Control and operations of tokamaks
Exercise Session 5 - Free boundary evolution
Solutions

Lecturer: F. Felici
Instructors: Cassandre Contré, Reinart Coosemans, Antonia Franck,
Michele Marin, Simon Van Mulders
EPFL - SPC

February 2023

1 Poloidal flux in a tokamak

a,b) Draw a poloidal cross section of a generic tokamak. Sketch the magnetic field generated by the central solenoid.

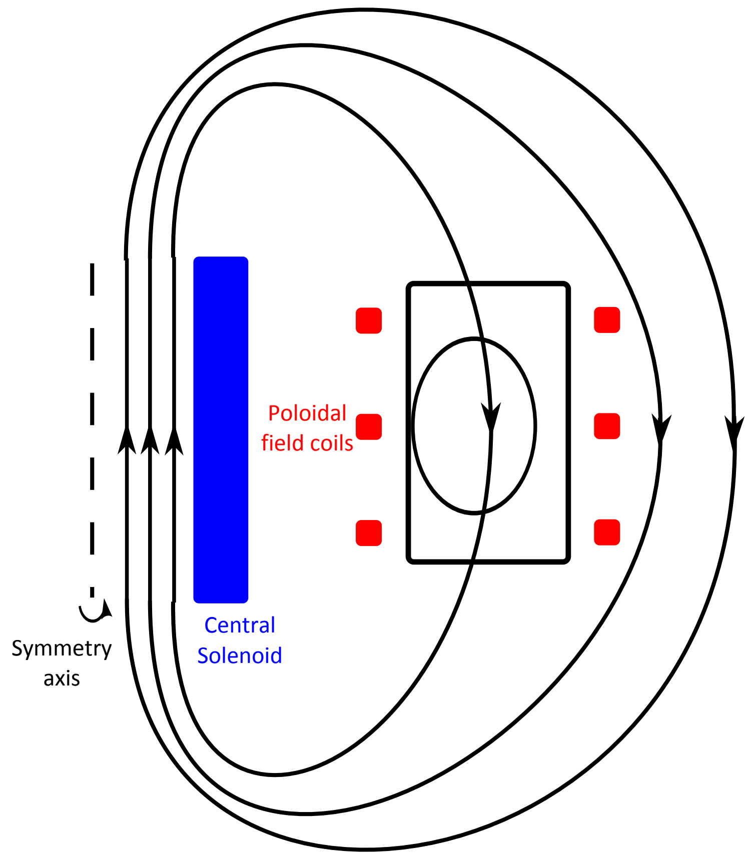

Figure 1: Poloidal cross section of a generic tokamak. Note that the height of the plasma is 1.5 times the width due to $\kappa = 1.5$. The magnetic field is similar to the field produced by a magnetic dipole. Inside the central solenoid the magnetic field is constant. A limited plasma is depicted which touches the wall or limiter.

c) Sketch the magnetic field generated by the solenoid assuming the tokamak has an iron core.

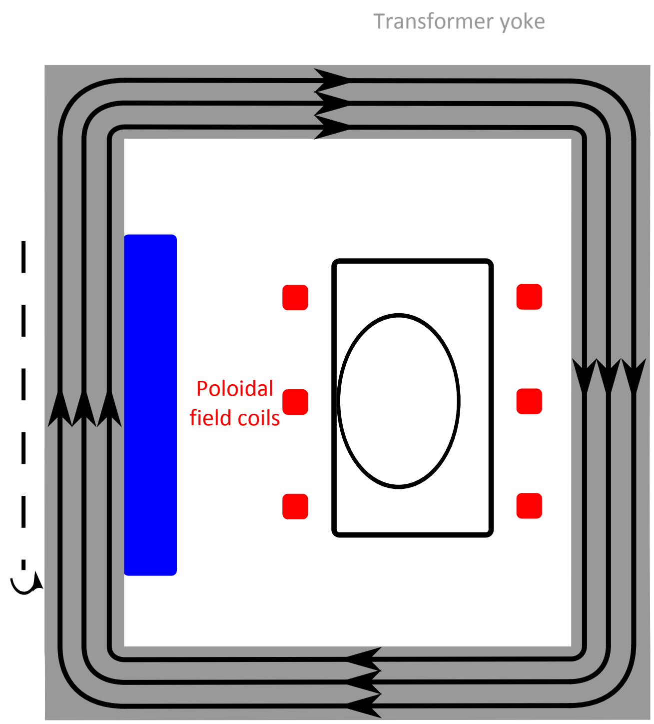

Figure 2: Poloidal cross section of a generic tokamak with a iron transformer yoke. The magnetic field lines are contained within the transformer yoke.

d) Sketch the flux surface distribution inside the vacuum vessel.

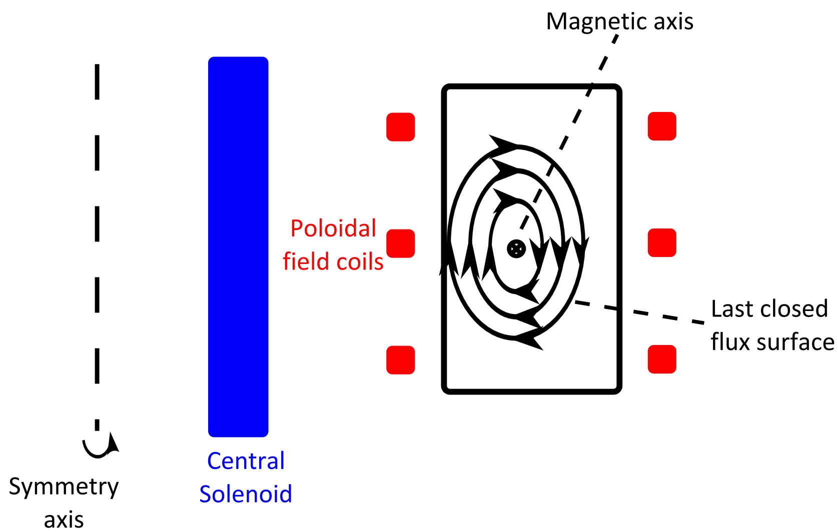

Figure 3: Poloidal cross section of a generic tokamak with flux surfaces. Several flux surfaces are indicated with arrows indicating the direction of the magnetic field lines (projected in the poloidal plane) due to the current in the plasma. The current is directed inside the paper.

e) Sketch the value of the poloidal flux as a function of R. First consider a case without a current in the plasma.

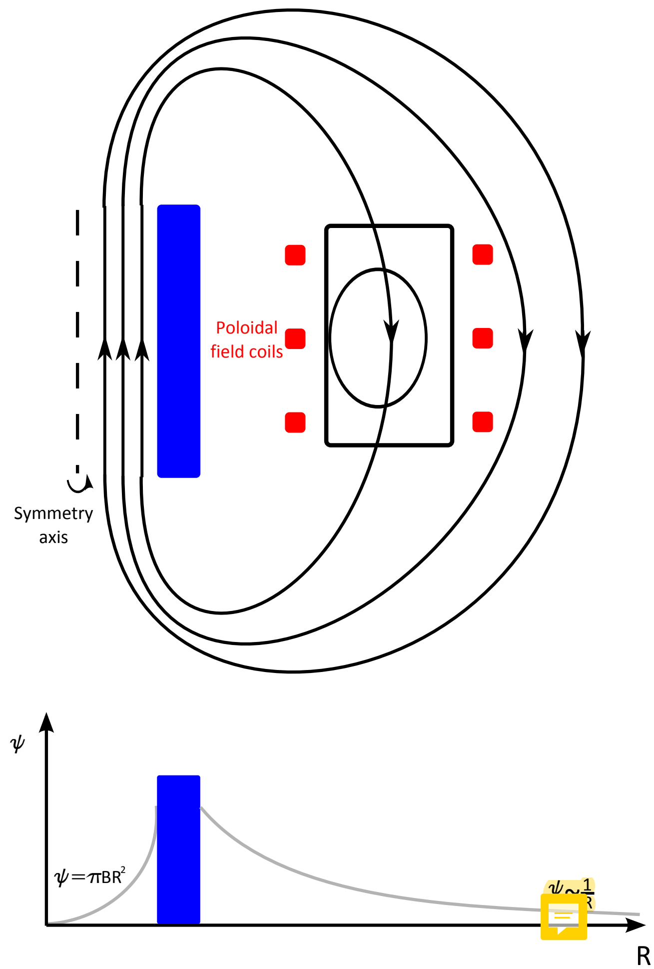

Figure 4: Poloidal cross section of a generic tokamak with the poloidal flux as a function of the major radius R. The magnetic field for small R is constant and, therefore, the poloidal flux increases with R^2. For larger R the magnetic field decays as $\frac{1}{R}$.

f) Sketch the value of the poloidal flux and take into account a positive plasma current.

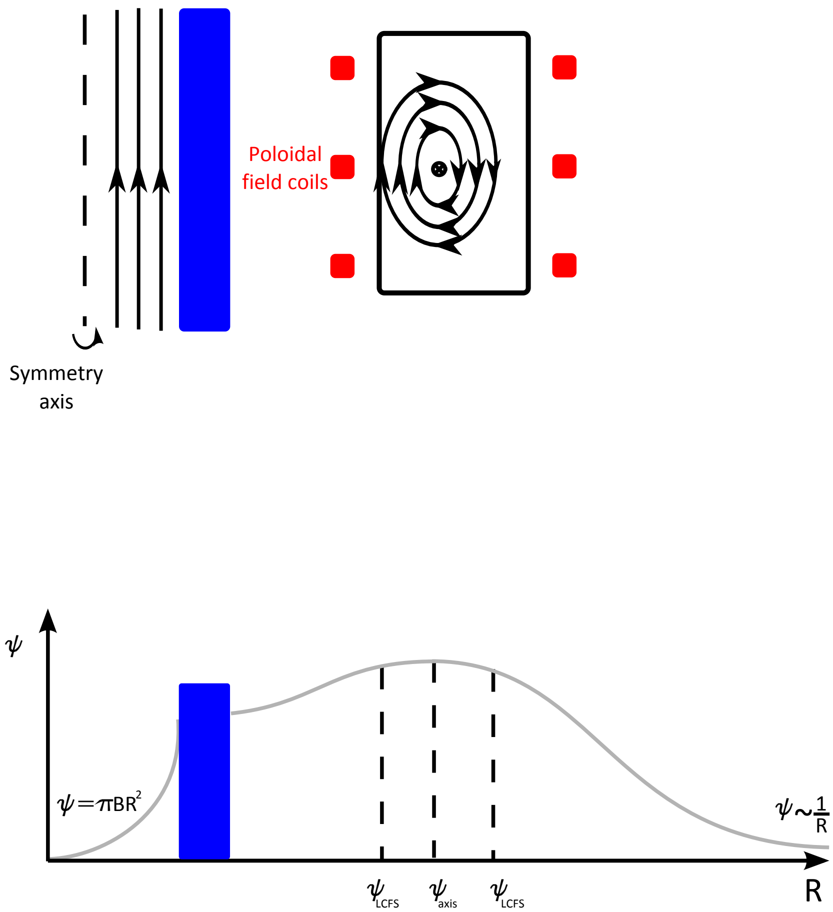

Figure 5: Poloidal cross section of a generic tokamak with the poloidal flux as a function of the major radius R. Due to the magnetic field in positive direction generated by the plasma current, the poloidal flux increases between R and R_0 at the magnetic axis. For R larger than R_0, the magnetic field is in negative direction and the poloidal flux decreases. At very large R the magnetic field also decays as $\frac{1}{R}$.

2 The q profile

a) What value does q have at the last closed flux surface for a plasma with an X-point?

The last closed flux surface contains the X-point and at this point $B_p = 0$. Therefore, if the safety factor is considered in the midplane at radius R, the limit for the last close flux surface is unbounded.

$$q(R_{LCFS}) = \lim_{R \to R_{LCFS}} q(R) \to \infty$$

b) Derive the expression for the 'engineering q' q^*. This is the value of q at the last closed flux surface assuming a large aspect ratio tokamak for which $R \approx R_0$ and assuming the toroidal magnetic field to be external and constant such that $B_\phi = B_0$. Furthermore it is assumed that the poloidal field is constant on the flux surface and determined by the enclosed plasma current.

Assume a large aspect ratio for which $R \approx R_0$ and that the toroidal magnetic field is external and constant to $B_\phi = B_0$, such that $T(\psi) = R_0 B_0$. Furthermore assume that the poloidal field is constant on the flux surface and determined by the enclosed plasma current

$$|B_p| = \frac{\mu_0 I_p}{c}.$$

For the circumference c, note that $b = \kappa a$ and therefore

$$c = 2\pi\sqrt{\frac{a^2 + b^2}{2}} = 2\pi\sqrt{\frac{a^2(1+\kappa^2)}{2}} = 2\pi a\sqrt{\frac{1+\kappa^2}{2}}.$$

Combining all the elements gives

$$q^* = \frac{T}{2\pi} \oint \frac{1}{R^2} \frac{d\ell}{|B_p|} = \frac{R_0 B_0}{2\pi} \frac{1}{R_0^2} \frac{1}{|B_p|} \oint d\ell = \frac{2\pi a^2 B_0}{\mu_0 R_0 I_p} \left(\frac{1+\kappa^2}{2}\right).$$

c) Estimate the maximum plasma current and compare with typical value for ASDEX Upgrade and TCV.

Using the equation

$$I_p = \frac{2\pi a^2 B_0}{\mu_0 R_0 q^*} \left(\frac{1+\kappa^2}{2}\right).$$

the maximum plasma current can be calculated for q^* as is given in the table below.

For ASDEX Upgrade, a typical current of 2 MA is reported which is close to the limited reported in the table above. For TCV, a maximum current of 1.02 MA which also matches the calculated limit. This slightly higher value is likely to be achieved at a smaller value of κ.

3 Full magnetic control simulation using MEQ (optional)

No solution developed yet...

	ASDEX Upgrade	TCV
B_0 [T]	2.5	1.5
a [m]	0.6	0.25
R_0 [m]	1.65	0.88
κ	1.5	2.5
$I_p [MA]$	2.2	0.97