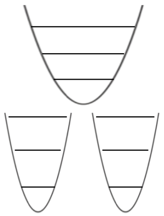
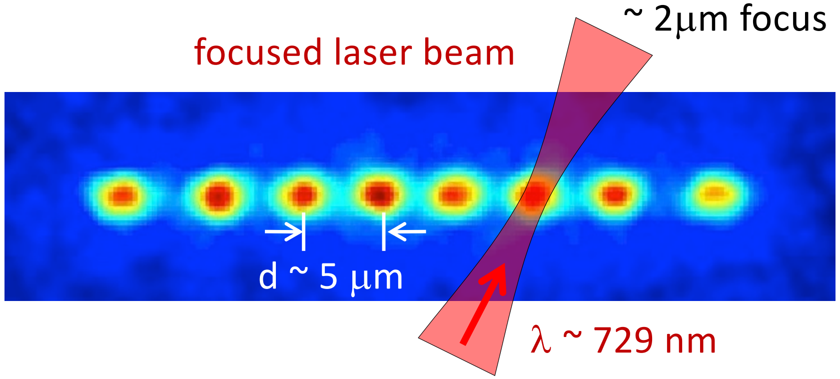
Interactions in linear ion strings

Trap frequencies:

$\nu_z \propto 1\ \mathrm{MHz}$

$\nu_{x,y} \propto 5\ \mathrm{MHz}$





Length scales

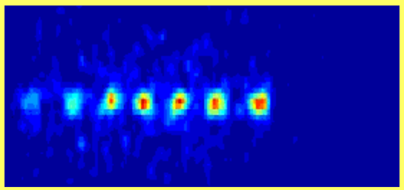
ion distance		$^{40}Ca^+$ wavelengths		ion localisation		Bohr radius
d	>	λ	>>	z_0	>>	a_0
5 μm		397 nm 729 nm		10-20 nm		50 pm

Vibrational modes

"all-to-all connectivity"

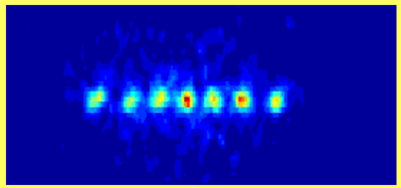
centre-of-mass mode

$\nu = \nu_z$



breathing mode

$\nu = \sqrt{3}\,\nu_z$



"quantum bus to connect qubits"

(Reminder) Transformation to an interaction picture

goal: look at evolution of a driven system, separate out time evolution of unperturbed system

time dependent SE: $i\hbar \frac{\partial}{\partial t} |\psi\rangle = H |\psi\rangle$ $|\psi(0)\rangle \xrightarrow{\boxed{U}} |\psi(t)\rangle$

$H = H_0 + H_1$

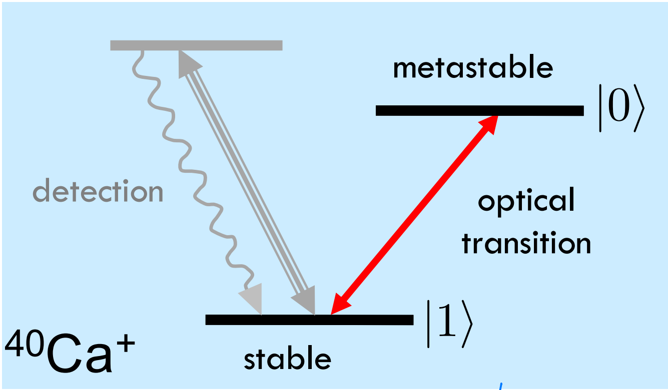
time indep. (H_0) time dependent (H_1)

$$\boxed{H_{int} = U_0^\dagger H_1 U_0 \quad \text{where } U_0 = e^{-iH_0 t/\hbar}}$$

$$\psi(t) = U_0 U_{int} U_0^\dagger |\psi(0)\rangle$$

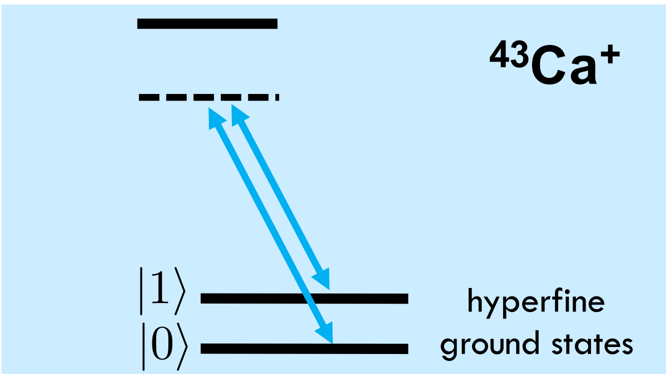
Laser-ion interaction: two level atom and laser light

optical qubit



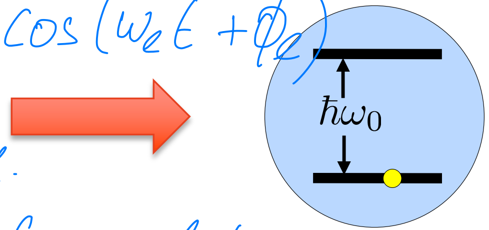
$W_0 = 2\pi \times 411\ THz$

hyperfine qubit



$W_0 = 2\pi \times 2.87\ GHz$

$H_\ell = \hbar \Omega \sigma_x \cos(\omega_\ell t + \phi_\ell)$



Atom

$H_a = \frac{\hbar \omega_0}{2} \sigma_z$

$\Omega \propto E_0$ Rabi freq.

ω_ℓ, ϕ_ℓ ... laser freq and phase

$$\boxed{H = H_a + H_\ell}$$

transform to interact. picture + rotating wave approx (RWA)

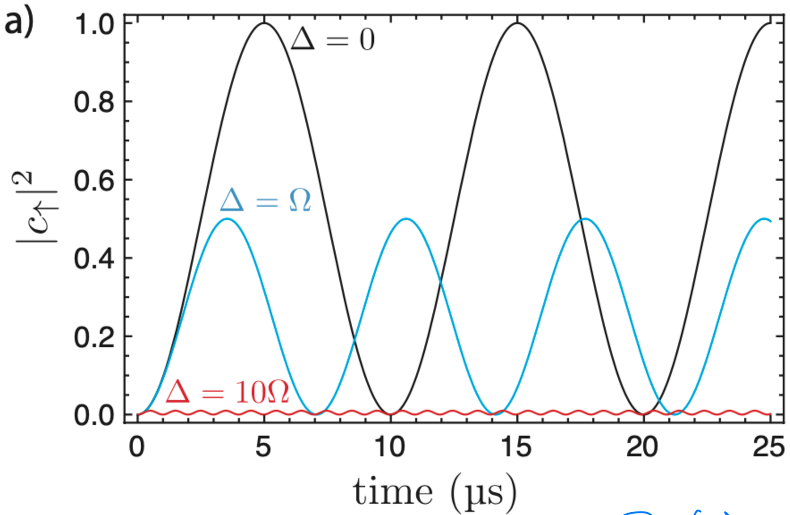
$\Delta = \omega_\ell - \omega_0$ (drop $\omega_\ell + \omega_0$ terms)

$$H_{int} = \hbar \frac{\Omega}{2} \left[\cos(\Delta t + \phi_\ell)\, \sigma_x + \sin(\Delta t + \phi_\ell)\, \sigma_y \right]$$

$$\sigma_\pm = (\sigma_x \pm i \sigma_y)/2$$

$$H_{int} = \hbar \frac{\Omega}{2} \left(e^{-i(\Delta t + \phi_\ell)} \sigma_+ + e^{i(\Delta t + \phi_\ell)} \sigma_- \right)$$

Laser-ion interaction: two level atom and laser light



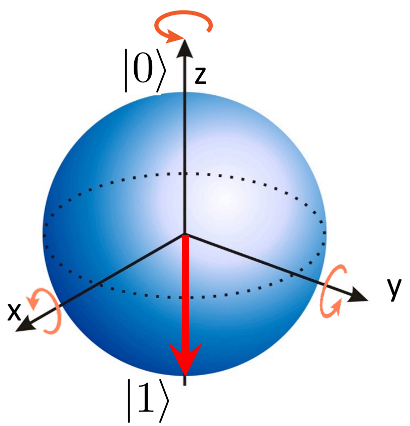
Rabi oscillations

on resonance ($\Delta = 0$)

$$U = e^{-i\frac{H_{int} t}{\hbar}} = \begin{pmatrix} \cos\frac{\Theta}{2} & -ie^{-i\phi_L}\sin\frac{\Theta}{2} \\ -ie^{i\phi_L}\sin\frac{\Theta}{2} & \cos\frac{\Theta}{2} \end{pmatrix}$$

$\Theta = \Omega \cdot t$ pulse area

$$P_\uparrow = |c_\uparrow|^2 = \frac{\Omega^2}{\Omega^2+\Delta^2}\sin^2\left(\frac{1}{2}\sqrt{\Omega^2+\Delta^2}\,t\right)$$



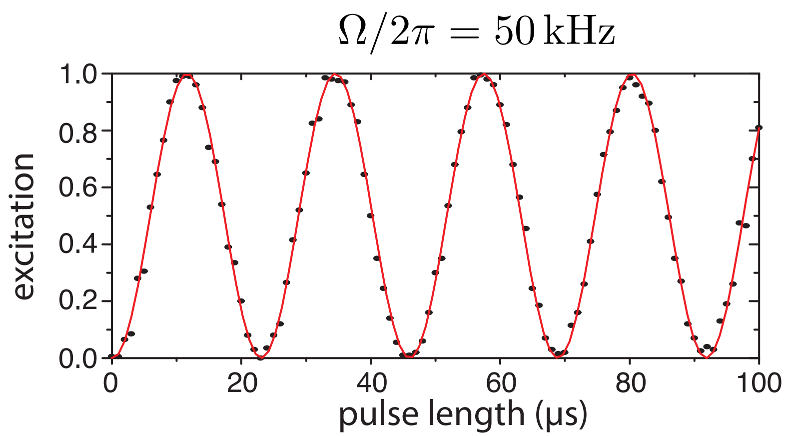
$\omega_{laser} = \omega_0$

$H \propto \sigma_x$

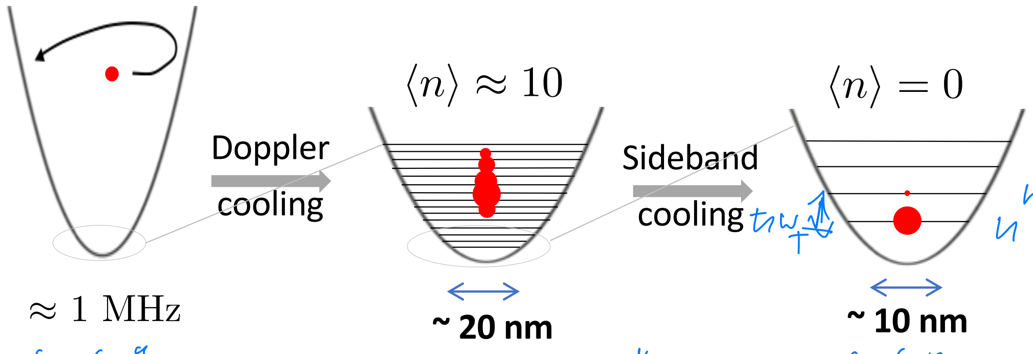
$H \propto \sigma_y$ for $\phi_L = 90°$

$|\omega_{laser} - \omega_0| \gg \Omega$

$H \propto \sigma_z$



From ion trap to quantum harmonic oscillator



"Fock states"

$n=2$

$n=1$

$n=0$

$\hbar\omega_T$

"particle" classical $\longrightarrow$ "wave packet" quantum

$\downarrow$

quantized excitation of energy $\hbar\omega_T$

$T_D = 0.5\,mK$

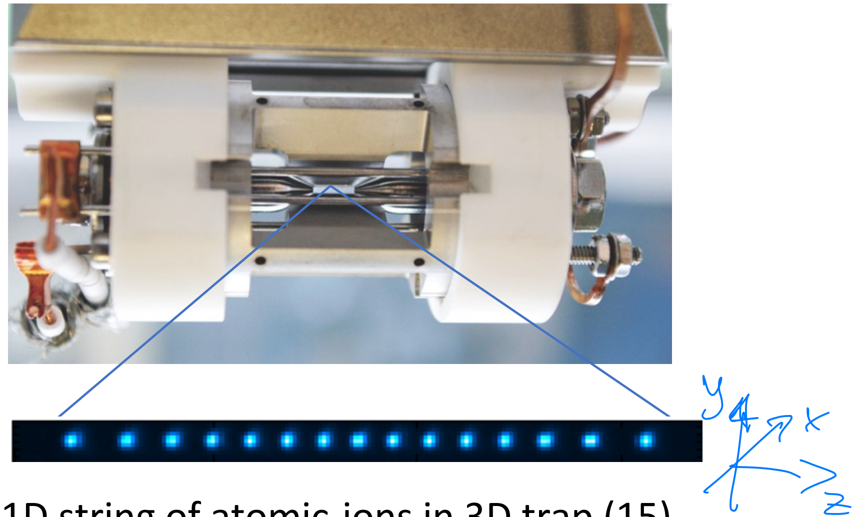
$\downarrow$

mean phonon number $\langle n \rangle$

$k_B T_D = \hbar \omega_T$

typical $\omega_T/2\pi \approx 1\,MHz \rightarrow \langle n \rangle \approx 10$

"Lamb-Dicke regime"



1D string of atomic-ions in 3D trap (15)

for each ion 3 motional modes

3N for N ions

example 2 ions

z, y, x ... $\Big\}$ Center of mass COM

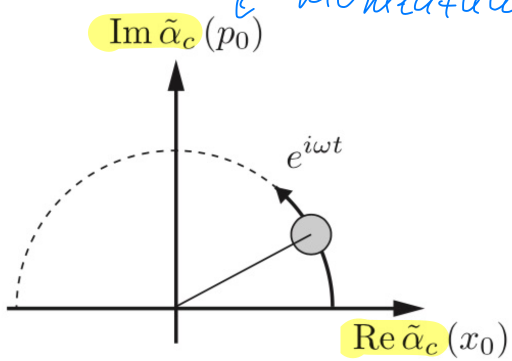
"stretch"

"rocking" mode

$\omega_{stretch} = \sqrt{3}\,\omega_{com}$

~ 1.7

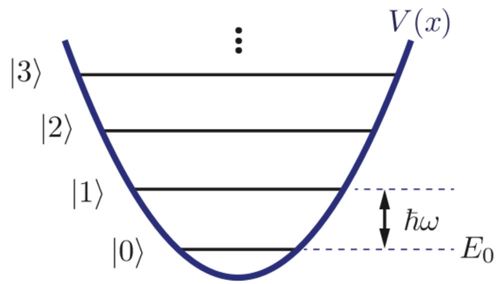
Quantum harmonic oscillator



momentum

position

phase space



$\hat{N}$

↑ creation operator

$$a^+ = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$$

↓ annihilation operator

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)$$

classical oscillator $\alpha_c = x_c + i p_c$

$$\alpha(t) = |\alpha_c| e^{i\omega t}$$

quantum oscillator

$$H_{h.o.} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2$$

$$\hat{x} = (a + a^+)\, x_0 \quad ; \quad \hat{p} = i(a - a^+)\frac{\hbar}{2x_0}$$

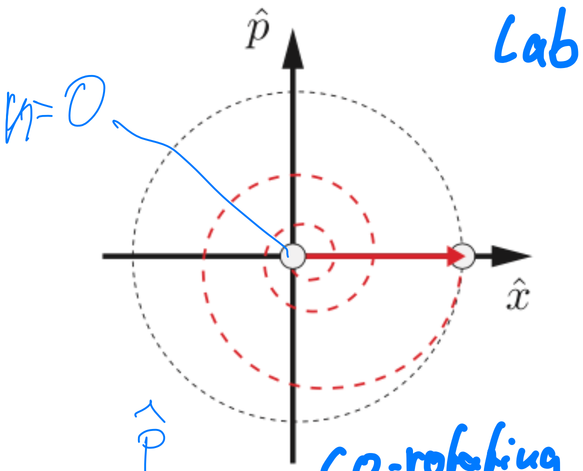
with $x_0 = \sqrt{\frac{\hbar}{2m\omega}}$ ground state wave packet size (rms) $\simeq 10\,nm$ for Ca^+ at $1\,MHz$

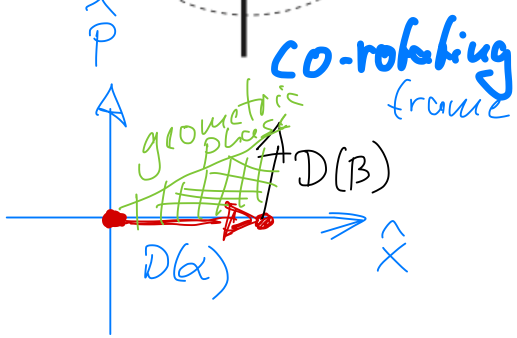
$$H_{h.o.} = \hbar\omega\left(a^+ a + \frac{1}{2}\right) = \hbar\omega\left(\hat{N} + \frac{1}{2}\right)$$

number operator

apply time-dependent force

Driving the quantum harmonic oscillator





$$F(t) = A\sin(\omega_d t + \phi_d) = \frac{A}{2i}\left(e^{i(\omega_d t+\phi_d)} - e^{-i(\omega_d t+\phi_d)}\right)$$

interaction energy $H_I(t) = -\hat{x}\,F(t)$

$$H = H_{h.o.} + H_I(t)$$

$$H_{int} = \left(a e^{-i\omega t} + a^\dagger e^{i\omega t}\right) x_0 F(t)$$

if drive resonant $(\omega_d = \omega)$ + RWA

$$H_{int} = \frac{A x_0}{2i}\left(a e^{i\phi_d} + a^\dagger e^{-i\phi_d}\right)$$

Displacement operator

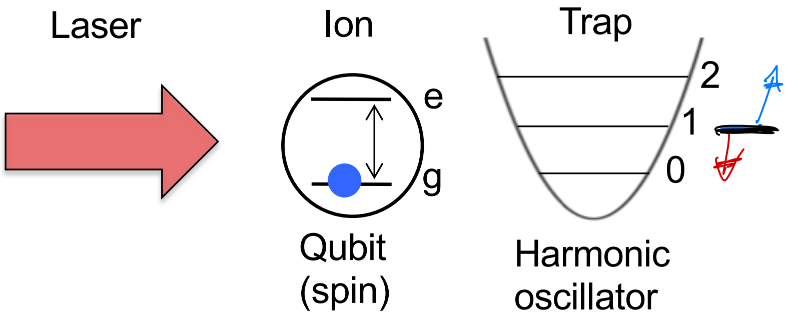
$$\boxed{D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}}$$

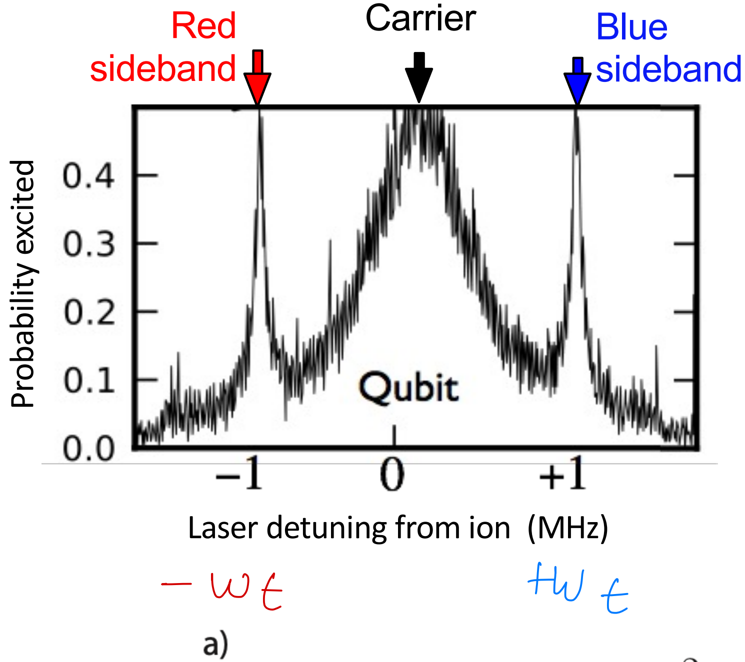
Define $\alpha = e^{i\phi_d} A x_0 t / 2\hbar$

↳ dimensionless complex ampl.

$$D(\alpha)D(\beta) = D(\alpha+\beta)\underbrace{e^{\frac{1}{2}(\alpha\beta^* - \alpha^*\beta)}}_{\text{geometric phase}} = D(\alpha+\beta)e^{i\,\mathrm{Im}(\alpha\beta^*)}$$

Laser-ion interaction: putting it all together





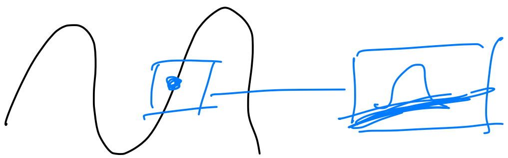
$-\omega_t$ $\qquad$ $+\omega_t$

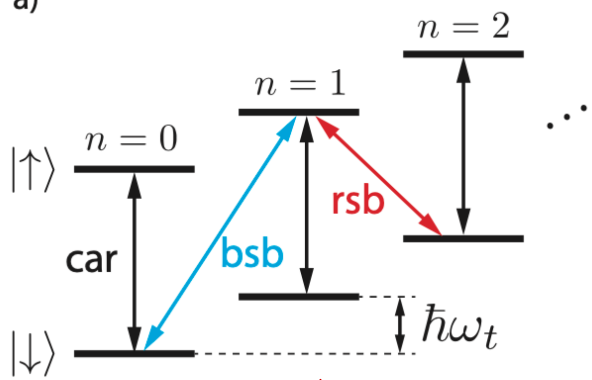
$$H_0 = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega_t\left(a^\dagger a + \frac{1}{2}\right)$$

ion wave packet ~10um, light 400 − 700 nm

λ_L

Lamb-Dicke factor η





Fock space

$$\eta = k x_0 \cos\Theta$$

$$= k\sqrt{\frac{\hbar}{2m\omega_t}}\cos\Theta$$

$k \quad \frac{2\pi}{\lambda}$

light (k) Θ

motion (ion x_0)

Laser-ion interaction: putting it all together

$$H_1 = \hbar \frac{\Omega}{2}(\sigma_+ + \sigma_-)\left(e^{i\eta(a+a^\dagger)} e^{-i(\omega_\ell t - \phi_\ell)} + e^{-i\eta(a+a^\dagger)} e^{i(\omega_\ell t + \phi_\ell)}\right)$$

→ interaction picture w.r.t. H_0 + RWA

$$H_{int} = \hbar\frac{\Omega}{2}\left(e^{-i(\Delta_\ell t - \phi_\ell)}\sigma_+ \exp\left(i\eta\left(a e^{-i\omega_\ell t} + a^\dagger e^{i\omega_\ell t}\right)\right) + h.c.\right)$$

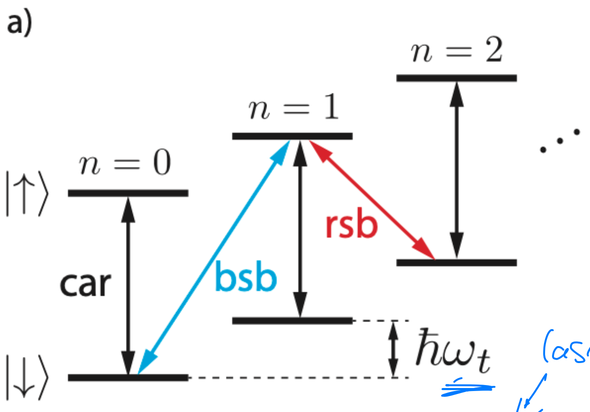
$x_0 \ll \lambda_{laser}$ → simplify w. Taylor expansion

$$\exp\{i\eta(\ldots)\} = 1 + i\eta\left(a e^{-i\omega_t t} + a^\dagger e^{i\omega_t t}\right) + O(\eta^2)$$

ok if $\langle n \rangle \leq 20$

$$\rightarrow H_{int} = \hbar\frac{\Omega}{2}\left(e^{-i(\Delta t - \phi_\ell)}\sigma_+\left\{1 + i\eta(\;) + h.c.\right\}\right.$$

Laser-ion interaction: putting it all together



laser

assume $\Omega \ll \underline{\omega_t}$ second RWA "adiabatic regime"

$\Omega \sim 50\,\mathrm{kHz}$, $\omega_t \sim 1\,\mathrm{MHz}$

$$\Omega_n = (1-\eta^2 n)\,\Omega_0$$

$$\Omega_{n,n-1} = \eta\sqrt{n}\,\Omega_0$$

$$\Omega_{n,n+1} = \eta\sqrt{n+1}\,\Omega_0$$

effective Rabi freq.

for $\Delta = 0$ "carrier"

$$H_{car} = \hbar\frac{\Omega}{2}\left(e^{i\phi_e}\sigma_+ + e^{i\phi_e}\sigma_-\right)$$

for $\Delta = -\omega_t$ "red sideband" (RSB) Jaynes-Cummings H

$$H_{RSB} = i\hbar\eta\frac{\Omega_{n,n-1}}{2}\left(\dots \right)$$

for $\Delta = +\omega_t$ "blue sideband" (BSB) Anti-Jaynes Cummings

$$H_{BSB} = i\hbar\eta\frac{\Omega_{n,n+1}}{2}\left(\dots \right)$$