# Gauge Theories and the Standard Model

## Problem Set 2

Due Tuesday, October 1, in class (BSP 727)

**Lecture:** Tim Cohen
**Exercises:** Majid Ekhterachian, Stefan Stelzl

## Problem 1: Ward identity for QCD?

In the first problem set you considered the process of electron-positron scattering into two photons. For this problem, consider a similar process in a non-Abelian gauge theory: scattering of quark-antiquark into two gluons.

(i) Draw all the Feynman diagrams contributing to the amplitude for this process.

(ii) Writing the amplitude as $\mathcal{M} = \mathcal{M}_{\mu\nu}\epsilon_1^\mu\epsilon_2^\nu$, where $\epsilon$'s are the gluon polarization vectors, find the expression for $\mathcal{M}_{\mu\nu}$.

(iii) Check if the Ward identity, $\mathcal{M}_{\mu\nu}\epsilon_1^\mu k_2^\nu = 0$, holds for the expression you have obtained, for $\epsilon_1$ being the polarization vector for gluon with (a) transverse and (b) non-transverse polarizations. Also check the above for the following two contributions separately: (1) contribution of the diagrams not involving the non-Abelian gluon self-interaction vertex (2) contribution of the other diagram(s).

(iv) How do you interpret the results you obtained in the previous part?

## Problem 2

Taking the matrix element you found in Problem 1, in this problem you will compute the differential cross section for quark-antiquark annihilation into two gluons with transverse polarizations, $\frac{d\sigma}{dt}(q\bar{q} \to gg)$. You can ignore all masses and use explicit polarization vectors and spinors.

(i) Which combination of initial state helicities give vanishing contribution?

(ii) Compute the amplitudes for all combination of initial and final state helicities.

(iii) Find the various helicity cross sections.

(iv) BONUS: Find the total differential cross section averaged over initial spins and colors.