Control and operation of tokamaks
Exercise 2 - Plasma current control and plasma position determination

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Ex 2.1: Plasma current control

Assume a simple model describing the Ohmic coils, plasma current, and current in the first (slowest) eigenmode of the vacuum vessel current distribution

$$L_{OH}\dot{I}_{OH} + M_{p,OH}\dot{I}_p + M_{p,OH}\dot{I}_e + R_{OH}I_{OH} = V_{OH} \tag{1}$$

$$M_{e,OH}\dot{I}_{OH} + M_{e,p}\dot{I}_p + L_e\dot{I}_e + R_eI_e = 0 \tag{2}$$

$$M_{p,OH}\dot{I}_{OH} + M_{p,p}\dot{I}_p + M_{p,e}\dot{I}_e + R_pI_p = 0 \tag{3}$$

a) Describe the spatial distribution of current in the slowest vacuum vessel eigenmode.

b) Calculate the loop voltage required to drive a plasma current of 300kA assuming $R = 3 \times 10^{-6}\Omega$.

c) Calculate the current ramp rate in the OH coils required to sustain this plasma current if $M_{p,OH} = 0.8748 \times 10^{-4}$.

d) Assume the OH coils have a current range of $\pm$20kA. Compute the maximum flux swing in Wb if $M_{p,OH} = 0.8748 \times 10^{-4}$.

e) For how long can this plasma current be sustained at most?

Ex 2.2: Plasma position determination

In this exercise you will construct a linear estimator for the plasma current, vertical and radial position based on measurements.

In the file `Plasma_position_determination.m`, a current distribution `Ix` for a TCV equilibrium is given on the plasma grid `x` (with grid coordinates `rx,zx`) with a given total current radial and vertical position. The associated set of poloidal field coil currents `Ia` is also given.

The measurements of magnetic probes and flux loops corresponding to this current distribution are calculated and time-dependent noise with standard deviation `eBm,eFf` is added to these measurements (`Bm,Ff`). The coil currents `Ia` are assumed to have no measurement noise.

Also, it is shown how to define a much smaller plasma grid `h` and how to compute a matrix `Txh` such that `Ix=Txh*Ih` allows to convert current distributions on one grid to another.

a) Write a weighted least-squares problem to determine the current distribution `Ix` on the full `x` grid from the measurements of magnetic probes and flux loops, in the form $\min ||AI_x - b||_2$. Does this least-squares problem have a unique solution?

b) Write a least-squares problem to determine the current distribution `Ih` on the reduced grid. What is A and b? Does this problem have a unique solution?

c) Write the solution of least-squares problem in (b) in the form `Ih=Ahy*y` where `y` is the vector of measurements `[Bm;Ff;Ia]`. Calculate `Ahy` and plot the current distribution `Ix` obtained for the first time slice of the noisy measurement on the (r, z) grid.

d) Recall that the plasma current can be determined from the current distribution as $I_p = \sum_{j=1}^{n_x} I_{x,j}$. Write a linear estimator for the plasma current from the measurements such that $I_p = H_{I_p} \cdot y$

e) We can define an estimator for the plasma radial and vertical position, by assuming the position corresponds to the centroid of the current distribution:

$$\hat{r} = \frac{\sum_j^{n_x} r_{x,j} I_{x,j}}{\sum_i^{n_x} I_{x,i}} \tag{4}$$

$$\hat{z} = \frac{\sum_j^{n_x} z_{x,j} I_{x,j}}{\sum_i^{n_x} I_{x,i}} \tag{5}$$

Use these expressions to compute an estimate of r and z based on the measurements of magnetic probes and flux loops. Is this a linear estimator?

f) Write a estimator for the plasma rI_p and zI_p i.e. the product of radial/vertical position and plasma current. Is this linear?

g) Plot the outputs of the linear estimators and compare to the known values of $r_0 I_{p0}, z_0 I_{p0}, I_{p0}$. Attempt to improve the estimate of $z_0 I_{p0}$ and I_{p0} by changing the number of degrees of freedom in `Ih` and explain the results.

h) (Optional) Plot the mean and standard deviation of I_p, zI_p, rI_p estimates for various values of nr, nz (and $nh = nz * nr$). Does the estimator quality increase, decrease or stay the same when increasing nh? Explain why.

i) (Optional) Now assume that the circuit current measurements `Ia_meas` is also affected by measurement noise of standard deviation `eIa=100[A]`. Check the quality of your estimator and compare it to the non-noisy case. Reformulate the least squares problem to jointly estimate the plasma current distribution and circuit currents from the measurements and compare the estimation results.