Gauge Theories and the Standard Model

Problem Set 5

Due Tuesday, October 29, in class (BSP 727)

Problem 1

Starting the model with a $U(1)$ global symmetry

$$|\partial\Phi|^2 - V(\Phi),$$

with

$$V = m^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4,$$

and $m^2 < 0$.

(i) Find the Noether current $\hat{J}^\mu$ in this theory in terms of Φ.

(ii) Express the Noether current using the parameterization

$$\Phi = \frac{1}{\sqrt{2}}\big[v + \sigma(x)\big]e^{i\phi(x)/v}.$$

(iii) Use this to compute the Goldstone boson decay constant f_ϕ in terms of the Lagrangian parameters using the relation.

$$\langle\phi(p)|\hat{J}^\mu(x)|0\rangle = ie^{ip\cdot x}p^\mu f_\phi.$$

You should see that $f_\phi \neq 0$ is exactly related to there being spontaneous symmetry breaking.

Problem 2

This problem explores the linear versus non-linear representation of the Goldstone boson by computing Goldstone self-scattering in both descriptions. Take the theory with a global $U(1)$ symmetry from the previous problem.

(i) Express the Lagrangian in the broken phase using the non-linear parameterization

$$\Phi(x) = \frac{1}{\sqrt{2}}\big[v + \sigma(x)\big]e^{i\phi(x)/v}.$$

(ii) Using the resulting Lagrangian, compute $\phi\phi \to \phi\phi$ scattering amplitude.

(iii) Now express the same Lagrangian in the broken phase using the linear parameterization

$$\Phi = \frac{1}{\sqrt{2}}\big(v + \Phi_1 + i\Phi_2\big).$$

(iv) Compute $\Phi_2\Phi_2 \to \Phi_2\Phi_2$ scattering amplitude using this parameterization. Notice that it is only clear that Φ_2 is effectively derivatively coupled at the end of the calculation.