Gauge Theories and the Standard Model

Problem Set 6

Due Tuesday, November 5, in class (BSP 727)

Problem 1: Spontaneous breaking in the linear SU$(2)\times$SU(2) σ-model

Consider the theory of four real scalar fields, Φ_1, Φ_2, Φ_3 and Φ_4, that is invariant under a global SO(4) symmetry. Using the isomorphism $\mathrm{SO}(4) \hat{=} \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ we can rewrite the real scalars in terms of the 2×2 matrix

$$\Sigma = \frac{1}{\sqrt{2}}(\mathbb{I}_2 \Phi_4 + i\sigma_a \Phi_a) \,.$$

Σ transforms as a $(\mathbf{2}, \mathbf{2})$ under the global symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ via

$$\Sigma \longrightarrow U_L \Sigma U_R^\dagger \,,$$

with $U_L \in \mathrm{SU}(2)_L$ and $U_R \in \mathrm{SU}(2)_R$.

(i) Show that Σ is "pseudo-real", namely that

$$\Sigma^* = \sigma_2 \Sigma \sigma_2$$

and verify that this property is respected by the transformation above.

(ii) Show that at the dimension-four level the most general Lagrangian that is invariant under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ is

$$\mathscr{L} = \frac{1}{2}\mathrm{Tr}[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger] + \frac{1}{2}\mu^2 \mathrm{Tr}[\Sigma\Sigma^\dagger] - \frac{1}{4}\lambda \mathrm{Tr}[\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger] \,.$$

What about invariants involving the determinant and terms with epsilon-tensor contractions?
(Hint: Work with the components ${\Sigma_{i_L}}^{i_R}$ and remember that SU(2) *has two invariant tensors.)*

(iii) If μ^2 is positive the theory exhibits spontaneous-symmetry breaking. Pick a vacuum of your liking and expand the Lagrangian around it. How many massless states (Goldstone bosons) do you find in the theory?
(Hint: It is easier to choose the vacuum if you express Σ in terms of Φ_is.)

(iv) For the vacuum that you picked, find $\langle \Sigma \rangle$ and find the broken and unbroken generators of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ by proceeding similarly to the proof of the Goldstone Theorem.

(v) What is the symmetry breaking pattern that you found, which symmetry is the vacuum still preserving?

Problem 2: Branching fractions for the decay of the Z boson

(i) Re-derive the Feynman rules for the Standard Model (SM) interaction vertices involving the electroweak gauge bosons, Higgs and the SM fermions. Ignore the lepton masses and quark masses (and mixings), except for the top and bottom quarks.

(ii) Compute the decay width of the Z boson to the SM fermions and find the branching fractions of Z to various two fermion final states.

(iii) Compare your result with the data published by the Particle Data Group for the Z branching ratios.