Gauge Theories and the Standard Model

Problem Set 12

Due Tuesday, December 17, in class (BSP 727)

Problem: Neutrino-nucleon scattering

In this problem, we will first (re-)derive the expression we saw in the lecture for deep inelastic electron scattering cross section in terms of the structure functions. We then repeat the similar analysis for neutrino-nucleon scattering.

(i) Show that the electron nucleon scattering cross section from photon exchange can be written in terms of two Lorentz-invariant structure functions W_2 and W_2 as

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right)$$

where E (E') is the energy of the incoming (outgoing) electron in the nucleon rest frame and θ is the electron scattering angle.

(ii) Now consider the neutrino-nucleon scattering process, $\nu_l + N \to l^- + X$. Seperate the (charged) weak current as a sum of the leptonic and hadronic parts,

$$J^\lambda = J_l^\lambda + J_h^\lambda.$$

What is the explicit form of the leptonic part, J_l^λ?

(iii) Considering the hadronic part, define

$$W_{\alpha\beta}^{(\nu)} = \frac{1}{4M} \Sigma_\sigma \int \frac{d^4x}{2\pi} e^{iq.x} \langle p, \sigma | \left[J_{h\beta}^\lambda(x), J_{h\alpha}^{\lambda\dagger}(0) \right] | p, \sigma \rangle.$$

Express $W_{\alpha\beta}^{(\nu)}$ in terms of Lorentz invariant structure functions. How many such functions are there in general?

(iv) Show that the cross section for neutrino scattering can be written in terms of only three of the Lorentz-invariant structure functions

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{G_F^2}{2\pi^2} E'^2 \left(2W_1^{(\nu)} \sin^2 \frac{\theta}{2} + W_2^{(\nu)} \cos^2 \frac{\theta}{2} - W_3^{(\nu)} \frac{E + E'}{M} \sin^2 \frac{\theta}{2} \right).$$