Normalization of states (massive case)

- $|p,\sigma\rangle \equiv U(\Lambda_p)|\bar{p},\sigma\rangle \qquad U(\Lambda_p) \equiv H(p)$

- $P^\mu|p,\sigma\rangle = p^\mu|p,\sigma\rangle \implies \langle p',\sigma'|p,\sigma\rangle = A_{\sigma'\sigma}(p)\,\delta^3(p-p')$

- $p = \Lambda_p \bar{p} \qquad p' = \Lambda_p K \qquad K \neq \bar{p}$ in general $\qquad K \equiv \Lambda_p^{-1} p'$

$$\implies \begin{cases} U(\Lambda_p)|\bar{p},\sigma\rangle \equiv |p,\sigma\rangle \\ U(\Lambda_p)|K,\sigma'\rangle = |p',\sigma''\rangle\, D(W(\Lambda_p,K))_{\sigma''\sigma'} \end{cases}$$

$$\langle K,\sigma'|\bar{p},\sigma\rangle = \langle K,\sigma'|U(\Lambda_p)^\dagger U(\Lambda_p)|\bar{p},\sigma\rangle = \langle p',\sigma''|p,\sigma\rangle\, D^*(W(\Lambda_p,K))_{\sigma''\sigma'}$$

$$\langle k\sigma' | U(\Lambda_p)^\dagger U(\Lambda_p) | \bar{p}, \sigma\rangle = \underline{\langle k\sigma' | \bar{p}\,\sigma\rangle}$$

$$\boxed{= D(W(\ldots))^*_{\sigma''\sigma'} \langle p', \sigma'' | p, \sigma\rangle = \langle k\,\sigma' | \bar{p}\,\sigma\rangle}$$

$$D(W(\Lambda_p, \bar{p}))_{\sigma''\sigma'} \langle k\sigma' | \bar{p}\,\sigma\rangle = \boxed{\langle p'\sigma'' | p\,\sigma\rangle}$$

$$U(\Lambda_p) | \bar{p}, \sigma\rangle = | \Lambda p, \sigma'\rangle D_{\sigma'\sigma}(W(\Lambda_p, \bar{p}))$$

$$\equiv | \Lambda p, \sigma\rangle \longrightarrow$$

$$D_{\sigma'\sigma}(W(\Lambda_p, \bar{p})) = \delta_{\sigma'\sigma}$$

$$\langle p', \sigma' | p, \sigma \rangle = \boxed{\langle k, \sigma' | \bar{p}, \sigma \rangle}$$

$$k = \Lambda_p^{-1} p'$$

$$\langle k, \sigma' | \bar{p}, \sigma \rangle = A_{\sigma'\sigma}(\bar{p}) \, \delta^3(\vec{k})$$

$$= c \, \delta_{\sigma'\sigma} \, \delta^3(\vec{k})$$

$$= c \, \delta_{\sigma'\sigma} \, \delta^3((\Lambda_p^{-1}(p' - p))$$

$$= c \, \delta_{\sigma'\sigma} \cdot \frac{E}{\omega} \, \delta^3(p - p')$$

$$A_{\sigma'\sigma} = \langle \sigma' | \sigma \rangle = \langle \sigma' | U^{\dagger}(R)\, U(R) | \sigma \rangle$$

$$= D^{*}_{\sigma'''\sigma'} \langle \sigma''' | \sigma'' \rangle D_{\sigma''\sigma}(R)$$

$$= \left(D^{\dagger} \; A \; D \right)_{\sigma'\sigma}$$

$$\left[D(R)\, A \right]_{\sigma'\sigma} = \left[A\, D(R) \right]_{\sigma'\sigma}$$

$$A \propto \mathbb{1}$$

$\underline{\underline{EX}}$ $\langle i\, p' | j\, p \rangle$

$\varepsilon_{ijk}\, p_k$

$$\Rightarrow \langle p',\sigma' | p,\sigma\rangle = \langle k,\sigma'' | \bar{p},\sigma\rangle \, D(W(\Lambda_p,k))_{\sigma''\sigma'}$$

$$= A_{\sigma''\sigma}(0)\,\delta^3(\vec{k})\,\underbrace{D(W(\underline{\Lambda_p},\bar{p}))_{\sigma''\sigma'}}_{\delta_{\sigma''\sigma'}}$$

$$U(\Lambda_p)|\bar{p},\sigma\rangle = \underline{|p,\sigma'\rangle}\,\underline{D(W(\Lambda_p,\bar{p}))_{\sigma'\sigma}}$$

$$= A_{\sigma'\sigma}(0)\,\delta^3(\vec{k})$$

$$= c\,\delta_{\sigma'\sigma}\,\delta^3(k)$$

$$= c\,\delta_{\sigma'\sigma}\,\delta^3((\Lambda_p^{-1}p')^i)$$

$$= c\,\delta_{\sigma'\sigma}\cdot\frac{E}{\omega}\,\delta^3(p-p')$$

Standard convention: $C = (2\pi)^3 \, 2m$

$$\Rightarrow \boxed{\langle p', \sigma' | p, \sigma \rangle = 2E_p \, (2\pi)^3 \, \delta_{\sigma'\sigma} \, \delta^3(p' - p)}$$

Need Jacobian for $(\Lambda_p^{-1} p')$

$\Lambda_p = e^{i\eta k} R$

$\Lambda_p^{-1} = R^{-1} \underline{e^{-i\eta k}}$

$p' = p'^0, \; p'^\perp, \; p'^L$

$\gamma = \frac{p_0}{m} \equiv \frac{E}{m}$

$(\Lambda_p^{-1} p')^\perp = p'^\perp \longrightarrow \delta^2(p'^\perp)$

$E_{p'} \equiv p_0'$

$(\Lambda_p^{-1} p')^L = \gamma p'_L - \gamma\beta p'^0 = \underbrace{\frac{E_p p'_L - p E_{p'}}{m}}$

$0 \Longrightarrow 0 \qquad p'_L = p_L$

$$\Longrightarrow \quad \frac{\delta(p'_L - p_L)}{\left| \frac{\partial (\Lambda^{-1} p')^L}{\partial p'_L} \right|}$$

$$\left| \frac{\partial (\Lambda^{-1} p')^i}{\partial p'_i} \right| = \frac{m}{E} \equiv \frac{m}{p_0} \equiv \frac{m}{p'_0}$$

Synthesizing

$$U(\Lambda)\,|p,\sigma\rangle = |\Lambda p,\sigma'\rangle\, D_{\sigma'\sigma}(\Lambda,p)$$

$$|\Lambda p,\sigma'\rangle = U(\Lambda)\,|p,\sigma\rangle\, D^{*}_{\sigma\sigma'}(\Lambda,p)$$

$$|p',\sigma'\rangle = |\Lambda_P k,\sigma'\rangle = U(\Lambda_P)\,|k,\sigma''\rangle\, D^{*}_{\sigma''\sigma'}(\Lambda_P,k)$$

$$|p,\sigma\rangle = U(\Lambda_p)\,|\bar{p},\sigma\rangle$$

$$\langle p',\sigma'|p,\sigma\rangle = \langle k,\sigma''|\bar{p},\sigma\rangle\, D_{\sigma''\sigma'}(\Lambda_p,k)$$

$$= \delta^3(k)\, A\, \delta_{\sigma''\sigma}\, D_{\sigma''\sigma'}(\Lambda_p,k)$$

$$= \delta^3(k)\, A\, \underbrace{D_{\sigma\sigma'}(\Lambda_p,\underline{0})}_{\delta_{\sigma\sigma'} \text{ by construction}}$$

$$= \delta^3(k)\, A\, \delta_{\sigma\sigma'}$$

$$= \delta^3(\underbrace{\Lambda_p^{-1} p'}_{\text{space}})\, A\, \delta_{\sigma\sigma'}$$

$$(\Lambda_p^{-1})(p') = p'_\perp + \vec{n}_L \frac{E p'_L - p E'}{m}$$

$$\delta^3(\Lambda^{-1} p') = \delta^2(p'_\perp)\, \delta\left(\frac{E p'_L - p E}{m}\right)$$

check

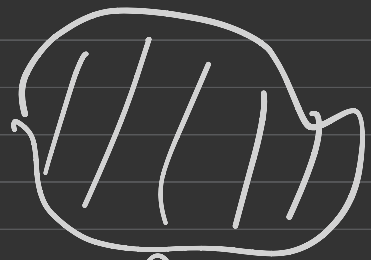
$$\equiv \frac{E}{m} \delta^2(p'_\perp)(p'_L - p)$$

$$\equiv \frac{E}{m} \delta^3(\underline{p}' - \underline{p})$$

Locality, causality, cluster property : Fields

Simple qualitative remarks

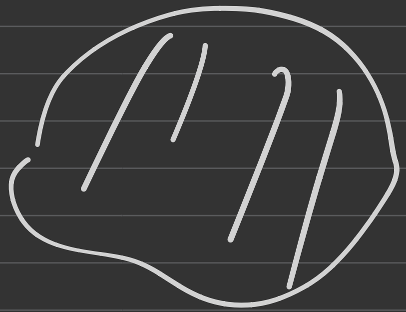
- empty space can exist
- local "excitations of empty space" exist



A

- local excitations can be expected elsewhere

A



A'

- if A and A' sufficiently seperated

$\Rightarrow$ no relative influence

- causality is satisfied

"Axioms"

I) Hilbert space contains one vacuum state $|0\rangle$
"singlet of $ISO(3,1)$"

II) $\exists$ local observable fields $O_\alpha(x)$ (operator)

- $e^{iPa} O_\alpha(x) e^{-iPa} = O_\alpha(x+a)$
- $O_\alpha(x)$ create local excitations

III $\langle 0 | \mathcal{O}_{\alpha_1}(x_1) \cdots \mathcal{O}_{\alpha_n}(x_n)\, \mathcal{O}_{\beta_1}(y_1+a) \cdots \mathcal{O}_{\beta_m}(y_m+a) | 0 \rangle$

limit

$a \to$ spacelike ∞

$$\xrightarrow{a\to\infty} \langle 0 | \mathcal{O}_{\alpha_1} \cdots \mathcal{O}_{\alpha_n} \rangle \cdot \langle 0 | \mathcal{O}_{\beta_1} \cdots \mathcal{O}_{\beta_n} \rangle$$

IV $[\mathcal{O}_\alpha(x), \mathcal{O}_\beta(y)] = 0 \qquad x \times y$

◎ Fields can be organized into irreps of $SO(3,1)$

- $\mathcal{O}_{\alpha, A} \equiv (j_-, j_+) \qquad A = 1, \dots, (2j_-+1)(2j_++1)$

- $U^\dagger(\Lambda)\, \mathcal{O}_{\alpha, A}(x)\, U(\Lambda) = D^{j_-, j_+}_{AB}(\Lambda)\, \mathcal{O}_{\alpha, B}(\Lambda^{-1}x)$

- $U^\dagger(\Lambda)\, \mathcal{O}_{\alpha, A}(0)\, U(\Lambda) = D^{j_- j_+}_{AB}\, \mathcal{O}_{\alpha, B}(0)$

- $\vec{J} = \vec{J}_+ + \vec{J}_- \implies \mathcal{O}(0) = \bigoplus_{j = |j_+ - j_-|}^{j = j_+ + j_-} \tilde{\mathcal{O}}_j$

$$\boxed{\vec{J}_\pm = \vec{J} \pm i \vec{K}}$$

Clebsch-Gordan

$$\mathcal{O}_A(0) = \sum_{j, m} T^j_{Am}\, \tilde{\mathcal{O}}_{jm}(0)$$

$$U(R)^\dagger O_A U(R) = D_{AB}(R)\, O_B$$

$$= \sum_{j,m} T_A^{j}\, U(R)^\dagger \tilde{O}_{j,m} U(R) = \sum_{j,m} T^{j}_{Am}\, \tilde{O}_{jm'}\, D^{j}_{mm'}(R)$$

$$\Rightarrow O_A = \sum_{j,m} D^{-1}_{AB}(R)\, T^{j}_{Bm}\, D^{j}_{mm'}(R)\, \tilde{O}_{jm'}$$

$$\Rightarrow \boxed{T^{j}_{Am} = D^{-1}_{AB}(R)\, T^{j}_{Bm'}\, D^{j}_{m'm}(R)}$$

$T^{j}_{Am} \equiv SO(3)$ invariant tensor

Relativistic wave functions

$$|p, \sigma\rangle$$

$$(M, s)$$

$$\mathcal{O}_A(x)$$

Lorentz index

spacetime coordinates

$$\langle 0| \mathcal{O}_A(x) | p, \sigma\rangle \equiv \langle A, x | p, \sigma\rangle = \psi_{A\sigma}(x, p)$$

$$\bullet\ \langle 0|\mathcal{O}_A(x)|p,\sigma\rangle = \langle 0|e^{iPx}\,\mathcal{O}_A(0)\,e^{-iPx}|p,\sigma\rangle = \langle 0|\mathcal{O}_A(0)|p,\sigma\rangle\, e^{-ipx}$$

$$H(p) \equiv U(\Lambda_p)$$

$$= \langle 0|H(p)\,H(p)^{-1}\mathcal{O}_A(0)\,H(p)|\bar{p},\sigma\rangle\, e^{-ipx}$$

$$= D_{AB}(\Lambda_p)\,\langle 0|\mathcal{O}_B(0)|\bar{p},\sigma\rangle\, e^{-ipx} \equiv \psi_{A\sigma}(p)\, e^{-ipx}$$

$$\bullet\ C_{A\sigma}^{j_+,j_-,s} \equiv \langle 0|\mathcal{O}_A(0)|\bar{p},\sigma\rangle = \sum_{j,m} T_{Am}^{j}\,\underbrace{\langle 0|\tilde{\mathcal{O}}_{jm}(0)\,|\bar{p},\sigma\rangle}$$

$$\langle j,m|s,\sigma\rangle = \delta_{js}\,\delta_{m\sigma}\,\sqrt{Z}$$

$$\boxed{C_{A\sigma}^{j_+,j_-,s} = \sqrt{Z}\, T_{A\sigma}^{s}}$$

$$C_{A\sigma} = \langle 0 | O_A | \sigma \rangle$$

$$C_{A\sigma} = \langle 0 | U(R)^{\dagger} U(R) \, O_A \, U(R)^{\dagger} U(R) | \sigma \rangle$$

$$= \langle 0 | \, U(R) \, O_A \, U(R)^{\dagger} \quad U(R) | \sigma \rangle$$

$$= D^{0}_{AB}(R)^{-1} \langle 0 | O_B | \sigma' \rangle \, D^{s}_{\sigma'\sigma}(R)$$

$$C_{A\sigma} = D_{AB}^{-1}(R) \; C_{B\sigma'} \; D^{s}_{\sigma'\sigma}(R)$$

$$D^{\vartheta}_{AB}(R)\, C_{B\sigma} = C_{B\sigma'}\, D^{s}_{\sigma'\sigma}(R)$$

$$\langle j, j_3 | s, \sigma \rangle = A_{j_3 \sigma} = \langle j, j_3 | U(R)^\dagger U(R) | s, \sigma \rangle$$

$$= R^s_{\sigma' \sigma} \, R^{j\,*}_{j_3' j_3} \, \langle j, j_3' | s, \sigma' \rangle$$

$$A = (R^j)^\dagger A \, R^s$$

Schur

$$\begin{cases} \cdot \; j \neq s \implies A = 0 \\ \cdot \; j = s \implies A \propto \mathbb{1}_{j_3, \sigma} \end{cases}$$

$$C^{j_+, j_-, s}_{A\sigma} = \sqrt{Z}\, T^s_{A\sigma} \implies \psi_{A\sigma}(p) = D_{AB}(\Lambda_p)\, T^s_{B\sigma} \cdot \sqrt{Z}$$

$$(2j_+ + 1)(2j_- + 1) \qquad 2s+1$$

- $\delta_{AA'} = \sum_j \Pi^j_{AA'}$

$$\Pi^j_{AA'} T^s_{A'\sigma} = \delta^{js} T^s_{A\sigma}$$

$$\Pi^j \Pi^k = 0 \qquad j \neq k$$

$$(\Pi^j)^2 = \Pi^j$$

$$D_{AB}(R)\, \Pi^j_{BC} = \Pi^j_{AB} D_{BC}(R)$$

$$(\delta_{AA'} - \Pi^s_{AA'})\, T^s_{A'\sigma} = 0$$

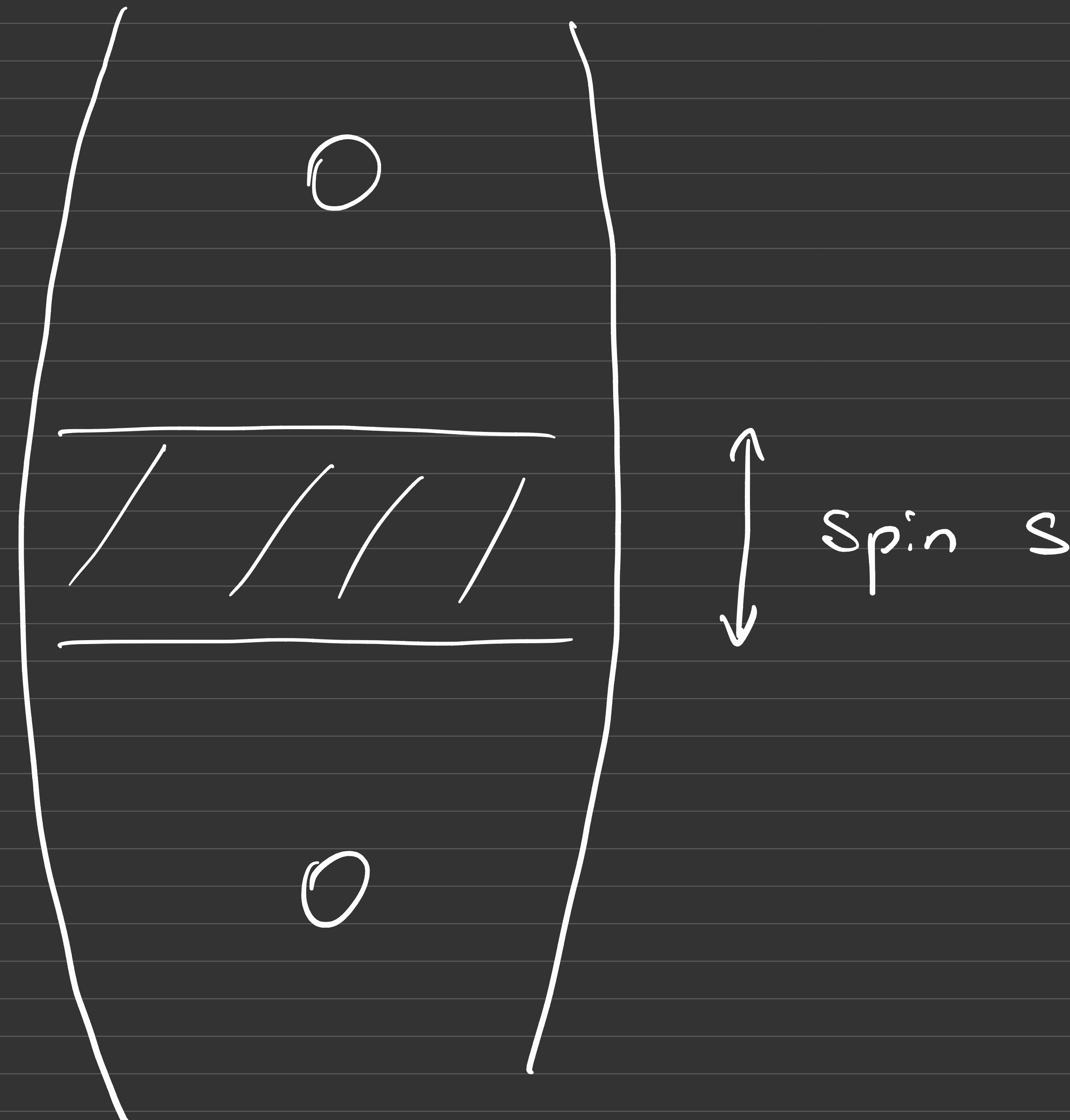
$T_A \longrightarrow$ up to basis choice

$$A = (2j_+ + 1)(2j_- + 1)$$

$$\begin{bmatrix} \left[\,|j_+ - j_-|\,\right] & & & \\ & \left[\,|j_+ - j_- + 1|\,\right] & & \\ & & \vdots & \\ & & & \left[\,|j_+ + j_-|\,\right] \end{bmatrix}$$

invariant subspaces

$$T^{j_- j_+ s}_{A \sigma} =$$



In the above basis can define projectors Π^j on each individual spin-j subspaces. We have

$$\mathbb{1} = \sum_{j=|j_+-j_-|}^{j_++j_-} \Pi^j$$

Obviously these projectors commute with rotations

The same structure can be written in the "A"-index basis

$$\boxed{\delta_{AB} = \sum_j \Pi^j_{AB} \qquad \begin{array}{l} \Pi^J_{AB}\, T^S_{B\sigma} = \delta^{JS}\, T^S_{A\sigma} \\ \Pi^J_{AB}\, \Pi^K_{BC} = 0 \quad \text{if } J \neq K \\ (\Pi^J)^2 = \Pi^j \end{array}}$$

$\ast$

- Π^{S}_{AB} can be constructed explicity by using the parity conjugate wave-function

$$T^{j_+ j_- S}_{\bar{A}\sigma} = \mathcal{P}_{\bar{A}B}\, T^{j_- j_+ S}_{B\sigma}$$

in short $$T^{R\ S}_{\bar{A}\sigma} = \mathcal{P}_{\bar{A}B}\, T^{L\ S}_{B\ \sigma}$$

- $\mathcal{P} \equiv$ parity matrix $\qquad \mathcal{P}\vec{J}_{\pm} = \vec{J}_{\mp}\mathcal{P}$

$$\Longrightarrow \qquad [\mathcal{P}, \vec{J}\,] = 0$$

- Up to normalization

$$\Pi^{S}_{AB} = \sum_{\sigma} T^{LS}_{A\sigma} \left(T^{RS}_{\bar{B}\sigma}\right)^*$$

△ Relativistic wave eq. $\equiv$ covariant trasposition of (*) constraints

- $\Pi^{j}_{AB}\, C^{j\text{-}j+s}_{B\sigma} = 0$ for $j \neq s$ or equivalently
- $\left(\delta_{AB} - \Pi^{s}_{AB}\right) C^{j\text{-}j+s}_{B\sigma} = 0$

$$\circledcirc \quad (\mathbb{1} - \Pi^{s})_{AA'} \, C^{j_+, j_-, s}_{A'\sigma} = 0$$

$$\left[D(\Lambda_p) (\mathbb{1} - \Pi^{s}) D^{-1}(\Lambda_p) \right]_{AA'} \left[D(\Lambda_p) C^{j_- j_+ s} \right]_{A'\sigma} = 0$$

$$\boxed{\Pi^{\perp}_{AA'}(p) \, \psi^{j_+, j_-, s}_{A'\sigma}(p) = 0}$$

- We can also prove covariance $\equiv$ $\boxed{D(\Lambda) \, \Pi^{\perp}(p) \, D^{-1}(\Lambda) = \Pi^{\perp}(\Lambda p)}$

$$D(\Lambda) \, D(\Lambda_p) \quad \Pi^{\perp} \, D(\Lambda_p)^{-1} \, D(\Lambda)^{-1} =$$

$$= D(\Lambda_{\Lambda p}) \left[D^{-1}(\Lambda_{\Lambda p}) \, D(\Lambda) \, D(\Lambda_p) \right] \Pi^{\perp} \left[D(\Lambda_p^{-1}) \, D(\Lambda^{-1}) \, D(\Lambda_{\Lambda p}) \right] D(\Lambda_{\Lambda p})^{-1}$$

$$= D(\Lambda_{\Lambda p}) \quad D(W(\Lambda,p)) \, \pi^{\perp} \, D(W^{-1}(\Lambda,p)) \; D(\Lambda_{\Lambda p})^{-1}$$

$$D(W)\,(\mathbb{1} - \pi^{S})\, D(W^{-1}) = \mathbb{1} - \pi^{S}$$

$$= D(\Lambda_{\Lambda p})\,(\pi^{\perp})\, D^{-1}(\Lambda_{\Lambda p}) \equiv \pi^{\perp}(\Lambda p)$$

$$\dashv \begin{cases} \Pi^{\perp}{}_{A}{}^{B}(p)\,\psi_{B\sigma}(p) = 0 \\[2ex] (p^2 - m^2)\,\psi_{B\sigma}(p) = 0 \end{cases}$$

covariant wave eq.

Ex

- $s = 1 \qquad p^2 = m^2$
- $j_+, j_- = \left(\frac{1}{2}, \frac{1}{2}\right) \Rightarrow A_\mu$

$$\Bigg\} \begin{cases} \partial^\mu A_\mu = 0 \\ (\Box - m^2) A_\mu = 0 \end{cases}$$

Examples

- $S=0$

$$\langle 0|\, \phi(0)\, |\bar{p}\rangle = C \qquad \bar{p}^2 = M^2$$

$(0,0)$

$$\psi(p) \equiv \langle 0|\, \phi(0)\, |p\rangle = C$$

- $S=1$

$\left(\frac{1}{2},\frac{1}{2}\right) \longrightarrow 0 \oplus 1$

$$\langle 0|\, A_\mu(0)\, |\bar{p},\sigma\rangle = C_{\mu\sigma} \qquad \sigma = \pm, 0$$

$s=1$

$\mu = 0$ $\quad \langle 0|\, A_0\, |\bar{p},\sigma\rangle = C_{0\sigma} = 0$

$\mu = i$ $\quad \langle 0|\, A_i\, |\bar{p},\sigma\rangle = C_{i\sigma} = \sqrt{Z}\, \varepsilon_{i\sigma}$

$$\varepsilon_{i+} = (1, i, 0)\frac{1}{\sqrt{2}}$$

$$(i,-) = (-1, +i, 0)\frac{1}{\sqrt{2}}$$

$$\varepsilon_{i,0} = (0,0,1)$$

rest frame

$$\psi_{\mu\sigma}(\bar{p}) \begin{cases} \psi_{0\sigma}(\bar{p}) = 0 \\ \psi_{i\sigma}(\bar{p}) = \sqrt{Z}\,\varepsilon_{i\sigma} \end{cases}$$

$$\bar{p}^{\mu}\psi_{\mu\sigma}(\bar{p}) = 0$$

$$\psi_{\mu\sigma}(p) \equiv \langle 0|A_\mu|p,\sigma\rangle$$

$$\left\{ \; p^{\mu}\psi_{\mu\sigma}(p) = 0 \right.$$

$$\left\{ (p^2 - M^2)\, t_{\mu\sigma}(p) = 0 \right.$$

$$\left\{ \begin{array}{l} \overline{\partial^\mu A_\mu = 0} \\ \underline{(\Box + M^2)\, A_\mu = 0} \end{array} \right.$$

$$\langle 0 | A_\mu(0) | \bar{P} \rangle = C_\mu$$

$\mu = 0$ $$\langle 0 | A_0 | \bar{P} \rangle = \sqrt{z}$$

$\mu = i$ $$\langle 0 | A_i | \bar{P} \rangle = 0$$

$$C_\mu \propto \bar{P}_\mu$$

$$A_\mu \propto \partial_\mu \varphi$$

$$\epsilon_{\mu\nu\rho\sigma}\, \bar{P}^{\rho} C^{\sigma} = 0$$

$$\bar{P}^{\rho} C^{\sigma} - \bar{P}^{\sigma} C^{\rho} = 0$$

$$P^{\rho}\psi^{\sigma} - P^{\sigma}\psi^{\rho} = 0$$

$$\boxed{\partial^{\rho} A^{\sigma} - \partial^{\sigma} A^{\rho} = 0} \qquad \boxed{(\Box + \mu^2) A^{\rho} = 0}$$

$$A^{\sigma} = \partial^{\sigma}\varphi$$

- $s=2$ $\qquad h_{\mu\nu} = \underbrace{(1,1)}\oplus\underbrace{(0,0)} \longrightarrow h^{\mu}{}_{\mu}$

$(1,1) \longrightarrow 0 \oplus 1 \oplus 2$

$1 \quad 3 \quad 5$

$$\underline{\langle 0|\, h_{\mu\nu}\, |\bar{p},\sigma\rangle = C_{\mu\nu,\sigma}} \qquad \sigma=-2,\dots,2$$

- Wigner-Eckart $\qquad 00,\ ii,\ 0i \longrightarrow 0 \qquad 0 = C_{00,\sigma} = C_{0i,\sigma} = C_{ii,\sigma}$ sum

$ij - (ii) =$ symmetric traceless $\longrightarrow$ 5 components

$C_{ij,+2} = \varepsilon_{i+}\varepsilon_{j+}$ and so on

- constraints

$00 \longrightarrow \qquad C_{00,\sigma}=0 \qquad \bar{p}^{\mu}\bar{p}^{\nu} C_{\mu\nu,\sigma}=0$

$ii \longrightarrow \qquad C_{ii,\sigma}=0 \qquad \underline{C_{\mu\nu,\sigma}\,\eta^{\mu\nu}=0}$

$0i \longrightarrow \qquad C_{0i,\sigma}=0 \qquad \underline{\bar{p}^{\mu} C_{\mu\nu,\sigma}=0}$

$$\begin{cases} p^{\mu}\, \psi_{\mu\nu,\sigma} = 0 \\ \psi^{\mu}{}_{\mu,\sigma} = 0 \\ (\Box + M^2)\, \psi_{\mu\nu,\sigma} = 0 \end{cases}$$

$$\begin{cases} \partial^{\mu} h_{\mu\nu} = 0 \\ h^{\mu}{}_{\mu} = 0 \\ (\Box + M^2)\, h_{\mu\nu} = 0 \end{cases}$$