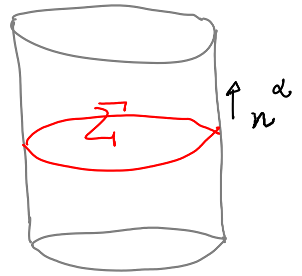
• the conserved charges are given by



$$Q_\xi = \int_{\partial\Sigma} d\varphi \, \sqrt{\sigma} \, n^\alpha \, T_{\alpha\beta} \, \xi^\beta$$

ξ^β ← asympt. diffeo

n^α ↗ timelike unit normal

$T_{\alpha\beta}$ ← renormalized BY stress tensor

• the induced metric on $\partial\Sigma$, cut off @ $z=\varepsilon$, scales as $\frac{\hat{\sigma}_{\alpha\beta}}{\varepsilon^2}$ (e.g, $\frac{d\varphi^2}{\varepsilon^2}$), while the unit timelike normal scales as $n^\alpha = \varepsilon \, \hat{n}^\alpha$ (unit timelike normal w.r.t. $g^{(0)}$). Letting $\xi^\alpha \big|_{z=\varepsilon} \equiv \chi^\alpha$, assumed finite, the the expr. for the charge becomes

$$Q_\xi = \int_{\partial\Sigma} d\varphi \, \sqrt{\hat{\sigma}} \, \hat{u}^\alpha \, T_{\alpha\beta} \, \chi^\beta$$

in the $\varepsilon \to 0$ limit

• the charges are conserved due to the fact that $\nabla^\alpha_{(0)}(T_{\alpha\beta}\chi^\beta) = 0$ on the boundary ⇐ χ^α is a conformal Killing vector of the bnd. metric

$$\nabla_\alpha(T^{\alpha\beta}\chi_\beta) = \underbrace{(\nabla_\alpha T^{\alpha\beta})}_{0 \text{ by e.o.m.}}\chi_\beta + T^{\alpha\beta}\nabla_\alpha\chi_\beta = T^{\alpha\beta}\underbrace{\nabla_{(\alpha}\chi_{\beta)}}_{\propto\, g^{(0)}_{\alpha\beta}} \propto \operatorname{tr} T = 0$$

• w/ this, the charges become

$$Q_\xi = \int_0^{2\pi} d\varphi \, \hat{u}^t \, T_{t\alpha} \, \chi^\alpha = \frac{1}{8\pi G \ell}\int_0^{2\pi} d\varphi \left(g^{(2)}_{+\alpha} - g^{(2)}_{-\alpha}\right)\chi^\alpha$$

and so

$$Q_{\xi_L} = \frac{1}{8\pi G\ell}\int_0^{2\pi} d\varphi \, \mathcal{L}(x^+)\,\chi_L(x^+) \; ; \qquad Q_{\xi_R} = -\frac{1}{8\pi G\ell}\int_0^{2\pi} d\varphi \, \bar{\mathcal{L}}(x^-)\,\chi_R(x^-)$$

- the commutator of the charges is

$$\{Q_{\eta_L}, Q_\xi\} \equiv \delta_\xi Q_{\eta_L} = \frac{1}{8\pi G\ell}\int_0^{2\pi} d\varphi\ \delta_\xi \mathcal{L}(x^+)\ \underbrace{\chi_{\eta_L}(x^+)}_{\text{bnd. restriction of } \eta_L}$$

- if ξ is a L.M. diffeomorphism, the change in the backgnd. value of $\mathcal{L}(x^+)$ under its action is (obtained from $\delta_\xi g_{\mu\nu}$)

$$\delta_{\xi_L}\mathcal{L}(x^+) = 2\mathcal{L}(x^+)\chi'_{\xi_L}(x^+) + \chi_{\xi_L}(x^+)\mathcal{L}'(x^+) - \frac{\ell^2}{2}\chi'''_{\xi_L}(x^+)$$

and thus

$$\{Q_{\eta_L}, Q_{\xi_L}\} = \frac{1}{8\pi G\ell}\int_0^{2\pi} d\varphi\left[2\mathcal{L}\chi'_{\xi_L}(x^+) + \chi_{\xi_L}\mathcal{L}'(x^+) - \frac{\ell^2}{2}\chi'''_{\xi_L}(x^+)\right]\chi_{\eta_L}(x^+)$$

$$= \underbrace{\frac{1}{8\pi G\ell}\int_0^{2\pi} d\varphi\ \mathcal{L}(x^+)\left(\chi'_{\xi_L}(x^+)\chi_{\eta_L}(x^+) - \chi'_{\eta_L}(x^+)\chi_{\xi_L}(x^+)\right)}_{Q_{[\eta_L,\xi_L]_{LB}}} - \underbrace{\frac{\ell}{16\pi G}\int_0^{2\pi} d\varphi\ \chi'''_{\xi_L}(x^+)\chi_{\eta_L}(x^+)}_{\text{central charge } K_{\eta_L,\xi_L}}$$

(check of the repr. thm.)

- if ξ is a R.M. diffeo, then $\delta_{\xi_R}\mathcal{L}(x^+) = 0 \Rightarrow \{Q_{\eta_L}, Q_{\xi_R}\} = 0$
- taking $\chi_{\eta_L} = e^{imx^+}$ $\quad \chi_{\xi_L} = e^{inx^+}$, then

$$\{Q_L^m, Q_L^n\} = -i(m-n)\,Q_L^{m+n} + \frac{i\ell n^3}{16\pi G}\int_0^{2\pi} d\varphi\ \underbrace{e^{i(m+n)x^+}}_{2\pi\delta_{m+n,0}}$$

$$= -i(m-n)\,Q_L^{m+n} - \frac{i\ell}{8G}m^3\delta_{m+n,0}$$

- passing from Dirac brackets to commutators $[\ ,\ ] = i\{\ ,\ \}_{D.B}$

classical $\rightarrow$ semi-classical

and letting $Q_L^m = L_m + \# c\,\delta_{m,0}$, we find

$$\boxed{[L_m, L_n] = (m-n) L_{m+n} + \frac{c}{12} m(m^2-1)\,\delta_{m+n,0}}$$

Virasoro algebra w/

conformal anomaly $\rightarrow$ $\boxed{c = \frac{3\ell}{2G}}$ $\gg 1$

$(\ell_{AdS} \gg \ell_{Planck})$ for the classical gravity approx. to hold

- this is the quantum symmetry algebra of a 2d CFT
- obtained from a purely classical GR calculation

Remarks : in higher dimensional AdS, the ASG = $SO(d,2)$ ≈ isometries of the AdS vacuum solution. This is a rather common coincidence

- for AdS_3, the isometries of the vacuum are $SO(2,2) \simeq SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ which is just a small subgroup (generated by $L_0, L_{\pm 1}$ & $\bar{L}_0, \bar{L}_{\pm 1}$) of the ∞-dim'l ASG.
- thus, in general ASG $\neq$ vacuum isometries. The symmetries of the theory are encoded in the ASG.

- the calculation of the ASG of AdS_3 was first performed in 1986 by Brown & Henneaux, long before AdS/CFT was discovered ('97), or even BTZ black holes ('92).

- in the original paper, two sets of possible bnd. cond. on the asympt. AdS_3 metric were proposed : i) very strict ones $\Rightarrow$ ASG $= \mathbb{R} \times U(1)$ and ii) bnd. cond. allowing for all $SO(2,2)$ vacuum isometries $\Rightarrow \underbrace{Vir \times Vir}_{ASG}$.

- of course, after the discovery of the BTZ black hole soln's & the general understanding of the behaviour of the metric near the AdS bnd (FG exp.) it is clear that the 2nd set of bnd. cond. are the correct ones