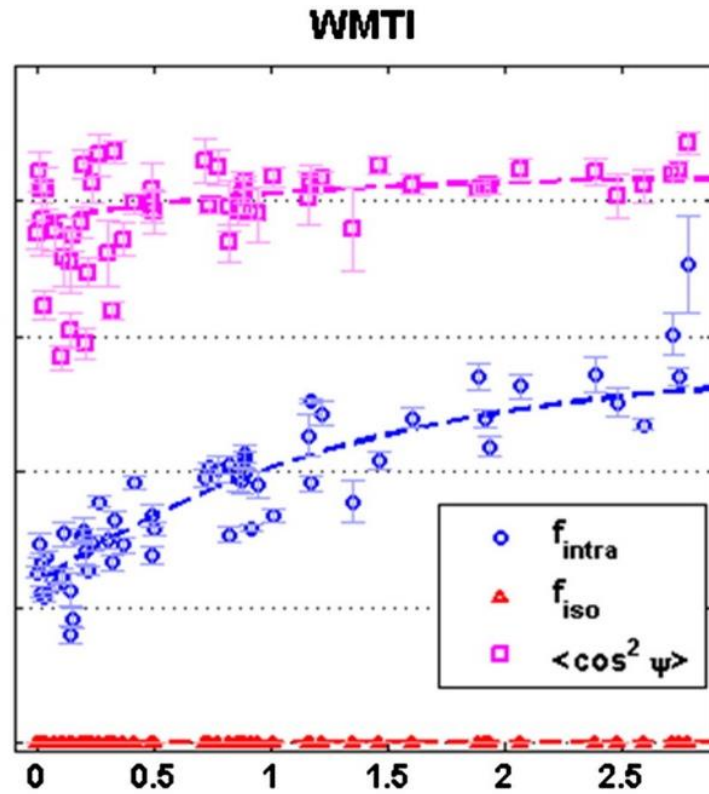
Competition between inflammation & neurodegeneration

WM demyelination and remyelination in cuprizone-fed mice (model for Multiple Sclerosis)



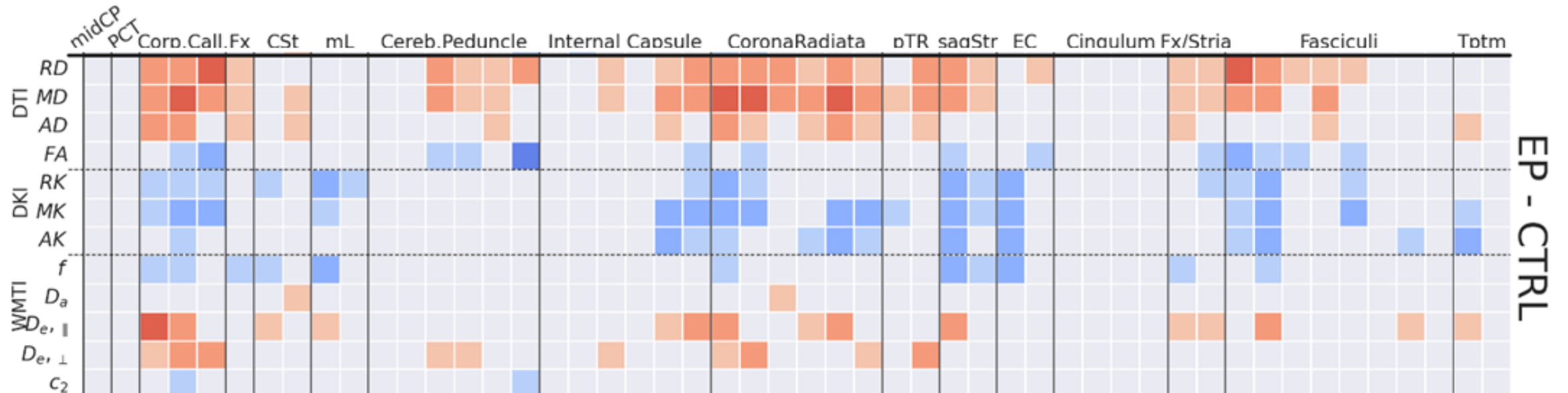
Example applications

Human WM development



Example applications

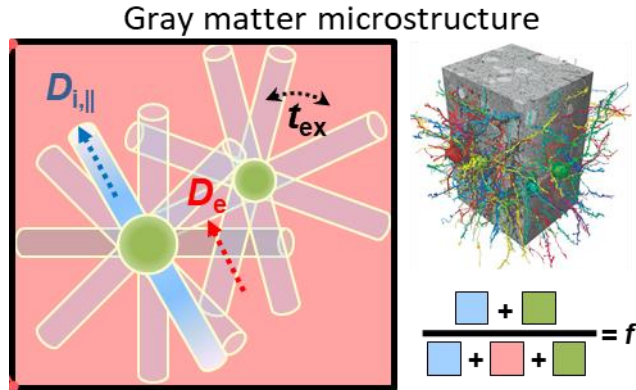
Early Psychosis



Outline

- I. Diffusion-weighting
- II. Gaussian approximation regime: ADC and DTI
- III. Beyond Gaussian:
 - i. Signal representations: DKI
 - ii. White matter models & applications
 - iii. Gray matter models & applications
 - iv. Thinking outside the brain

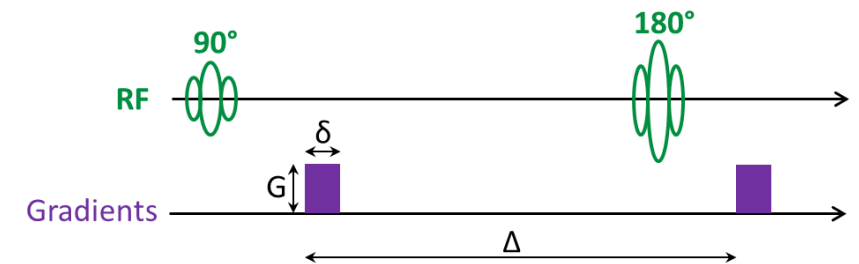
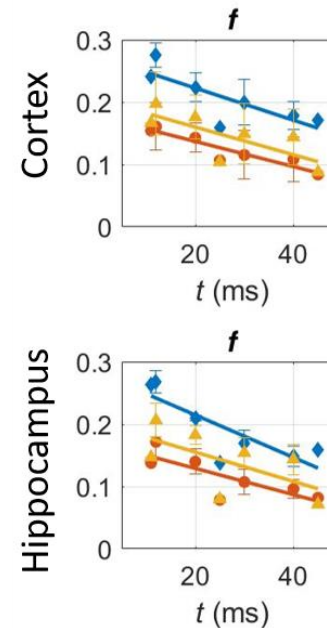
What is a relevant model for gray matter?



- Cell bodies: a third compartment?
- Little myelin: water exchange between compartments (t_{ex})?
- “Structural disorder”, i.e. non-Gaussian diffusion (spines, beading...)?

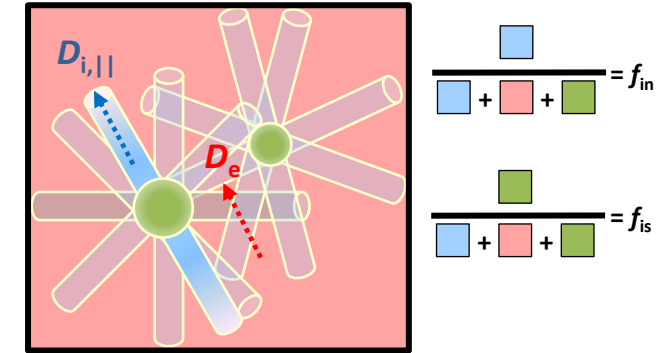
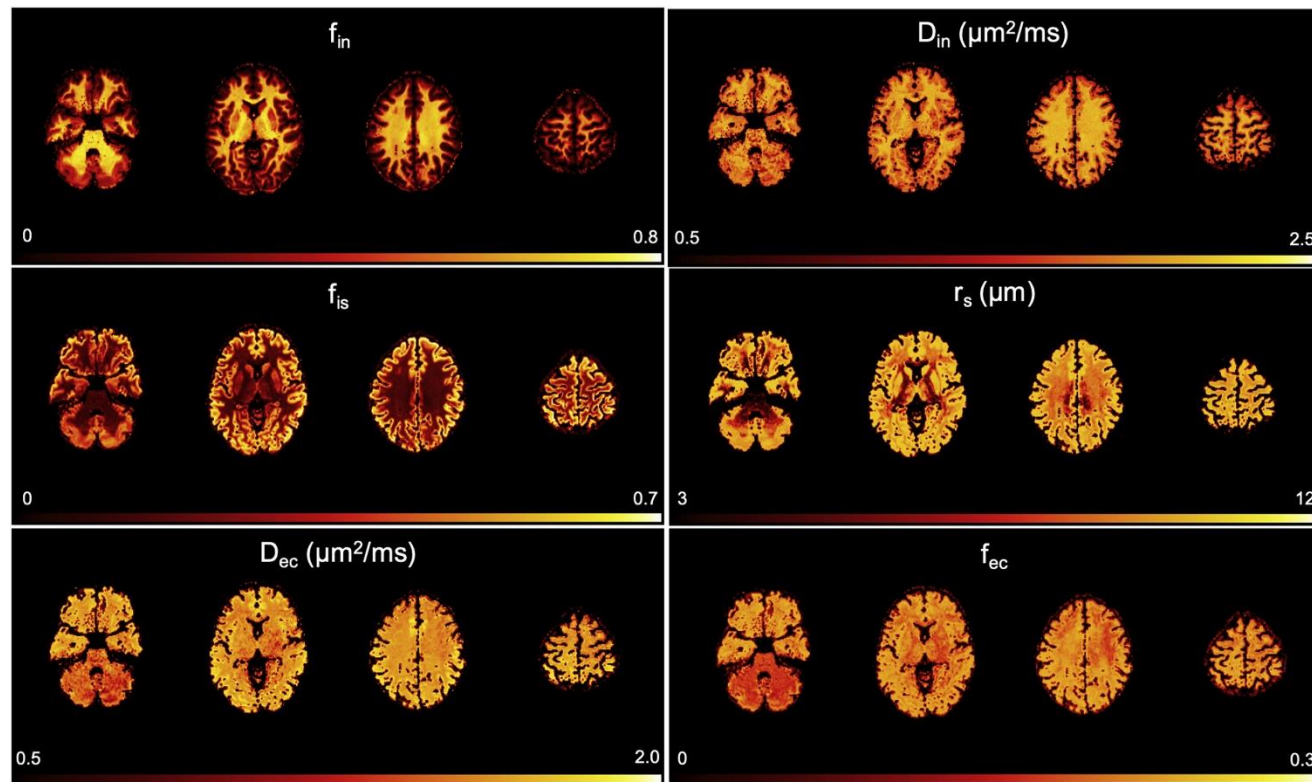
➡ Exploit both q-t dimensions to probe dynamic processes

- Time-dependent estimates of model parameters
 - Neurite fraction decreases



Modeling soma: SANDI – Soma and Neurite Density Imaging

- Assumption: non-exchanging compartments
 - $t_d \leq 20$ ms



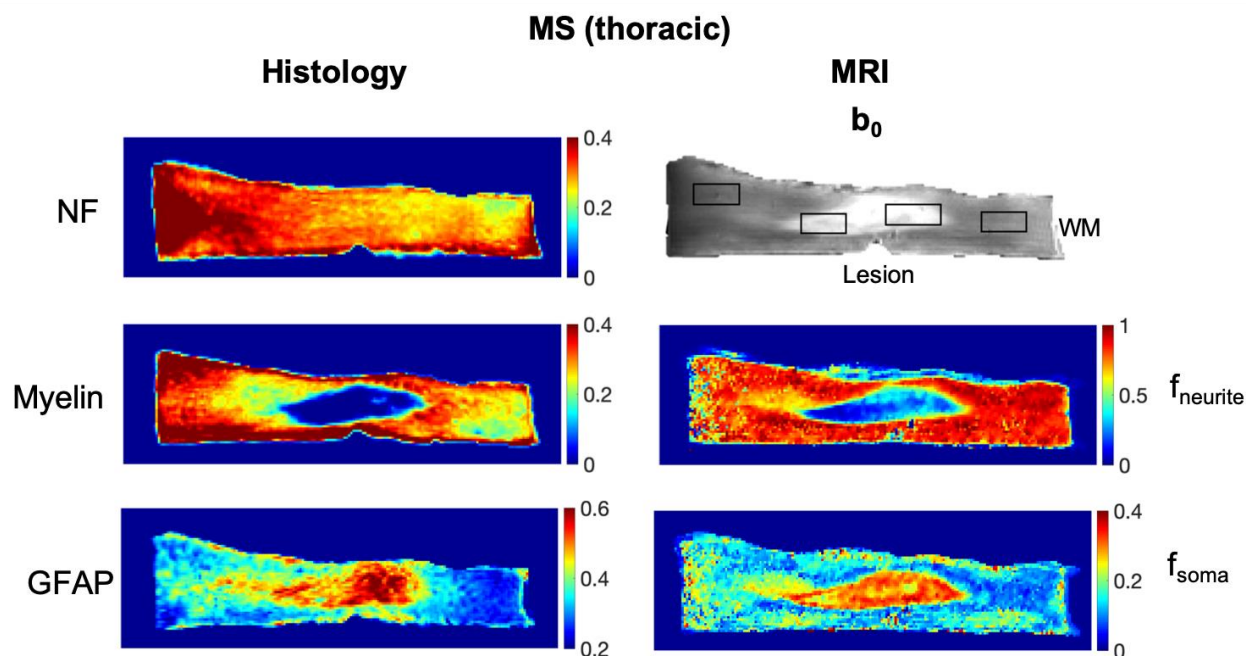
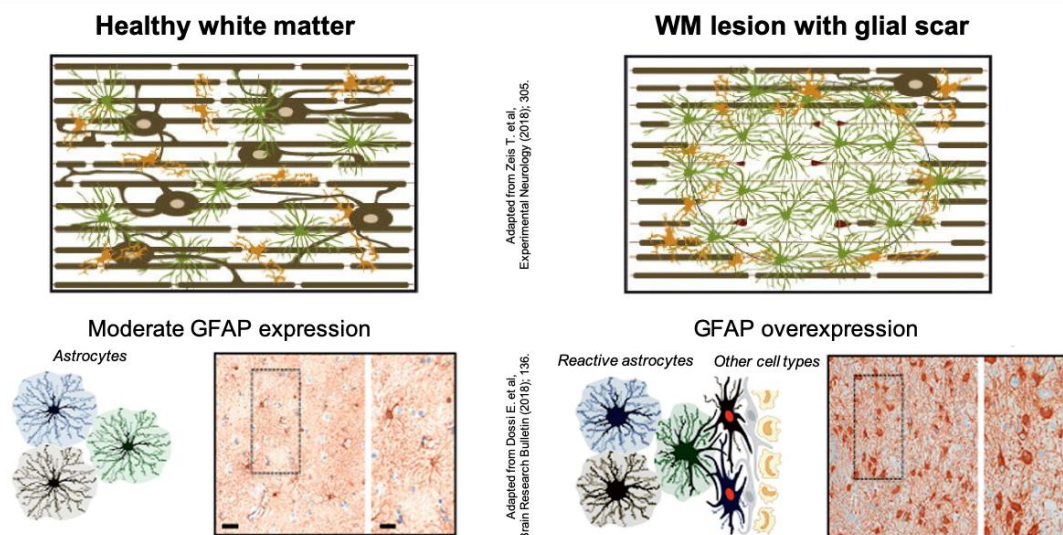
- ☺ Reasonable estimate of soma density & size
- ☹ Not feasible on a typical MRI clinical scanner

$$b = (\gamma \delta G)^2 \cdot \left(\Delta - \frac{\delta}{3} \right)$$

If $\left(\Delta - \frac{\delta}{3} \right)$ is to be kept short,
Need very high G to achieve high b -values ($b > 5$ ms/ μm^2)

Glial scarring in Multiple Sclerosis

3-compartment SANDI model
Short Diffusion time



- $f_{neurite}$ captures demyelination and loss of axons?
- f_{soma} captures increase in astrocyte reactivity?

Modeling exchange: 2-compartment anisotropic Kärger model

NEXI – Neurite EXchange Imaging

Kärger Assumption: Barrier-limited exchange

Additionally:

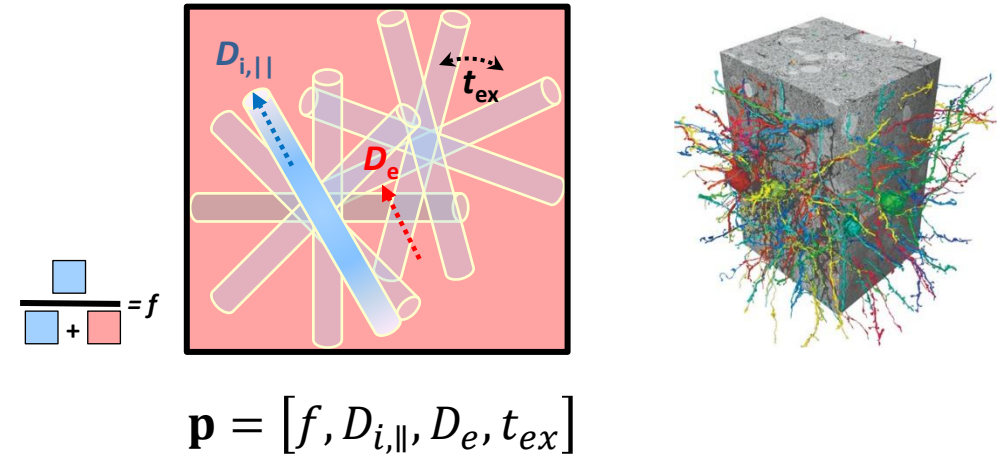
Isotropic distribution of neurites

$$D_{e,\parallel} = D_{e,\perp} \equiv D_e$$

Data acquisition requirements:

Multi-shell (multiple q-values), multi-t (multiple diffusion times)

→ Vary G and Δ



NEXI (Neurite EXchange Imaging) – preclinical in vivo

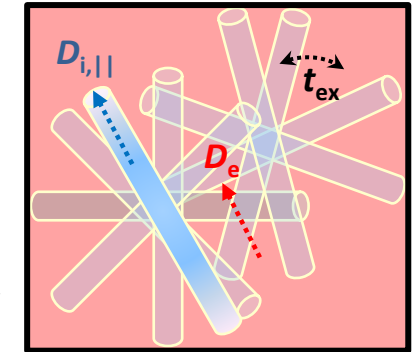
Model exchange, ignore soma

14 Tesla MRI, 1 T/m gradients

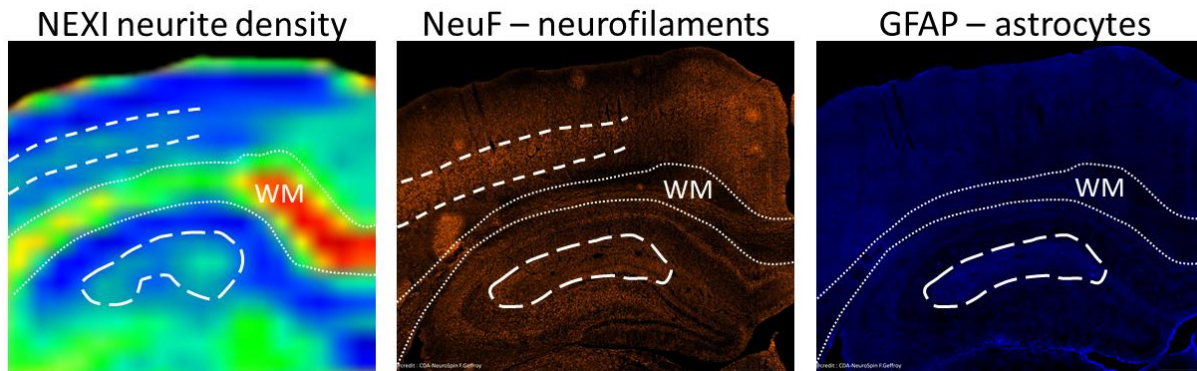
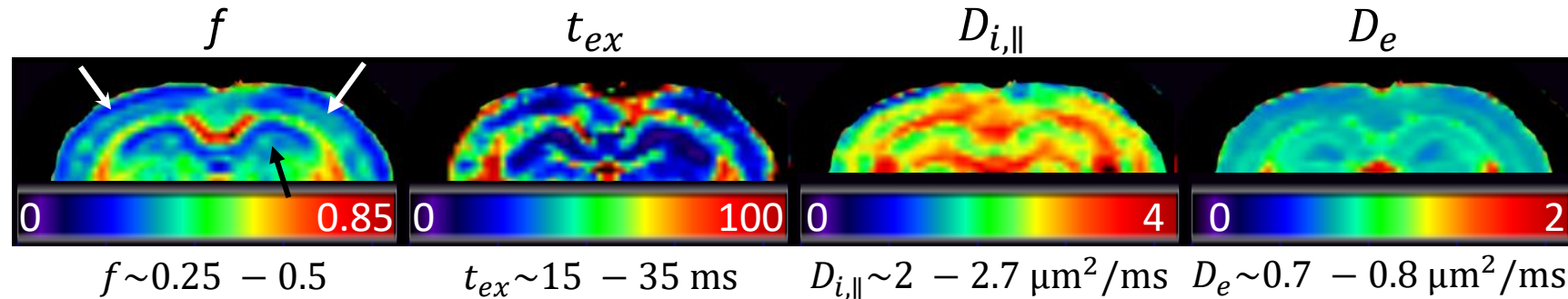
$b = 0 - 10 \text{ ms}/\mu\text{m}^2$

$\Delta = 10 - 45 \text{ ms}$

$$\frac{\text{blue box}}{\text{blue box} + \text{red box}} = f$$



$$\mathbf{p} = [f, D_{i,||}, D_e, t_{ex}]$$



$$t_{ex} \cong \frac{d}{4P}$$

$$P \sim [2 - 33] \times 10^{-3} \mu\text{m}/\text{ms}$$

Vs literature:

$$P \sim [6 - 30] \times 10^{-3} \mu\text{m}/\text{ms}$$

Harkins, Magn Reason Med 2009

Finkelstein, 1987

Latour, PNAS 1994

Stanisz, Magn Reason Med 1997

NEXI – human *in vivo*

3T Connectom scanner ($G_{\max}$ 300 mT/m)

$$b = 1 - 7.5 \text{ ms}/\mu\text{m}^2$$

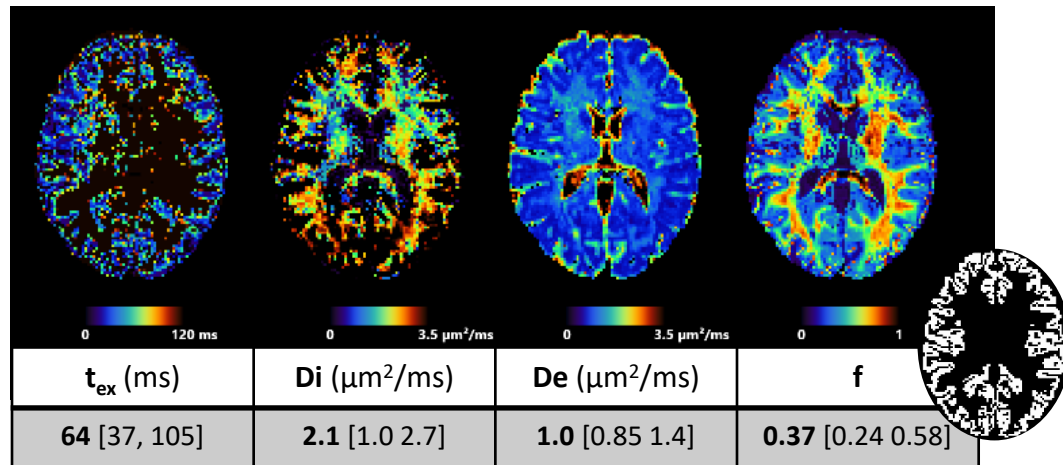
$$\Delta = 20 - 40 \text{ ms}$$

$$\delta = 9 \text{ ms}$$

1.8-mm isotropic resolution

Scan time: 45 min

Estimation: Non-linear least squares



3T Clinical scanner ($G_{\max}$ 80 mT/m)

$$b = 1 - 5 \text{ ms}/\mu\text{m}^2$$

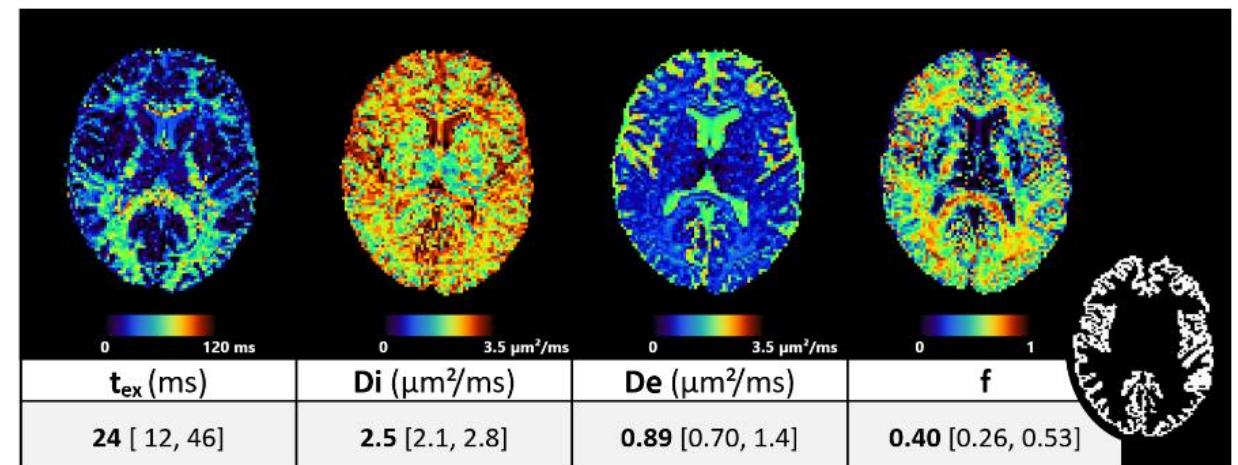
$$\Delta = 28 - 65 \text{ ms}$$

$$\delta = 16.5 \text{ ms}$$

2-mm isotropic resolution

Scan time: 35 min

Estimation: Multi-layer perceptron

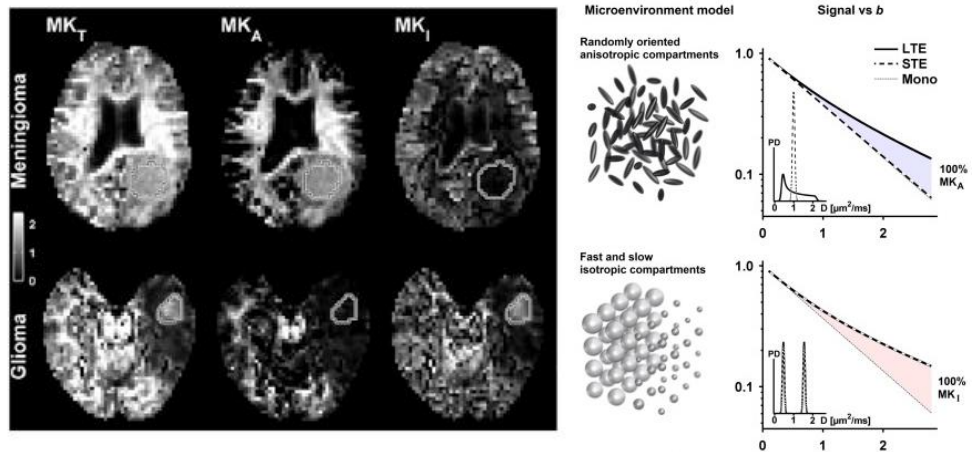


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Microstructure models (almost) outside the brain

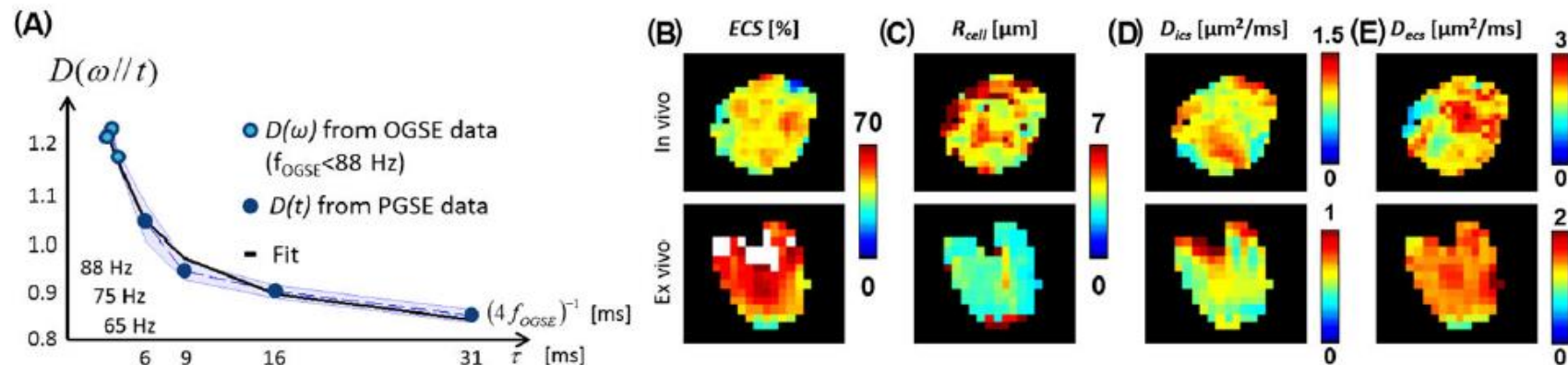
A. Diffusional variance decomposition



Szczepankiewicz, NeuroImage 2016

Glioma

Quantification of cell density, radius, and compartment diffusivities



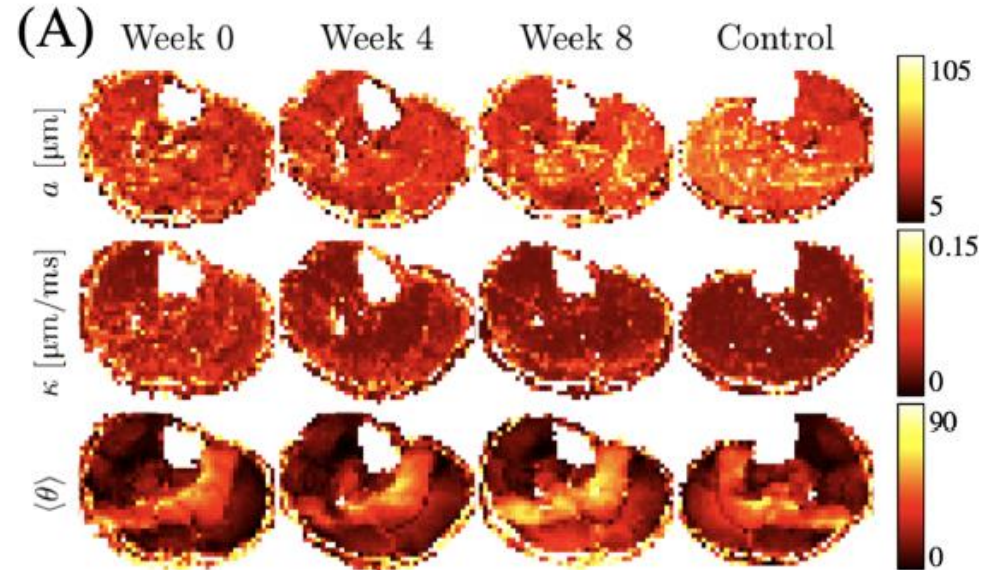
Reynaud, NMR in Biomed 2016

Microstructure models outside the brain

Random Permeable Barrier Model

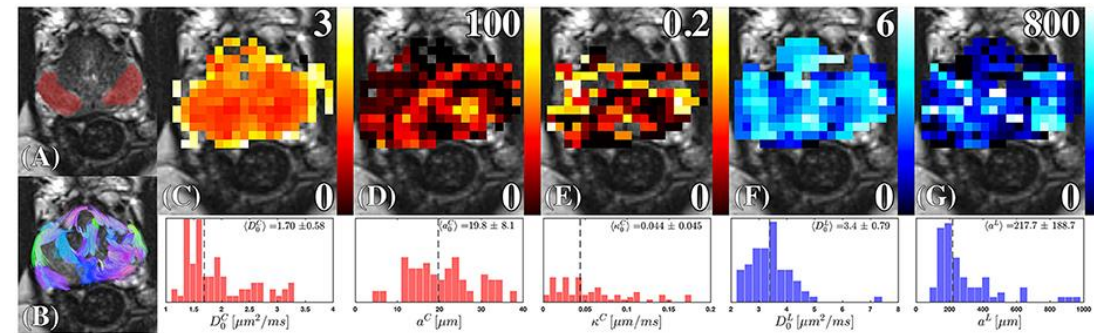
Muscle

Quantification of myofiber size, permeability and orientation



Lemberskiy, NMR in Biomed 2021

Prostate



Cellular diffusivity, fiber diameter, and membrane permeability

Luminal diffusivity and diameter

Lemberskiy, Front Phys 2018

Summary

- Biophysical tissue models:
 - In theory: specific metrics of microstructure
 - In practice: dependent on validity of model assumptions & fitting procedure
 - Interpret output with caution!
- Substantial progress in white matter modeling
 - Estimation of main Standard Model parameters comprehensively
 - Accessible applications and translation to preclinical & clinical studies of disease
- Current focus: gray matter modeling
 - Accounting for inter-compartment exchange appears critical
 - Parameter estimation may require (q-t) coverage
 - Challenge for retrospective studies & prospective clinical studies
 - Eventually: propose a more comprehensive model (exchange + soma + ...)
- Other organs / tissue types
 - High interest

Reviews:

Jelescu and Budde, Front Phys 2017
Novikov, MRM 2018
Nilsson, NeuroImage 2018
Alexander, NMR in Biomed 2019
Novikov, NMR in Biomed 2019
Jelescu, J Neurosci Meth 2020

Guideline Papers:

Jelescu, [...], Schilling, arXiv 2022
Schilling, [...], Jelescu, arXiv 2022

Questions?

