Gauge Theories and the Standard Model

Problem Set 4

Due Tuesday, October 15, in class (BSP 727)

Problem 1

(Problem 14.1 of the book by Schwartz) Show that for complex scalar fields

$$\int \mathcal{D}\phi^* \mathcal{D}\phi \exp\left[i \int d^4x (\phi^* M \phi + J^* \phi + \phi^* J)\right] = \mathcal{N} \frac{1}{\det M} \exp\left(i J^* M^{-1} J\right)$$

for some (infinite) constant $\mathcal{N}$.

Problem 2

Consider the Lagrangian of scalar QED, the theory of a charged (complex) scalar field ϕ interacting with the electromagnetic field A_μ:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* (D^\mu \phi) - m^2 |\phi|^2,$$

where $D_\mu = \partial_\mu + i q_\phi e A_\mu$ and q_ϕ is the charge of the scalar.

(i) Using the functional methods, find the propagator of the complex scalar field. Then derive the Feynman rules for the interactions between the photon and the scalar particles.

(ii) Compute the differential cross section for $e^+ e^- \to \phi \phi^*$, ignoring the electron mass (but not the scalar mass) and averaging over the initial polarizations. Find the angular dependence and total cross section. Compare your results with the corresponding results for $e^+ e^- \to \mu^+ \mu^-$ which you can find in Chapter 5 of the QFT book by Peskin and Schroeder.