Gauge Theories and the Standard Model

Problem Set 7

Due Tuesday, November 12, in class (BSP 727)

Problem 1: Parity and charge conjugation in $W^\pm$ decays

Consider the two-body decays of polarized W^+ and W^- bosons to $e^+\nu_e$ and $e^-\bar{\nu}_e$, respectively. Similarly to $u(p,s)$ and $v(p,s)$ for fermions a massive vector propagates three degrees of freedom described by three polarization vectors. We can choose the basis vectors to describe spin-eigenstates along the z direction. In this case, in the rest frame of the boson ($k = (M_W, \vec{0})$) they can be chosen to be

$$\epsilon^\mu_{W^-}(k, s=+1) = \epsilon^\mu_{W^+}(k, s=+1) = \frac{1}{\sqrt{2}}(0,-1,-i,0) \qquad \text{(transverse } W^+ \text{ or } W^-\text{)}\,,$$

$$\epsilon^\mu_{W^-}(k, s=-1) = \epsilon^\mu_{W^+}(k, s=-1) = \frac{1}{\sqrt{2}}(0,1,-i,0) \qquad \text{(transverse } W^+ \text{ or } W^-\text{)}\,,$$

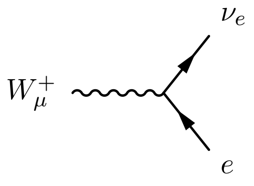
$$\epsilon^\mu_{W^-}(k, 0) = \epsilon^\mu_{W^+}(k, 0) = (0,0,0,1) \qquad \text{(longitudinal } W^+ \text{ or } W^-\text{)}\,,$$

(i) Suppose somebody measures for you the six differential decay widths

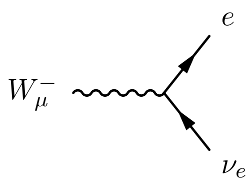
$$\frac{d\Gamma}{d\cos\theta}(W^+_{T/L} \to e^+\nu_e)\,, \qquad \frac{d\Gamma}{d\cos\theta}(W^-_{T/L} \to e^-\bar{\nu}_e)\,,$$

inclusive in all fermion polarizations. List all seven independent relations among those differential rates that would be fulfilled if C and P were exact symmetries. What are the relations if only the combination CP was a symmetry?

(ii) Using the Feynman rules of the SM



$i\frac{e}{\sqrt{2}s_{\mathrm{w}}}\gamma_\mu P_L$



$i\frac{e}{\sqrt{2}s_{\mathrm{w}}}\gamma_\mu P_L$

compute the six partial widths neglecting the mass of the electron. Which of the relations from question (i) are not satisfied?

Problem 2: Muon decay in the Electroweak theory

(i) Consider the (dominant) three-body decay of a muon to an electron and neutrinos

$$\mu^-(P) \to e^-(p)\bar{\nu}_e(q_1)\nu_\mu(q_2)\,.$$

Draw the Feynman diagram(s) that contribute to this process in the SM. Neglect the electron mass with respect to the muon mass and compute the amplitude square by averaging over initial and summing of final fermion polarizations.

(ii) In the rest frame of the muon the differential width is then obtained by the matrix-element square times the three-body phase-space element

$$d\Gamma = \frac{(2\pi)^4}{2m_\mu}|\mathcal{M}|^2\delta^4(P-p-q_1-q_2)\frac{d^3\boldsymbol{p}}{(2\pi)^3 2E_e}\frac{d^3\boldsymbol{q}_1}{(2\pi)^3 2E_1}\frac{d^3\boldsymbol{q}_2}{(2\pi)^3 2E_2}\,.$$

Show that for the muon decay the result is

$$d\Gamma = \frac{4G_\mathrm{F}^2}{(2\pi)^5 m_\mu}(q_2\cdot p)(q_1\cdot P)\delta^4(P-p-q_1-q_2)\frac{d^3\boldsymbol{p}}{E_e}\frac{d^3\boldsymbol{q}_1}{E_1}\frac{d^3\boldsymbol{q}_2}{E_2}\,.$$

(iii) Integrate first over the phase space of the neutrino momenta. To this end define and evaluate the corresponding tensor integral

$$I^{\mu\nu} = \int\frac{d^3\boldsymbol{q}_1}{E_1}\frac{d^3\boldsymbol{q}_2}{E_2}q_1^\mu q_2^\nu\delta(P-p-q_1-q_2)\,.$$

What are the transformation properties of $I^{\mu\nu}$ under Lorentz? On which linear combination of momenta can the integral only depend on? To evaluate $I^{\mu\nu}$ decompose it in terms of independent Lorentz structures.
(Hint: Remember that Lorentz scalars are independent of the reference frame, so you may evaluate them without loss of generality in the centre-of-mass frame. You should find that

$$I^{\mu\nu} = \frac{\pi}{6}(\eta^{\mu\nu}q^2 + 2q^\mu q^\nu)$$

with q the only momentum on which $I^{\mu\nu}$ can depend on.)

(iv) Integrate also over the allowed phase space of the electron and show that the width is

$$\Gamma_\mu = \frac{G_\mathrm{F}^2 m_\mu^5}{192\pi^3}$$

(v) Use the experimental input for muon lifetime from `pdglive.lbl.gov` to determine G_F. Compare your value of G_F to the value that PDG lists and discuss which are the approximations that could account for the difference in the values.

(vi) Compute the partial width of τ for two of its decay modes using the previous result.

(vii) Now assume that the muon decay was instead (dominantly) mediated by a scalar particle rather than by the W boson. Consider the following interaction term with and a $SU(2)_L$-doublet scalar with mass m_Φ

$$\mathcal{L}_\mathrm{int} \supset Y_\Phi \bar{L}_i \Phi e_i + h.c$$

for simplicity assume that the Yukawa coupling Y_Φ is diagonal in the SM mass basis and is flavor-universal (independent of lepton flavor). Find the decay rate of the muon in this theory.

(viii) Argue that using only the polarization averaged/summed differential decay rates and assuming that the neutrinos are not directly detected, one cannot distinguish wether the muon decay was mediated by a scalar or the SM W boson.

(ix) Propose an observable that can tell apart Φ-mediation from W-mediation in a muon decay experiment.