Control and operation of tokamaks
Exercise 4 - FBTE & LIUQE

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February 2023

Prerequisites

This exercise comes with a `Matlab` skeleton `exercise04_skeleton.m` to start with and three additional scripts, related to the different tasks. In case you do not have access to LAC you will have to download the `tcv_data_ex4.zip` and `meq_code.zip` from the Moodle on a PC at CO5/CO6. Those will provide you with the TCV data and the MEQ toolbox, respectively.

Exercise 1 - Design of the vacuum equilibrium field

From the integral Shafranov equilibrium we know that we must impose a vertical field given by

$$B_{z0} = -\frac{\mu_0 I_p}{4\pi r_0}\left(\ln\frac{8r_0}{a\sqrt{\kappa}} + \beta_p + \frac{\ell_i}{2} - \frac{3}{2}\right), B_{r0} = 0, \tag{1}$$

and a field index

$$n \equiv -\frac{r_0}{B_{z0}}\frac{\partial B_z}{\partial r} = -1.44. \tag{2}$$

So let us choose currents in the shaping poloidal field coils (PFC) E and F of TCV that would satisfy those requirements for the following plasma parameters:

$$I_p = -400\text{kA}, r_0 = 0.87 \text{ m}, z_0 = 0, a = 0.25 \text{ m}, \beta_p = 0.3, \ell_i = 1, \kappa = 1.6$$

The magnetic field and its derivative at the point $(r0, z0)$ can be calculated with the appropriate Green's functions

$$
\begin{aligned}
B_{r0} &= \sum_a B_{ra} I_a \\
B_{z0} &= \sum_a B_{za} I_a \\
\frac{\partial B_{z0}}{\partial r} &= \sum_a C_a I_a.
\end{aligned}
$$

The field index condition can be replaced by a condition on

$$\frac{\partial B_{z0}}{\partial r} = -\frac{n B_{z0}}{r_0}. \tag{3}$$

A first option is to cast the requirements in a cost function that we will minimise

$$\chi^2 = (B_{za} \cdot I_a - B_{z0})^2 + (B_{ra} \cdot I_a)^2 + \left(C_a \cdot I_a - \left(-\frac{n B_{z0}}{r_0}\right)\right)^2. \tag{4}$$

Since we have 3 constraints but 16 degrees of freedom in the coil currents, we need an additional regularization term in the cost function that will keep the coil currents small.

$$\chi^2 = (B_{za} \cdot I_a - B_{z0})^2 + (B_{ra} \cdot I_a)^2 + \left(C_a \cdot I_a - \left(-\frac{n B_{z0}}{r_0}\right)\right)^2 + w_a \sum_a I_a^2. \tag{5}$$

a) Find the solution of this problem: the coil current combination I_a that minimises this cost function. To perform that task, rewrite the problem of minimizing χ^2 in the form of a least-squares problem $\min_x \|\mathbf{Ax} - \mathbf{b}\|_2^2$ and find the solution using the `Matlab` operator \. Choose an appropriate value for w_a by varying it and studying the obtained solutions. Plot the flux distribution generated by the optimal coil current combination in the poloidal plane.

   To obtain the needed Green's functions the provided `Matlab` function `exgf.m` is used. That function extracts the Green's functions from the `L.G` structure that you obtain running `FBTE`. In order to do so, you have two options.
   If you have access to LAC:

```
[L,LX,LY] = fbt('tcv',102735,[],'debug',0);
```

   Otherwise run from the provided `.mat` files:

```
prefix = 'tcv_102735_fbt';
fileargs = {'pfile',[prefix,'_P'],'gfile',[prefix,'_G'],...
    'xfile',[prefix,'_X']};
[L,LX,LY] = fbt('file',102735,[],'debug',0,fileargs{:});
```

b) A more elegant solution is to minimise the cost function

$$\chi^2 = w_a \sum_a I_a^2 \tag{6}$$

under specified constraints for B_{z0}, B_{r0}, n using the `lsqlin` `Matlab` function. Does the value of w_a influence the solution?

c) Run `FBTE` for a similar equilibrium stored in shot #102735. You can use the same code to run `FBTE` as given in part a) for the Green's functions. You can have a more detailed look at the code's outputs by running `FBTE` in debug mode, using the parameter `'debug'`> 0 in the function call.
The currents in the PFCs is stored in `LY.Ia`. Compare the vacuum field at $(r0, z0)$ and the poloidal flux distribution, generated by the PF coils for the `FTBE` solution to the case obtained in a) and b).

d) For the equilibrium found in c), compare the contribution to poloidal flux distributions from (i) currents in the PF coils (ii) the plasma current distribution (iii) the sum of plasma and PF coil contributions (`LY.Fx` in `FBTE`). Comment on what you see.

Exercise 2 - FBTE

In this exercise we will study the effect of the power dissipation weight on the coil currents obtained from `FBTE`. The power dissipation weight needs to be chosen in such a way that the coil currents are minimized while maintaining the plasma shape within the required tolerances. For shot #102734 choose different power dissipation weights in the cost function `dissi`. A typical value for this parameter on TCV is 10^{-15}. Values above 10^{-12} can lead to unconverged solutions. Observe its effect on the coil current values and on the discrepancy between the desired LCFS and the equilibrium LCFS. Comment on what you see.

You can use the provided `Matlab` function `exfbte.m` that runs `FBTE` for different dissipation weights.

Exercise 3 - LIUQE Reconstruction

In this exercise we will study the effect of the basis functions to parametrize the p' and TT' source terms. In `LIUQE` the parameters `naas` and `nbbs` select how many basis functions are used for p' and TT' respectively. The first 3 basis function are of the form,

$$g_1 = \psi_N - 1 \quad g_2 = (\psi_N - 1)\psi_N \quad g_3 = (\psi_N - 1)\psi_N(2\psi_N - 1)$$

and if the parameter is set to 0 a null basis function is used.

You can use the provided `Matlab` function `exliuqe.m` to obtain data for the given equilibrium. It calls `LIUQE` by the command `[L,LX,LY] = liu(source,shot,time,'par',value)`. By varying the parameter `'bfp',[naas,nbbs]` the number of basis functions is selected.

a) Investigate the effect of various combinations of basis function numbers (`naas` and `nbbs`) and observe their effect on the q profile, the p profile and the T profile and LCFS for a diverted plasma equilibrium shot #40000 at $t = 0.3\,\mathrm{s}$. Comment on the relevance of the different combinations.

b) Repeat the above steps for a limited plasma equilibrium shot #40000 at $t = 0.1\,\mathrm{s}$.