Control and operations of tokamaks
Exercise 1 - Control of PF coils

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1.1 Analysis of PF coil controllers for TCV

Recall the coil circuit equation (neglecting the vacuum vessel)

$$\mathbf{M_{aa}}\dot{\mathbf{I}}_\mathbf{a} + \mathbf{R_a}\mathbf{I_a} = \mathbf{V_a} \tag{1}$$

Assume that there is a modeling error such that the true resistivity matrix is $\mathbf{R_a} = \mathbf{R_{a,0}} + \mathbf{\Delta R_a}$, with $\mathbf{\Delta R_a}$ a diagonal matrix. We define the *nominal* case as the case where $\mathbf{\Delta R_a} = 0$.

a) Compute the MIMO transfer function from $\mathbf{V_a}(\mathbf{s})$ to $\mathbf{I_a}(\mathbf{s})$ including the $\mathbf{\Delta R_a}$ term.

b) How could the poles of the nominal transfer function (assuming $\mathbf{\Delta R_a} = 0$) be calculated for given $\mathbf{M_{aa}}$ and $\mathbf{R_{a,0}}$? Is this transfer function always stable?

c) Does a voltage applied to coil i also affect currents in other coils ($I_j,\ i \neq j$)? Explain your answer.

d) Now, assume (as in the lecture) that the coils are controlled by a controller of the form: $\mathbf{V_a} = \mathbf{M_{aa}}\mathbf{u} + \mathbf{R_{a,0}}\mathbf{I_a}$ with $\mathbf{u} = k_p(\mathbf{I_{ref}} - \mathbf{I_a})$. We showed that the controller had zero steady-state tracking error for step references. Compute the steady-state tracking error for the case including model mismatch. Is it still zero? Explain your results.

e) An alternative PF coil controller formulation has the form:

$$\mathbf{V_a}(s) = (s\mathbf{M_{aa}} + \mathbf{R_{a,0}})\frac{k_p}{s}(\mathbf{I_{ref}}(s) - \mathbf{I_a}(s)) \tag{2}$$

Compute the transfer function from $\mathbf{I_{ref}}$ to $\mathbf{I_a}$ in this case. Analyze the behavior in the limit $t \to \infty$ in the case of model mismatch. What is the advantage of choosing this controller?

f) For both controllers, compute (for the nominal case) the tracking error in response to a unit ramp reference signal (having Laplace transform $\mathbf{I_{ref}}(s) = \frac{1}{s^2}$).

g) In theory, is there an upper limit to the value of k_p to guarantee stability?

h) Give two reasons why we do not want to choose a too high value for k_p in practice.

1.2 Vacuum vessel response

a) You are given a Matlab script `PF_coil_design_given.m` showing how to perform the eigenmode reduction of the vacuum vessel. Use this model to simulate the response to an open-loop step of 100V in the voltage of current F001. Plot the time histories of all the coil currents.

b) Plot the time evolution of the contribution of each eigenmode.

c) Plot the time evolution of the poloidal flux, the B_r and B_z field at the coordinates $(R, Z) = (0.8, 0)$. Explain what you see.

d) Plot the time evolution of the (induced) voltage at these same coordinates and explain.

e) Plot a contour map of the poloidal flux at $t = 0.01$s, 0.1s and 1s, comment what you see.

f) Close the PF coil controller loop with the second controller proposed in Eq.(2). Compute the closed-loop transfer function from the reference to the coil currents and from the reference to the input control voltages[1]. Plot the step responses $I_{ref} \rightarrow V_a$ and $I_{ref} \rightarrow I_a$ for a 100A current request on coil F001, for the case $k_p = 100$.

g) Analyze the effect of increasing k_p on the speed of the step response and on the voltage command signal. Compute the current and voltage response in time to a 1kA step request on coil F001 for increasing values of k_p and plot it for coils F001, F002 and E008. What is the maximum value of k_p that ensures that all the command voltages stay below 500V?

h) Introduce now a delay of $\tau = 0.5\ ms$ in the inputs of the system[2]. Plot in the same figure the $I_{ref} \rightarrow I_a$ step response of 1kA for coil F001 selecting $k_p = 500$, with and without the delay. Comment and show the different responses of the system for increasing values of k_p and find what value of k_p drives the system unstable.

i) *Optional:* Repeat points f) and g), assuming a 10% modeling error in the resistivity such that $\mathbf{R_{a,0}} = 0.9\mathbf{R_a}$. Compare the response of this case with the one obtained in the absence of the model mismatch.

[1] Hint: draw the block diagrams corresponding to the interconnection calculated by the matlab command `feedback(K * P, eye(size(K)))` and `feedback(K, P)`.

[2] Hint: set in matlab the property `InputDelay` of the transfer function object.